Grand Unification Theories

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Abstract

In this report the basic features and the most relevant observable consequences of some Grand Unifying Theory are discussed, specially SU(5), SU(10), their SUSY versions and String Theory. Emphasis is given to commenting on specific predictions rather then theoretical technicalities, the reader is invited to refer to the references for a detail exposure of the topics addressed here.

1 Introduction

The Standard Model of Particle Physics is arguably one of the most successful theories of the history of science. Given a set of initial "tuning" parameters, its predictive power encompasses particle masses, scattering amplitudes, decaying processes, and etc., in many cases to striking precision.

In spite of such great success, physicists have always known that the Standard Model (SM for short) is incomplete for it ignores gravity completely, furthermore, new experimental results (e.g. Neutrino Oscillations) suggests that other modifications are also necessary. On top of this incompleteness issue one could argue that the SM is rather unnatural when compared to, for example, Electroweak theory alone, moreover the abundance of free parameters is often a source or criticism.

Motivated by some (but not only) of the ideas above one of the holy grails of modern theoretical physics is to find a unified description of particles physics, sometimes called Grand Unification Theory, or just GUT. Many different ingredients and approaches are used when constructing a GUT candidate, often times a fundamental theory is built using a large gauge group, e.g.: SU(5), that breaks spontaneously to SM's symmetry group $SU(3) \times SU(2) \times U(1)$ at some high energy scale. As it will be discussed further in the text, supersymmetry plays a central role in many of these unification models; other theories, such as those arising from string theory, are also very prominent candidates.

In the dawn of LHC era, physics are facing a unique opportunity to probe energies never probed before, hopefully narrowing down, if not completely, quite dramatically, the possible GUT candidates, therefore, rather than focusing on the machinery of each GUT candidate, this notes will favour presenting only the basics concepts of various models, what makes them unique, their problems and experimental consequences.

This paper is organized as follows:

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2 Georgi-Glashow SU(5) Model

2.1 Group Theory Motivation

Let $\Phi = \Phi^a t^a$ be a scalar field transforming under the adjoint representation of a $SU(N)$ gauge group. Suppose Φ acquires some vacuum expectation value (VEV for short) through some process not relevant to the present discussion, this VEV is a traceless hermitian $N \times N$ matrix that can be brought to diagonal form by a global $SU(N)$ transformation, which will be called V. In general V has eigenvalues v_i, with multiplicity N_i, that can be organized as:

$$\begin{bmatrix} v_1 I_{N_1\times N_1} & 0 & ... & 0 \\ 0 & v_2 I_{N_2\times N_2} & ... & 0 \\ 0 & 0 & ... & 0 \\ 0 & ... & 0 & v_n I_{N_n\times N_n} \end{bmatrix}, \tag{1}$$

all generators whose entries lies within the i-th block will commute with V, therefore forming an unbroken $SU(N_i)$ subgroup. In addition, the linear combination of all diagonal generators that is proportional to V, generates an unbroken $U(1)$ subgroup.

Motivated by the above discussion one could envision a situation in which a larger group is broken down to SM's gauge group by some scalar field subject to a given potential. This is the idea behind many of the attempts to formulate a unified theory of particle physics. In the following section Georgi-Glashow SU(5) Model, arguably the simplest example of such procedure, will be studied.

2.2 SU(5) 101

As pointed out above, let $\Phi = \Phi^a t^a$ be a scalar field transforming under the adjoint representation of SU(5), which by some process not relevant now, acquires a vacuum expectation value of the form: $\langle 0|\Phi|0\rangle = \mathrm{diag}\big(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\big)\, V$, such VEV will break the SU(5) gauge symmetry into precisely SU(3) × SU(2) × U(1). It is worth pointing out that this symmetry breaking process creates several other field that are not interesting to recreating the phenomenology of the SM, hence one should impose that the SU(5) breaking scale V is large compared to the electroweak scale.[1]

We can fit all the matter content of the SM into one representation of SU(5), namely: $\bar{5} \oplus 10$, the SU(5) gauge field matrix accommodates all standard model gauge fields, together with fields corresponding to the broken generators of SU(5) that after symmetry breaking acquire a mass of $M_x = \frac{5}{6\sqrt{2}} g_5 V$, with g_5 being SU(5)'s coupling.

