PHYS 528 Lecture Notes #10

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1 QCD at High Energies: Partons

We saw that the QCD coupling is large at low energies but grows smaller at high energies. For processes with characteristic momentum much larger than Λ_{QCD}, the QCD gauge coupling is small enough that processes in the theory can computed reliably in perturbation theory. Even so, the low-energy dynamics of QCD that leads to confinement must still be figured in if one wants to compare the predictions of QCD at high energies to experimental data [1].

In this context, QCD confinement enters in two ways. First, the hadronic initial states one collides in the laboratory consist of colour-singlet bound states of quarks and gluons, such as protons or pions. It turns out that the underlying collision can be treated as occurring between quarks or gluons, but we must relate these fundamental *parton* constituents to the colour-singlet composite hadrons from which they come. The second appearance of QCD confinement arises when a quark or gluon is produced in the final state. Confinement implies that we do not observe such quarks or gluons in isolation. Instead, a final-state of quarks or gluons *showers* by emitting soft QCD radiation and *hadronizes* into a collection of baryons and mesons. The net effect is that a final-state quark or gluon is reprocessed into a collimated *jet* of colour-singlet hadrons. In this section we will discuss partons, and in the next we will cover jets.

1.1 Deep Inelastic Scattering and the Parton Model

Much of what we know about the structure of protons (and neutrons) comes from deep inelastic scattering (DIS). In DIS, an energetic beam of leptons ($E \gg \Lambda_{QCD}$) is shot into a thin target of of nucleons. The leptons scatter off the nucleons, and the momentum of the outgoing scattered leptons is measured. A specific example is $e^-p \to e^-X$, where X is an unspecified (and unmeasured) hadronic final state. Hard scattering, with momentum transfer $|q^2| \gg \Lambda_{QCD}$, can be understood in terms of the electron scattering off one of the quarks in the proton through the exchange of a photon, as illustrated in Fig. 1. To relate theory to data we must do two things: i) calculate the electron-quark scattering matrix element; ii) relate this matrix element to the quark content of the proton.

The first step is straightforward and something we know how to do. Consider the process $e^-(k)q_i(p) \to e^-(k')q_i(p')$. Working in the quark-electron CM frame, it is straightforward to show that

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{2\pi\alpha^2 Q_i^2}{\hat{s}^2}\left[\frac{\hat{s}^2 + (\hat{s}+\hat{t})^2}{\hat{t}^2}\right], \tag{1}$$

where Q_i is the electric charge of the quark, and $\hat{s}$ and $\hat{t}$ are the quark-electron system

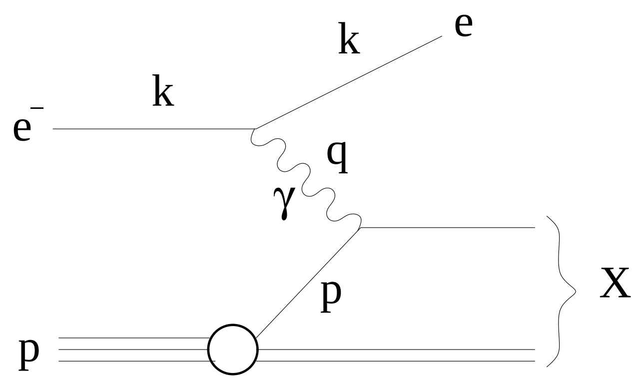

Figure 1: Deep inelastic scattering.

Mandelstam variables:

$$\hat{s} = (k+p)^2, \qquad \hat{t} = (k-k')^2. \tag{2}$$

The net electron momentum transferred in the event is $q = (k' - k)$. Since the Lorentz-invariant q^2 is spacelike, it is standard to define the related positive quantity $Q^2 = -q^2$.

To relate this perturbative quark-level scattering cross section to the underlying electron-proton cross section, as measured in the lab frame where the proton is essentially at rest, we must specify how the likely it is to get the quark species q_i from the proton with the given initial momentum. This is not something that can be done in perturbation theory. Instead, the QCD features of confinement and asymptotic freedom suggest a phenomenological *parton model* that is found to give an excellent descripton of data.

