Flavor mixing and CP violation

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Introduction

In the standard model, neutrinos are neutral, massless and colorless particles. They are only experienced the weak force, cannot interact to photons or gluons. These neutrinos are all left-handed, so the weak interactions are not invariant under the parity transformation. But if we combine this P transformation with charge conjugation C, then CP operator is back to be invariant in the weak interactions. That means a mirror image of a right-handed electron is a left-handed positron, not a left-handed electron. In other words, physics described by particles and antiparticles are same. Then how to expalain our matter dominant universe? If a system is invariant under exchanging particles and antiparticles, we should have equal amounts of each. But that is not ture.

In 1964, Cronin, Fitch and their coworkers found clear evidence that CP symmetry was not invariant in the weak interactions. That was too much small, but unmistakable effect. At a glace, it seemed physicists could avoid the problem, inconsistency between CP symmetry and matter dominant world. But they were faced with another question – why this happened?

In those days, there were three quarks in the standard model. And a three-quark model could say nothing about CP violation. Meanwhile, from a theoretical point of view, they had a feeling the existence of the fourth quark. But a four-quark model could not explain that effect either.

Kobayashi and Maskawa proposed a theoretical framework for this CP violation. They pointed out if we accepted six quarks, CP violation followed automatically. With six quarks, a mixing matrix for the quark sector in the standard model has a single complex phase. This is the only source of CP violation as long as we know so far.

It is quite interesting. While CP violation occurs in the weak interaction, the quarks come from the strong interaction sector. Of course, quarks couple to the weak interactions. But it is still unnatural CP is not conserving only under the weak interactions, not under the strong interactions.

In spite of these flaws, quark flavor mixing and CP violation are closely correlated. So let us start to consider flavor mixing first, and then move to CP violation.

Theory

What is flavor mixing? See following tree level Feynman diagrams in the weak interactions.

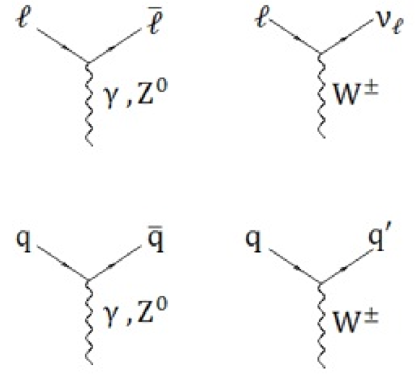

Figure 1: tree level Feynman diagrams in the weak interactions

As you can see, for leptons, flavors are not changed in both charged and neutral current processes. Otherwise, for quarks, no tree level flavor changing is in neutral current, but there is in charged current processes. Considering a lagrangian, we can get more theoretical intuition.

The standard model lagrangian consists of three parts.

$$\mathscr{L}_{SM} = \mathscr{L}_{kinetic} + \mathscr{L}_{Higgs} + \mathscr{L}_{Yukawa}$$

Among these, the Yukawa lagrangian arises fermion masses, and it is what makes fermion flavor mixing.

$$-\mathscr{L}_{Yukawa} = Y^d_{ij}\overline{Q^I_{Li}}\phi d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\tilde{\phi} u^I_{Rj} + Y^l_{ij}\overline{L^I_{Li}}\phi l^I_{Rj} + h.c.$$

Here 'I' denotes 'interaction eigenstate'. After symmetry breaking, fermion mass terms are appeared like

$$-\mathscr{L}_{Yukawa} = (M_d)_{ij}\overline{d^I_{Li}}d^I_{Rj} + (M_u)_{ij}\overline{u^I_{Li}}u^I_{Rj} + (M_l)_{ij}\overline{l^I_{Li}}l^I_{Rj} + h.c.$$

Always we can find matrices V_{fL}, $V^\dagger_{fR}$ such that

$$V_{fL} M_f V_{fR}^\dagger = M_f^{diag}$$

Then, fermion states are redefined as mass basis.

$$\begin{gathered} d_{Li} = (V_{dL})_{ij} d_{Lj}^I,\ d_{Ri} = (V_{dR})_{ij} d_{Rj}^I \\ u_{Li} = (V_{uL})_{ij} u_{Lj}^I,\ u_{Ri} = (V_{uR})_{ij} u_{Rj}^I \\ l_{Li} = (V_{lL})_{ij} l_{Lj}^I,\ l_{Ri} = (V_{lR})_{ij} l_{Rj}^I \\ \nu_{Li} = (V_{\nu L})_{ij} \nu_{Lj}^I \end{gathered}$$

Using these redefined states, we can rewrite kinetic terms.