If one define the following left-handed Weyl fields: ψ_i in the $\bar{5}$ representation, and χ_{ij} in the 10 representation, all standard model interaction follow from the lagrangian

$$\mathcal{L}_{int} = -g_5 \left[\psi_i^\dagger \left(A_\mu^T \right)_j^i \bar{\sigma}^\mu \psi^j + \chi^{\dagger ij} \left(A_\mu \right)_i^k \bar{\sigma}^\mu \chi_{kj} \right], \tag{2}$$

provided that ψ, χ and A_μ are organized accordingly. All the specific details of the model shall not be presented, rather its consequences and experimental implications will be discusses, the interested reader should refer to [8] for a introductory nevertheless more thorough approach.

To break the remaining SU(2) $\times$ U(1) gauge symmetry one need to add a scalar field in the representation $(1, 2, -\frac{1}{2})$, this is accomplished by introducing the field H^i in the $\bar{5}$ representation of SU(5):

$$H^i = \left(\phi^r, \phi^b, \phi^g, \varphi^-, -\varphi^0 \right), [2] \tag{3}$$

besides all possible Yukawa interactions, interesting on their own and discussed later in the text, this field is subject to the potential

$$\begin{aligned} V(\Phi, H) \quad = \quad & -\frac{1}{2} m_\Phi^2 Tr\Phi^2 + \frac{1}{4}\lambda_1 Tr\Phi^4 + \frac{1}{4}\lambda_2 \left(Tr\Phi^2 \right)^2 + m_H^2 H^\dagger H + \\ & + \frac{1}{4}\kappa_1 \left(H^\dagger H \right)^2 - \frac{1}{2}\kappa_2 H^\dagger \Phi^2 H, \end{aligned} \tag{4}$$

from which one can read the scalar masses:

$$m_\varphi^2 = M_H^2 - \frac{1}{8}\kappa_2 V^2, \tag{5}$$

$$M_\phi^2 = M_H^2 - \frac{1}{18}\kappa_2 V^2, \tag{6}$$

the value of these masses, as discussed below, present one of the toughest challenges to, not only SU(5), but various other GUTs.

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[1] Later it will be shown how this large scale arises naturally once experimental consequences of this model are analyzed.

[2] Note how this field also includes a scalar triplet.

2.3 Massive vector field interactions and proton decay

If one is to write all renormalizable interactions allowed by the underlying theory, one must consider interactions between massive vector fields (those arising from broken generators), called X, and the matter content of the theory. In the present case one have

$$\mathcal{L}_{Xint} = -\frac{1}{\sqrt{2}} g_5 X^{\dagger\alpha}_{i\mu} \left(\varepsilon^{ij} \bar{d}^{\dagger}_{\alpha} \bar{\sigma}^{\mu} l_j - \varepsilon^{ij} \bar{e}^{\dagger} \bar{\sigma}^{\mu} q_{j\alpha} + q^{\dagger\beta i} \bar{\sigma}^{\mu} \bar{u}^{\gamma} \varepsilon_{\alpha\beta\gamma} \right) + h.c. \qquad (7)$$

Note how the first and second terms inside the brackets have a different baryon and lepton number $(\frac{1}{3}, 1)$ than the third term $(-\frac{2}{3}, 0)$. Consequently baryon and lepton number conservation is no longer true in this model, by the exchange of one X boson one could have, for example, proton decay:

$$p^{+} \longrightarrow e^{+}\pi^{0},$$

the immediate issue one realize once facing this result is the absent experimental evidence for proton decay. Moreover, the lower bound for the proton life time is $\tau > 10^{33}$yr, a rough estimative from 7 gives:

$$\frac{1}{\tau} = \frac{g_5^4 m_p^5}{8\pi M_x^4} \quad \Longrightarrow \quad M_x > 10^{15} GeV \quad \Longrightarrow \quad V > 10^{15} GeV. \qquad (8)$$

If protons actually do decay, they do it in such a slow fashion that one is forced to push the unifying breaking scale to over $10^{15}GeV$, and face fine tuning and hierarchy problems.