The main features of the parton model are:

- Hadrons consist of quarks (and anti-quarks) and gluons that are collectively called *partons*. They are typically treated as being massless.
- The partons move along with the parent hadron with momentum components transverse to the direction of the parent smaller than the QCD scale: $p_T \lesssim \Lambda_{QCD}$.
- The momentum of the parent hadron is carried collectively by the constituent partons. If P is the momentum of the parent, each constituent parton carries (longitudinal) momentum $p_i = x_i P$ with $0 < x_i < 1$.
- The probability density that a parton of species i carries momentum fraction x is given by the *parton distribution function* (PDF) $f_i(x)$.

With these properties, it follows that the total cross section for a high-energy process involving a hadron N in the initial state $A + N \to B + C$ is:

$$\sigma_{tot} = \sum_i \int dx\; f_i^N(x)\, \hat{\sigma}(A + q_i(xP) \to B + C), \tag{3}$$

where the sum runs over all the partonic constituents of the hadron and $\hat{\sigma}$ is the parton-level cross section. The essential feature of the parton model is that it leads to a factorization of the perturbative hard parton-level matrix element and the non-perturbative dynamics embodied in the PDFs.

The PDFs in the parton model are universal (for a given hadron species). Parton densities are included for u, d, s, c, b, their anti-particles, and the gluon. They satisfy various sum rules such as

$$\int_0^1 dx \, \left[f_u^N(x) - f_{\bar{u}}^N(x)\right] = \begin{cases} 2; & N = p \\ 1; & N = n \end{cases} \tag{4}$$

$$f_u^p(x) = f_{\bar{u}}^{\bar{p}}(x) \tag{5}$$

$$1 = \int_0^1 dx \sum_i f_i(x) x. \tag{6}$$

The first two results reflect the net quark content of the nucleons while the third sum rule corresponds to the partons carrying the momentum of the parent hadron. Some popular sets of PDFs are CTEQ [2] and MRST [3].

Going back to DIS, it is possible to relate the parton momentum fraction x in each event to observables in that event. Treating all the constituents as massless (a good approximation for $E \gg \Lambda_{QCD}$), we have

$$\hat{s} = (k+p)^2 = 2k\cdot p = 2x\, k\cdot P = x(k+P)^2 = x\, s \tag{7}$$

where s is the electron-proton system Mandelstam variable. We also have

$$0 = p'^2 = (p+q)^2 = 2x\, P\cdot q - Q^2 \qquad \Rightarrow \qquad x = \frac{Q^2}{2P\cdot q}. \tag{8}$$

where the first condition comes from the relative masslessness of the outgoing quark.

Applying the parton model to DIS and using the parton-level cross section given in Eq. (1) (with a change of variables), we find

$$\frac{d^2\sigma}{dx dQ^2} = \sum_i f(x)\, Q_i^2 \left\{ \frac{2\pi\alpha^2}{Q^4} \left[1 + \left(1 - \frac{Q^2}{xs} \right) \right] \right\}, \tag{9}$$

where Q_i is the electric charge of the i-th parton (not to be confused with the momentum transfer variable Q^2). Note that all the kinematic dependence is contained within the curly braces, while the first factor (outside the braces) depends on x alone. This result is known as *Bjorken scaling*. It receives controlled corrections at higher orders in QCD. In Fig. 2 we show the predictions of the parton model for DIS (with QCD corrections) along with some experimental data – evidently the theory does very well over many orders of magnitude. While this result is specific to $e^- p \to e^- X$, DIS can also be performed using a neutron target or through the W-mediated process $\bar{\nu}_\ell p \to \ell^- X$.

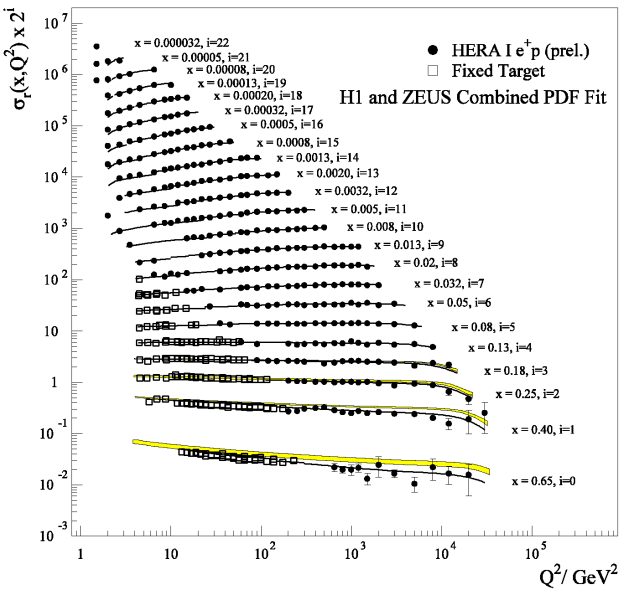

Figure 2: Predictions for deep inelastic scattering compared to data.