$$\mathscr{L}_{kinetic} = i\overline{f^I}\gamma^\mu D_\mu f^I = i\overline{f} V^\dagger \gamma^\mu D_\mu V f$$

Then neutral current interaction is universal in mass basis.

$$\mathscr{L}_{nc} = ieQA_\mu \overline{f} V^\dagger \gamma^\mu f + \tfrac{ie}{\sin\theta_W \cos\theta_W} \overline{f} V^\dagger [P_L T_3 - Q\sin^2\theta_W] V f$$

where

$$V_f^\dagger [P_L T_3 - Q\sin^2\theta_W] V_f = V_f^\dagger V_f [P_L T_3 - Q\sin^2\theta_W] = [P_L T_3 - Q\sin^2\theta_W]$$

How about charged current terms?

$$\begin{gathered} \mathscr{L}_{cc} = \tfrac{g}{\sqrt{2}} W_\mu^+ [\overline{\nu_l} V_\nu^\dagger \gamma^\mu V_l l] + h.c. \\ \mathscr{L}_{cc} = \tfrac{g}{\sqrt{2}} W_\mu^+ [\overline{u} V_u^\dagger \gamma^\mu V_d d] + h.c. \end{gathered}$$

Since neutrino is massless, we can always set $V_\nu^\dagger = V$. So no flavor mixing is for leptons. Otherwise, for quarks, $V_u^\dagger \gamma^\mu V_d$ is not unity. This is the flavor mixing. And

$$V_{CKM} = V_u^\dagger V_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

But how can the CKM matrix be related to CP violation?

Let consider other two parts of the stadard model lagrangian, kinetic and higgs. First, the kinetic part. There is no chirality, so it is always CP conserving. Next, the Higgs part. The part describes scalar sector in the standard model. So it does not violate CP symmetry either. Now go back to the Yukawa interactions. In this part, left-handed fermions are coupled to right-haded ones. For example

$$Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj}$$

And caused by the hermiticity of the lagrangian, there must be a hermition conjugation.

$$Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj}+Y_{ij}^{*}\overline{\psi_{Ri}}\phi^{dagger}\psi_{Lj}$$

After acting CP operator on those terms

$$Y_{ij}\overline{\psi_{Ri}}\phi^{dagger}\psi_{Lj}+Y_{ij}^{*}\overline{\psi_{Li}}\phi\psi_{Rj}$$

CP is conserved if and only if $Y_{ij}=Y_{ij}^{*}$. That means sources of CP violation are related to complex parameters of Yukawa couplings, Y_{ij}.

How many independent complex parameters are there?

Y^{u}, Y^{d}, Y^{l} , each matrix is 3×3 and complex. That is, $3\times 3\times 3=27$ real parameters and $3\times 3\times 3=27$ imaginary parameters. Too many. Fortunately, not all of them are physical. Considering global symmetries with and without Yukawa couplings, we can remove 15 real parameters and 26 imaginary ones. Finally, 12 real ones and a single complex phase are left.

As we have seen so far, the Yukawa couplings bring fermion masses and quark flavor mixing. So the real parameters in the couplings correspond to fermion masses and mixing angles. Actually

$$12 \text{ real} = 6 \text{ quark masses} + 3 \text{ lepton masses} + 3 \text{ quark mixing angles}$$

And the last one, a single phase, is the only source of CP Violation in the standard model.

Experiment

Since CP violation is closely correlated to flavor mixing, it is important to determine CKM parameters. There are three ways to do this.

(i) Direct measuerments
(ii) Indirect measuremets
(iii) CKM unitarity

7 of 9 matrix components can be measured directly; V_{ud},V_{us},V_{ub},V_{cd},V_{cs},V_{cb},V_{tb}. Figure 2 shows tree level diagrams for those kinds of processes. But remaining two,which are related to loop processes, can not be. They only can