2.4 Fine tuning, doublet-triplet splitting and hierarchy

Equations 5 and 6 show how the masses of the scalars depend on the GUT breaking scale V, and the mass of the field H. In the last section, though, it was presented an estimative for V based on proton decay. If one is to require the higgs mass in equation 5 to be of the order of the electroweak break scale one arrives at the problem of fine tuning:

$$m_{\varphi}^2 = M_H^2 - \frac{1}{8}\kappa_2 V^2 \sim -(100GeV)^2 \Longrightarrow$$

M_H^2 must be "equal" to $\frac{1}{8}\kappa_2 V^2$ to the first 16 digits, but **not** exactly equal. There is no physical reason to expect such a miraculous cancelation, or fine tuning, and the theory is rendered rather artificial unless this problem is solved in a more natural way.

On top of that, renormalization of this theory requires a new round of fine tuning of the parameters at each step, making the situation even more uncomfortable.

A similar problem arises when one requires $M_\phi^2 \gg m_\varphi^2$, again fine tuning is necessary to accomplish such a feat, and one finds oneself not being able to separate the masses of the triplet and doublet parts of H in a natural way.

Another unnatural aspect of this model (or maybe the one behind both discussed above) is the bewildering difference in GUT breaking and electroweak breaking scales, to put in another way, the higgs mass m_φ^2 is way too small when compared to the GUT scale. Again fine tuning is necessary when calculating radiative corrections to the (doublet) higgs mass if one wants to find the value predicted by electroweak theory.

2.5 Running of the couplings, renormalization scale and other predictions

Following the general idea of this report I will refrain my self from actually deriving all renormalization group equations contenting only with quoting the most relevant results.

After integrating out the heavy gauge fields, one finds for the beta function coefficients:

$$b_3 = -7, \quad b_2 = -\frac{19}{6}, \quad b_1 = \frac{41}{6}, \tag{9}$$

these can be used to evaluate M_x and $\sin^2\theta_W$, giving, with two-loop corrections:

$$M_x = 4 \times 10^{14} GeV, \quad \sin^2\theta_W\,(M_z) = 0.210, \tag{10}$$

M_x is roughly an order of magnitude smaller than the lower bound imposed by proton decay, as pointed out earlier. While $\sin^2\theta_W$ disagrees with the measured value of 0.231 by about 10%. One sees here two short comings of the Georgi-Glashow SU(5) Model, these can be considerably improved if one considers the super symmetric version of this model, although several other difficulties arise, as will be discussed shortly.

2.6 Fermion masses

The unified character of Yukawa couplings arising from the SU(5) model makes a sharp prediction about the relation between various fermion masses, they are:

$$m_b = m_\tau, \quad m_s = m_\mu, \quad m_d = m_e, \tag{11}$$

renormalization considerably improves the above result, turning it into the following mass ratios:

$$\frac{m_e}{m_\mu} = \frac{m_d}{m_s}, \quad \frac{m_\mu}{m_\tau} = \frac{m_s}{m_b}, \tag{12}$$

while experimental results for the above mass ratios are:

$$\frac{m_e}{m_\mu} \approx 0.0048, \quad \frac{m_d}{m_s} \approx 0.0269 - 0.0857, \tag{13}$$

$$\frac{m_\mu}{m_\tau} \approx 0.0595, \quad \frac{m_d}{m_s} \approx 0.0160 - 0.0315, \tag{14}$$

in clear disagreement with theoretical predictions. The situation can be greatly improved by various different methods including: more complicated Higgs mechanism, extra dimensions, non renormalizable terms.