1.2 Drell-Yan

An important process at high-energy hadron colliders is the production of leptons through the Drell-Yan process. This is the electroweak production of a lepton pair starting from a hadronic initial state. We show an example of a Drell-Yan process in Fig. 3. The lab frame typically has the colliding pair of colliding hadrons in their centre-of-mass (CM) frame. The remnants of the protons not involved in the hard collision are relatively collinear with the collider beam and are not usually seen.

Following the parton model, the total cross-section for Drell-Yan (at leading order) is

$$\sigma(pp \to \ell^+\ell^-) = \sum_{ij} \int dx_1 \int dx_2 \; f_i^p(x_1) f_j^p(x_2) \, \hat{\sigma}(q_i(p_1)\bar{q}_i(p_2) \to \ell^-\ell^+). \tag{10}$$

Here, we have $p_1 = x_1 P_1$ and $p_2 = x_2 P_2$. In the pp CM frame we have

$$p_1 = x_1(E, 0, 0, E), \qquad p_2 = x_2(E, 0, 0, -E). \tag{11}$$

where we have taken the z axis along the direction of the beam. From this we see that (at very high energies relative to Λ_{QCD}) the parton-level Mandelstam variable $\hat{s}$ is related to the lab-frame Mandelstam variable s via

$$\hat{s} = (p_1 + p_2)^2 = 2x_1 x_2 \, P_1 \cdot P_2 = x_1 x_2 \, s. \tag{12}$$

Note also that even though the collision is taking place in the pp CM frame, this lab frame does not coincide with the CM frame of the colliding partons. Instead, the parton CM frame has a net boost along the beam direction relative to the lab frame corresponding

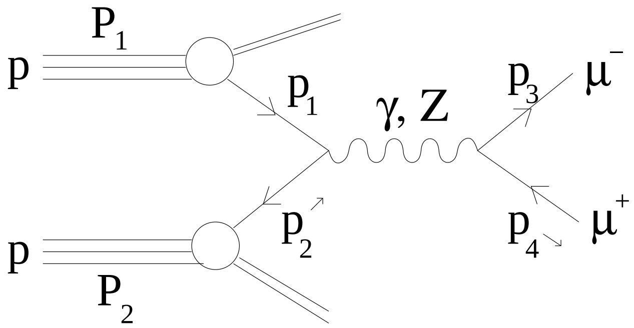

Figure 3: Drell-Yan production of $\mu^+\mu^-$ in a pp collision.

to the longitudinal momentum $(x_1 - x_2)\sqrt{s}/2$. This can make the kinematics of events at colliders more difficult to reconstruct than if the initial states were fundamental (as opposed to composite) particles. In many cases we focus entirely on the *transverse* momentum $\vec{p}_T$ of the particles that are produced, where $\vec{p}_T$ is the component of a particle's momentum orthogonal (or transverse) to the beam direction.

Historically, DIS has been used to determine parton distribution functions. Since the same PDFs apply to other processes such as Drell-Yan, we can use the measured PDFs from DIS to make predictions for Drell-Yan and other cross-sections.

1.3 Parton Evolution

Going beyond the leading order (LO), one encounters an additional complication when dealing with partons. At the next-to-leading order (NLO) and beyond (N^nLO), the PDFs pick up a dependence on a new dimensionful scale μ_F that we usually identify with the typical momentum scale of the underlying hard process, $\mu_F^2 \sim Q^2$. This scale μ_F is called the *factorization scale*, and it corresponds to where we choose to split up the dynamics of process into soft (low-energy and non-perturbative) and hared (high-energy and perturbative) pieces.