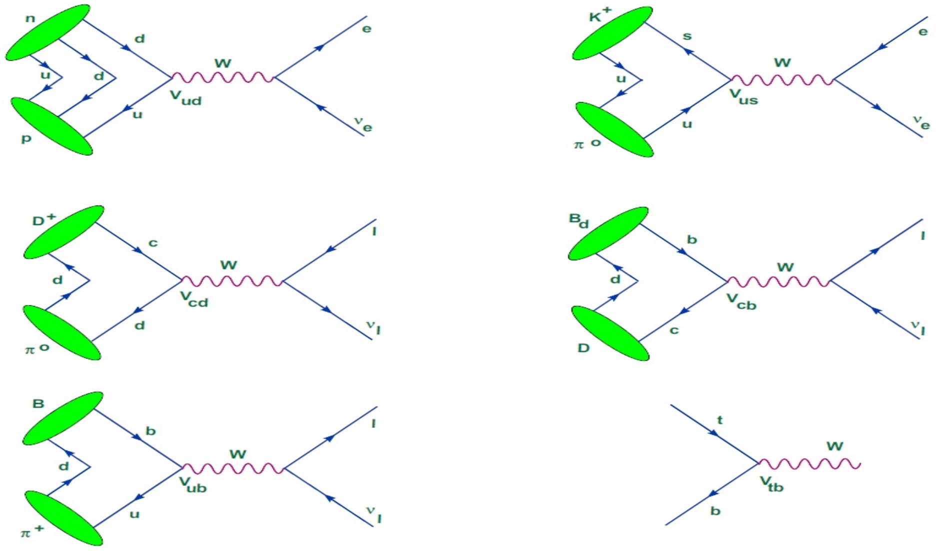

Figure 2: flavor mixing processes to determine CKM matrix parameters

be inferred or constrained indirectly. Besides, CKM Unitarity gives good boundaries of V_{td},V_{ts},V_{tb} and V_{cs}.

Now only the phase is left. We all know that phase is closely related to CP violation. That said we need to investigate CP violated processes, in order to determine the phase. As once it was showed in the first experimental evidence, CP violations effect is very small. So the more different processes– specially meson decays– you measure, the better result of the phase you will get. Why do we have specific interesting in meson decays? Electrically neutral and spin zero mesons are invariant under C and P separately. Such kinds of mesons are in states with CP symmetry, i.e. CP eigenstates. If CP is conserved, they only decay to another CP invariant state, which is a linear combination of eigenstates. Neutral K is one of such mesons, so should not decay to two pions. Only three pions decays is allowed to satisfy CP symmetry. The 1964-experiment was exactly what observed two pions process in neutral kaon decays. For a long time, it was believed all CP violations was occured in kaon physics. However, after B-Factory by now a large number of CP violation processes in B meson decays have been discovered. Becuase this B meson is relatively heavy than many other mesons, there are lots of way to go to lighter particles. The CP violation effect for

B mesons is relatively large, unlike the kaon decays. Several collaboration groups are working on investigation B meson decays.They produce huge number of $B^0/\overline{b^0}$ piars, to compare decay modes in particle and antiparticle sysem. For example, $B^0 \to K^+ + \pi^-$ occurs only the ratio of 1.82×10^{-5}, but it is more common that its its mirror image mode $\overline{B^0} \to K^- + \pi^-$. This experiments, called B-factories, is trying to understad the origins of CP violation. And they have given lots of wealth results so far.

I am not going to talk detailed techniques, but give general formalism.

As I said above, that is obvious we should measure CP violation. But how do we do that? What do we measure? What is the actual experimental observable?

To answer these question, let reparametrize the CKM matrix.

From $K^+ - \pi^0$ scattering, the V_{us} is well determined. Its value is $V_{us} = 0.22$. Setting $V_{us} = \lambda$, we can expand V in powers of λ. For example, a recent measurement of V_{cb} from B particle decays is 0.06. This suggests V_{cb} is of oder λ^2 so that $V_{cb} = A\lambda^2$. To order λ^2, the CKM matrix is rewritten

$$V_{CKM} = \left(\begin{array}{rcc} 1 - \frac{1}{2}\lambda^2 & \lambda & 0 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ 0 & -A\lambda^2 & 1 \end{array} \right)$$

Here is a problem. This reparametrized matrix does not have any complex phase. Insted of, there are two zero components. Extending up to λ^3-order and considering a complex phase, this matrix is slightly modified,

$$V_{CKM} = \left(\begin{array}{rcl} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{array} \right)$$

After adding three more equaions from the unitarity of the CKM matirx,

$$\begin{array}{c} V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \\ V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \\ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \end{array}$$

we can define a CP violating quatity J

$$Im[V_{ij}V_{kl}V_{jl}^*V_{kj}^*] = J \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln}$$

CP is violated if and only if $J \neq 0$ as Y^f was before

From one of the unitarity relations, setting $V_{cd}V_{cb}^*$ be real by convention,

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

It represents a triangle in complex plane. This triangle is only specified by the coordinates $(\overline{\rho}, \overline{\eta})$. One more interesting feature is all unitary triangles have same area. And the area is equals to $\|J\|/2$. Then, J is roughly $\lambda^6 A^2 \eta$.