3 Left Right Symmetry

Left-Right symmetry holds that physical laws should not discriminate left-handed and right-handed motion. The SM is clearly not left-right symmetric, nevertheless there are compelling arguments for believing that a broader theory, GUTs included, should be, here two are presented:

- The origins of parity violation are some what mysterious, while all other know interactions are parity conserving, low-energy weak-interaction processes apparently are not. Either weak interactions are truly parity violating and one must understand why, or we just do not understand it thoroughly and a broader theory is in need.
- There is now solid evidence for neutrino masses, and the urgent need for adapting the present description of fundamental particles to incorporate this new feature.

Left-right symmetry addresses both these points at once, therefore is very attractive from the point of view of unifying models.

The most immediate prediction of left-right symmetry is massive neutrinos. Other observables high energy consequences of the most basic L-R models are:

- A second neutral Z boson.[3]

[3] Actually an arbitrary number of neutral currents can be introduced.

- Right-handed charged currents.
- Right-handed neutrinos.

Many GUTs try to, or naturally do, incorporate L-R symmetry, the simplest example might be the unifying model based on the gauge group SO(10), which we turn our attention to now.

4 SO(10) GUT

The GUT based on the gauge group SO(10) has a few advantages over the simpler model based on SU(5):

- All fermionic content fits in just one 16 dimensional spinor representation, and only one of these representations has the correct quantum numbers.
- The model's gauge interactions are naturally parity conserving. Avoiding the cosmological domain wall problem.
- Is the minimal GUT model with L-R symmetry.

Furthermore, among the many ways one can discuss the algebra of SU(10), arguably the most useful for present purposes is in the spinor SU(5) basis. Using this basis most of the discussion is translated into SU(5) spinor language, and the similarities and differences of both models become evident.[4]

4.1 Fermion masses

As in the SM, fermion masses are also generated by Yukawa interaction with the Higgs field in the SO(10) model, with the difference now that the Higgs can be envisioned in different representations rendering quite different results, lets consider some cases separately.

4.1.1 10 dimensional Higgs

Let the 10-dimensional Higgs fields acquire two vacuum expectation values:

$$\langle 0|\psi_9|0\rangle = v_1, \quad \langle 0|\psi_{10}|0\rangle = v_2, \tag{15}$$

[4]See [6] for further details.

the mass terms arising from Yukawa interactions are

$$\mathcal{L}_{mass} = (v_2 - v_1)\left(\bar{d}_L d_R + \bar{e}_L e_R\right) + (v_2 + v_1)\left(\bar{u}_L u_R + \bar{\nu}_L \nu_R\right), \tag{16}$$

leading to:

$$m_d = m_e, \quad m_u = m_\nu. \tag{17}$$

Obviously this is a major flaw of this model, many approaches to fix such anomaly exist but I will refrain my self from addressing them.

4.1.2 120-dimensional Higgs

This representation of the Higgs breaks up into smaller representations under the sub-group SU(5),

$$\{120\} = \{45\} + \{45^*\} + \{10\} + \{10^*\} + \{5\} + \{5^*\},$$

and two are options of VEV

1. A linear combination of $\{45\}$ and $\{5\}$ acquire VEV, leading to

$$\begin{aligned} m_{d_a d_b} &= 3m_{e_a e_b} \\ m_{u_a u_b} &= 3m_{\nu_a \nu_b} \end{aligned} \tag{18}$$

2. Only the $\{45\}$ dimensional Higgs acquire VEV, leading to

$$\begin{aligned} m_{e_a e_b} &= 3m_{d_a d_b} \\ 3m_{\nu_a \nu_b} &= 0 \\ m_{u_a u_b} &\neq 0 \end{aligned} \tag{19}$$

Where $m_{d_a d_b}$ stands for mixing terms in the mass matrix between generations a and b.