To see how this works, consider the NLO correction to the Drell-Yan process in which a gluon is radiated off one of the initial quark legs. If perturbation theory is to be applicable, the probability to radiate such a gluon should be in some sense small. Computing the correction to the hard matrix element due to radiating a gluon with transverse momentum p_T, the correction to the hard matrix element goes like

$$\frac{d(\Delta\hat{\sigma})}{dp_T} \sim \frac{\alpha_s(p_T)}{p_T}\hat{\sigma}. \tag{13}$$

For large p_T we see that this correction is reasonable, being suppressed by both the large p_T and the perturbatively small $\alpha_s(p_T)$. However, for small $p_T \sim \Lambda_{QCD}$, the correction becomes very large since α_s blows up and the denominator becomes small. This would seem to invalidate our use of perturbation theory, even though the energy of the underlying Drell-Yan process is much larger than Λ_{QCD}.

This might look bad, but there is a way out. Note that the problem arises when the gluon emitted is either very soft or is collinear with the beam. In both cases, for $p_T \lesssim \Lambda_{QCD}$ the gluon continues to travel along with the incident hadron and can't be said to escape as an observable particle (or as well see below, a jet of particles). It is therefore sensible to include the effects of soft and collinear gluon (or quark) radiation within the PDFs since it is effectively just modifying the parton content of the initial state.

In contrast to the case of soft gluons, a gluon emitted with large $p_T \gg \Lambda_{QCD}$ is expected to escape from the hadron and lead to an additional observable particle in the final state. Clearly we do not want to include these hard emissions within the PDFs. Instead, we should keep them as perturbative corrections to the parton-level hard matrix element. More generally, this leads to the question of what p_T value one should use to divide between soft radiation that is included within the PDFs and hard radiation that is handled as a perturbative correction to the parton-level matrix elements. The answer is μ_F, the factorization scale, which represents the dividing line between PDFs and matrix elements. We can choose μ_F any way we like, but a judicious choice will help us optimize the perturbative expansion of the hard matrix elements. This best choice is usually $\mu_F^2 \sim Q^2$, the typical large momentum scale associated with the underlying LO hard process.

Different choices of the factorization scale lead to different sets of PDFs that incorporate varying amounts of NLO (and beyond) parton radiation. We therefore write $f_i(x, \mu_F)$ to account for this property. Even though the PDFs are inherently non-perturbative, it is possible to relate PDF sets at different values of μ_F ($\gg \Lambda_{QCD}$) using perturbation theory. The result is described by the DGLAP[1] equations:

$$\begin{aligned}
\frac{df_g(x,\mu_F)}{dt_F} &= \frac{\alpha_s(\mu_F)}{\pi}\int_x^1 \frac{dz}{z}\left(P_{g\to g}(z) f_g(\frac{x}{z},\mu_F) + P_{q\to g}(z)\sum_q \left[f_{q\to g}(\frac{x}{z},\mu_F) + f_{\bar{q}\to g}(\frac{x}{z},\mu_F)\right]\right), \\
\frac{df_q(x,\mu_F)}{dt_F} &= \frac{\alpha_s(\mu_F)}{\pi}\int_x^1 \frac{dz}{z}\left[P_{q\to q}(z) f_q(\frac{x}{z},\mu_F) + P_{g\to q}(z) f_g(\frac{x}{z},\mu_F)\right].
\end{aligned} \tag{14}$$

Here, $t_F = \ln(\mu_F)$, while $P_{g\to g}$, $P_{q\to q}$, $P_{q\to g}$, and $P_{g\to q}$ are called splitting functions. They can be computed in perturbation theory (for $\mu_F \gg \Lambda_{QCD}$). For example, $P_{g\to g}$ corresponds to diagrams in which a gluon splits into a gluon and something else, while $P_{g\to q}$ corresponds to diagrams where a gluon splits into the quark q and anything else.

2 QCD at High Enegies: Jets

In the previous section we discussed how to handle processes with composite hadrons in the initial state. The essential feature that allowed us to make quantitative predictions was the property of factorization, through which the cross section of a process can be split into a non-perturbative but universal part and a perturbative parton-level piece. We turn next to

[1]DGLAP = Dokshitzer, Gribov, Lipatov, Altarelli, and Parisi.

look at how to handle processes where quarks and gluons are present in the final state. Even though these particles might be produced with very high energies, they are not what one observes experimentally. Instead, energetic quarks and gluons *shower* by emitting (mostly) soft and collinear QCD radiation, and they combine together to form colour-neutral hadrons. This collection of hadrons tends to be very collimated, and is called a QCD jet. As for the case of hadronic initial states, there is a notion of factorization for processes with partons in the final state that allows us to separate the dynamics into a universal non-perturbative part and a perturbative hard matrix element specific to the process of interest.