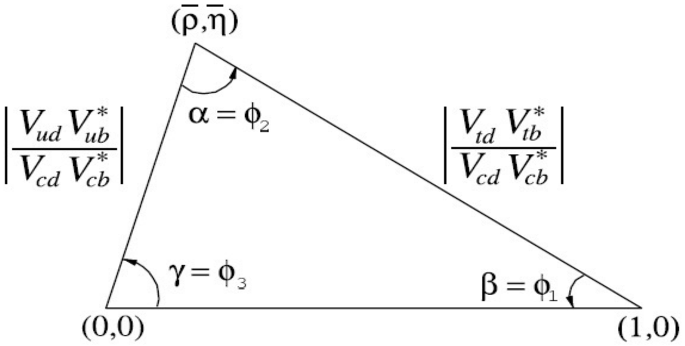

Figure 3: unitary triangle

Moreover, there are three angles $\alpha, \beta, and \gamma$. They can be written in terms of the CKM matrix components.

$$\alpha = arg[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}], \beta = arg[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}], \gamma = arg[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}]$$

They are physical observables for CP assymetries, which can be independently measured in B decays. The α is measure from $b \to u\overline{u}d$. The β is from $B_d \to J/\psi K_S$. The γ is from tree-level decays of B mesons. Because it does not depend on the top quark.

Figure 4. is the current global fit on $(\overline{\rho}, \overline{\eta})$ plane. The intersections are best fits for (λ, A, ρ, η), which are

$$\lambda = 0.2272 \pm 0.0010, A = 0.818^{+0.007}_{-0.017}, \rho = 0.221^{+0.064}_{-0.028}, \eta = 0.340^{+0.017}_{-0.045}$$

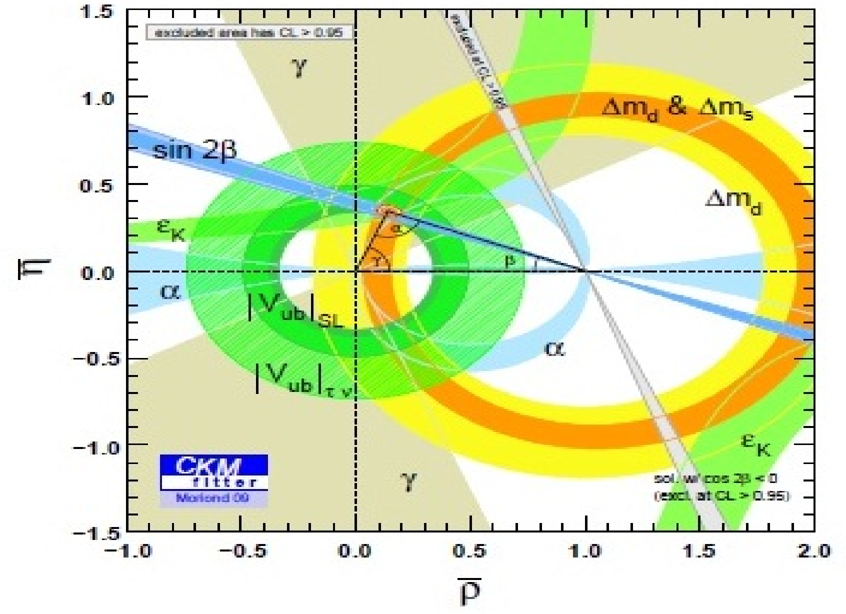

Figure 4: global fit to the mixing and CP violation

Conclusion

Finally, we have a whole set of well-defined parameters in the CKM theory. But everything is experimental measurment. No one knows how to calculate them theoreticaly. Furthermore, experimental values of ρ and η are 0.221 and 0.340 respectively. $\eta/\rho = \mathscr{O}(1)$ is not so small. Then why was the CP violation effect very small in kaon decays? In fact, this smallness came from small flavor mixing angles, not from a small phase. The standard model expects some cases of large enough CP asymmetry of order 1, even though it is not measured yet.

In addition, when we reparametrized the CKM matrix, we expanded it around λ. It was samll perturbation. But now non-perturbative corrections are required. It predicts another source of CP violation in QCD. This new parameter has been proved almost zero. while the theory still can not explain the reason why.

Is that all? No. Neutrino is not massless any more. Though the mass is small, it has mass. The reason why there is no flavor mixing in lepton sector was massless neutrinos. If they are massive, we must think about same thing, flavor mixing and CP violation, in lepton sector.

No one doubt particle physics has been excessively developed to this day. We have very powerful frameworks and those can explain many things very well. Nevertheless It is inevitable to need new physics beyound the stadard model. And there already has been a significant advance. In near future, we will be able to answer more fundamental questions.

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