4.1.3 126-dimensional Higgs

For reasons not relevant here there are six possible options of VEV for the Higgs field, the observable mass implications for one generation are:

$$\begin{aligned} m_e &= 3m_d \\ m_\mu &= 3m_\nu \end{aligned} \tag{20}$$

4.2 Second neutral Z boson and proton decay

In its most conventional form, the SO(10) grand unification model, due to its natural Left-Right symmetry, predicts a light right-handed extra neutral Z boson with mass ranging from 300GeV $\sim$ 1TeV[6], experimental bounds on the mass of new Z bosons can be found in [3], at the time of writing, effects from an extra neutral current were of the order of experimental error. The masses of extra W bosons are of the order 10^{11}GeV.

SO(10) also predicts proton decay, with a different branching ratio $\Gamma\left(p \rightarrow e^{+}\pi^{0}\right) = \Gamma\left(p \rightarrow \bar{\nu}\pi^{+}\right)$ and life time than the simpler SU(5). Proton decay in decoupled parity\L-R symmetry models that arise from SO(10) is predicted to be $\approx 10^{35\pm2}$yr. Within experimental bounds, and "low" enough to be tested.

5 Minimal Supersymmetric Standard Model

Supersymmetry provides a natural way for understanding why the Higgs mass is so small compared to the GUT scale[5]. By introducing a symmetric number of fermionic and bosonics degrees of freedom, one-loop corrections to the Higgs mass cancel exactly solving the Higgs stability problem, moreover, when turned into a local symmetry, supersymmetry naturally leads to gravity.

One of the most basic consequences of supersymmetry is the existence of bosonic and fermionic fields with precisely the same mass, ironically this is also the most clearly "wrong" prediction since it has never been observed, ergo supersymmetric **must** be broken at some high energy scale. How this symmetry breaking occurs is still subject of intense research and no consensus has been reached so far. Regardless of how it happens the consequences are somewhat similar and focus will be on discussing them.

The simplest possible way to introduce supersymmetry to the standard model leads to the so called: Minimal Supersymmetric Standard Model, or MSSM. In this section I shall present the basic features of this model and its particle content, paving the way for sections to come on supersymmetric unifying theories.

In this minimal supersymmetric extension every fundamental particles is either in a chiral supermultiplet, if it is a chiral fermion, or in a gauge supermultiplet, if it is a gauge boson. For every field present in the SM, there is now its supersymmetric partner, sharing the same mass, but with spin differing by 1/2.

All chiral fermions are now accompanied by a spin 0 complex field whose name is the same as the fermion in case with an extra s at the beginning (electron $\longrightarrow$

[5]Or at least why radiative corrections does not spoil the initial fine tuned choice of parameters.

selectron). The spin one gauge fields have now a spin 1/2 superpartner whose name is the same as the gauge field in question with an extra "ino" at the end (gluon $\longrightarrow$ gluino).

One novel features of the MSSM is the necessity for two gauge fields (together with two spin 1/2 superpartners)

$$\text{Higgs} = \begin{cases} H_u = \begin{pmatrix} H_u^+ & H_u^0 \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 & 2 & +\frac{1}{2} \end{pmatrix} \\ H_d = \begin{pmatrix} H_u^0 & H_u^- \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 & 2 & -\frac{1}{2} \end{pmatrix} \end{cases} \tag{21}$$

$$\text{Higgsinos} = \begin{cases} \tilde{H}_u = \begin{pmatrix} \tilde{H}_u^+ & \tilde{H}_u^0 \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 & 2 & +\frac{1}{2} \end{pmatrix} \\ \tilde{H}_d = \begin{pmatrix} \tilde{H}_u^0 & \tilde{H}_u^- \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 & 2 & -\frac{1}{2} \end{pmatrix} \end{cases} \tag{22}$$

The most obvious consequence of this minimal extension is the existence of supersymmetric partners (yet to be found). Speculations around the relation between light superpartners and dark matter are abundant, nonetheless clear evidence, or a solid theoretical model, have yet to be found or constructed.