To start, consider the process $e^-e^+ \to q\bar{q}$ in the electron-positron CM frame. At leading order the matrix element gets contributions from diagrams with a γ or a Z^0 in the s-channel. We can compute the cross-section perturbatively for $\sqrt{s} \gg \Lambda_{QCD}$. In this limit, and assuming $\sqrt{s} \ll m_Z$ so that the photon diagram dominates, the total cross-section to produce hadrons is approximted well by

$$\begin{aligned} R &\equiv \frac{\sigma(e^-e^+ \to hadrons)}{\sigma(e^-e^+ \to \mu^-\mu^+)} \\ &\simeq N_c \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \theta_c \left(\frac{2}{3}\right)^2 + \theta_b \left(\frac{-1}{3}\right)^2 \right] \\ &= \left(\frac{6 + 4\theta_c + \theta_b}{9}\right) N_c, \end{aligned} \tag{15}$$

where $N_c = 3$ is the number of colours and $\theta_f = \Theta(\sqrt{s} - 2m_f)$ (and we assume implicitly that $\sqrt{s} > 2m_s$). The contributions in this expression come from u, d, s, c, and b quarks respectively. This equation agrees well with data except near the masses of specific hadronic bound states.

The experimental success of Eq. (15) suggests that computing the cross section to produce quarks (and gluons) in the final state within perturbation theory is a sensible thing to do. However, the particles seen in the final state are not quarks or gluons, but rather a collection of hadrons (in the form of mesons and baryons). It is not immediately obvious why the parton-level calculation should match up so well with data.

A clue for to how to interpret this result is that the the hadronic products of e^+e^- collisions with $\sqrt{s} \gg \Lambda_{QCD}$ usually consist of two distinct back-to-back *jets*, each consisting of a collection of highly collimated mesons and baryons. The net kinematics of these jets matches very well with the expectations for the momenta carried by the quarks produced in parton-level collision. This strongly suggests that each energetic quark produced in the collision leads to a jet. A further piece of evidence for this picture is that the probability to see three hard jets in the final state coincides well with the probability to radiate a hard gluon from one of the quark legs.

The formation of QCD jets makes sense from the point of view of asymptotic freedom. The $q\bar{q}$ pair produced has no net colour, and the quark and anti-quark retain a *colour connection* in the form of exchanged soft gluons. However, for quark energies well above Λ_{QCD}, these soft exchanges are incapable of transferring large amounts of momentum. Heuristically, each soft exchange takes time Λ_{QCD}^{-1} and transfers momentum Λ_{QCD}. However, the energetic quarks separate beyond the range of these soft exchanges, also on the order Λ_{QCD}^{-1}, well before they can transfer any significant amount of momentum. Hard exchanges, on the other hand, are suppressed by powers of $\alpha_s(p_T)$ and can be computed in perturbation theory.

As each quark travels along, it radiates soft or collinear gluons which in turn split into more gluons and $q\bar{q}$ pairs. This is called a *parton shower*. The partons within the shower all travel along together since the initial boost of the quark is (by assumption) much larger than the transverse momenta $p_T \lesssim \Lambda_{QCD}$ of the radiated particles. These roughly collinear partons subsequently attract and bind to form hadrons in a process called *hadronization*. Both the parton shower and hadronization are non-perturbative processes in QCD. There exist a number of phenomenological models for them, with the most famous implementations encoded within the PYTHIA [4] and HERWIG [5] computer programs.

The essential point that allows us to make perturbative predictions for QCD processes is that the hard and soft dynamics factorize. Starting from the hard parton-level matrix element, the kinematics of the outgoing jets is set almost entirely by the momenta of the energetic quarks and gluons which give rise to them. Non-perturbative QCD effects simply dress up these parton final states into collimated jets of colour-singlet hadrons. This is analagous to processes with hadrons in the initial state, where we use PDFs to describe the parton content of the colliding colour-singlets. And as with PDFs, an important issue associated with this is choosing where to make the split between hard and soft. For jets, this process is sometimes called *matching* [6].

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