6 SUSY SU(5)

SUSY provides a partial solution to the hierarchy problem. While not completely fixing the parameters, it guarantees that after higher loop corrections further fine tuning is not necessary.

Naturally many unifying theories introduce SUSY resulting in not only new and interesting possible physical phenomena, but some pathologies as well, again SU(5) is arguably the simplest model to do so.

In order to be phenomenological consistent, not only SU(5) must be broken down to SM's gauge group, but SUSY must be broken as well, usually it is assumed that

$$\text{SUSY} \otimes \text{SU}(5) \longrightarrow \text{MSSM} \longrightarrow SM. \tag{23}$$

In its simplest realization one needs as basic ingredients the matter fields in the $\bar{5}$ and 10 representations ψ and T, two Higgs[6] H_u and H_d, and a Higgs superfield Φ, in the 24 representation. Moreover a superpotential $W(H_u, H_d, \Phi, \psi, T)$ that provides:

- realistic breaking of SU(5) to SU(3)⊗SU(2)⊗U(1) and subsequently to SU(3)⊗U(1),
- masses to quarks and leptons,

[6]For the same reasons two Higgs were necessary in the MSSM.

various potentials satisfy the above conditions, here I will discuss the consequences arising from:

$$\begin{aligned} W &= h_u \varepsilon^{ijklm} T_{ij} T_{kl} H_{u,m} + h_d T_{ij} \psi^i H_d^j + z \mathrm{Tr}\left[\Phi\right] + x \mathrm{Tr}\left[\Phi^2\right] \\ & +y \mathrm{Tr}\left[\Phi^3\right] + \lambda_1 \left(H_{u,i} H_d^j \phi_j^i + m' H_{u,i} H_d^i \right) . \end{aligned} \quad (24)$$

The above potential leads to a three-fold degenerate vacuum once SU(5) is broken, this is the so called Cosmological Domain Wall problem: there is no natural way to understand why the universe, when passing through the GUT breaking scale after the big bang, chose SM's gauge group as its preferred vacuum.

6.1 Proton decay in SUSY SU(5)

The beta function in the supersymmetric version of SU(5) is rather different from its non supersymmetric version:

$$\beta_i \left(g_i \right) = \left(-3N + T_F + T_H \right)_i \frac{g_i^3}{16\pi^2}, \quad (25)$$

this profoundly changes the GUT breaking scale (or the masses of the heavy X fields M_x), $\sin^2 \theta_W$, and proton lifetime predictions to:

$$M_x \approx 10^{16} \mathrm{GeV}, \quad (26)$$

$$\sin^2 \theta_W \left(m_W \right) = 0.236 \pm 0.003, \quad (27)$$

and

$$\tau_p \approx 10^{35} \mathrm{yr}. \quad (28)$$

The predicted value for $\sin^2 \theta_W$ improves considerably when compared to the non SUSY version. Proton decay, however, is not measurable since it surpass present experimental bounds, nonetheless other mechanisms for baryon non-conservation may result in interesting observable consequences.

6.2 Direct consequences of SUSY operators

An operator of the kind

$$\text{F type operator} \quad = \quad \frac{1}{M_x} \int d^2\theta \Phi^4, \quad (29)$$

can be responsible for Higgs mediated processes that are strongly not baryonic conserving. Proton decay through such an operator falls within experimental bounds, however, unlike SU(5), the SUSY version predicts that the dominant decay is:

$$p \longrightarrow K^+ \bar{\nu}_\mu, \tag{30}$$

the discovery of this decay could be strong evidence for the existence of SUSY.

7 SUSY SO(10)

Unlike SUSY SU(5), the supersymmetric version of SO(10) unifying model introduces naturally neutrino masses, furthermore, it make reasonable predictions on their possible values. Another desirable feature of this model is that it solves the SUSY CP problem, and R-parity problem, of MSSM.

For the sake of brevity I will only list some of the many important features of SUSY SO(10), and invite the reader to refer to the references for further details.

With specific Higgs mechanism chosen, SUSY SO(10) leads to the following mass relations at the GUT scale:

$$m_b = m_\tau, \quad \frac{m_e}{m_\mu} \approx \frac{1}{9}\frac{m_d}{m_s}, \tag{31}$$

as for neutrinos masses one finds:

$$m_{\nu_e} \approx \frac{m_u^2}{10 f v_{B-L}} \approx 10^{-8}\text{eV}, \tag{32}$$

$$m_{\nu_\mu} \approx \frac{m_c^2}{10 f v_{B-L}} \approx 10^{-4}\text{eV}, \tag{33}$$

$$m_{\nu_\tau} \approx \frac{m_t^2}{10 f v_{B-L}} \approx \text{eV}, \tag{34}$$

where $v_{B-L} \approx 10^{12}$GeV is the scale of B-L number symmetry breaking.

8 Extra Dimensions

Postulating the existence of extra dimensions offers a way out of the hierarchy conundrum. The quantum gravity scale is vastly diminished by assuming the gravity force to extend along extra dimensions, and one no longer has to deal with such disparity in energy scales (quantum gravity vs. electroweak). Although extra dimensions are not a unifying theory *per se* its consequences will quickly be mentioned for it plays a crucial role in String Theory, M-theory, and even standard QFTs.

Assume that besides the usual 1+3 dimensions of space-time, one also has n small, curled dimensions that are essentially invisible to any low energy observation. By allowing gravity to smear out across these new extra dimensions, and by demanding them to be really small (soon to be quantified), one is capable of dramatically lowering the plank scale, while keeping all observations done to date intact. The general idea is: given a scale R for the curled dimension, Newton's gravitation force equation reads:

$$F = \frac{m_1 m_2}{M_{pl}^2 r^2} \quad \text{for} \quad r \gg R, \tag{35}$$

$$F = \frac{m_1 m_2}{M_{pl(n+3+1)}^{2+n} r^{2+n}} \quad \text{for} \quad r \ll R, \tag{36}$$

leading to

$$M_{pl(n+3+1)}^2 = \frac{M_{pl}^2}{\left(M_{pl(n+3+1)} R\right)^n}. \tag{37}$$

If one requires the new plank scale to be $M_{pl(n+3+1)} \sim 30\text{TeV}$, one finds that $R \sim 2\mu m$. This is something one can actually probe in a particle accelerator, and indeed the LHC will look for such hidden (not so) small dimensions, also called "large extra dimensions"

9 String Theory

String theory starts from a completely different point of view than standard field theories, it assumes 1 dimensional strings as the fundamental object, and all the particles observed arise from string excitations. As direct consequence of this formalism a spin-2 massless field naturally appears, making of String Theory a strong candidate for a quantum description of gravity. Moreover one has plenty of room to fit all symmetries and matter content of the SM into a ST framework, nonetheless actually doing so is not so easy, and a realistic model has yet to be constructed.

Unlike other physical theories string theory fixes[7] the dimension of space time to 11[8], ergo, when building a realistic version of ST one must account for the fact that 7 of this 11 dimensions have not been observed yet. Curling up this extra dimensions in

[7] Direct consequence of imposing Lorentz invariance

[8] In its SUSY version.

a Kalusa-Klein fashion is probably the most common approach, making large extra dimension a **indirect** evidence for String Theory.

Other specific predictions arising from string theory vary vastly throughout different models, and addressing them is out of the scope of this notes, the reader is referred to [2] as a good place to start.

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