Electroweak Interactions
And
A Low Energy Test Of The Standard Model

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Introduction

As we all know, the Standard Model is the best known theoretical frame work we have to the date, within which all the interactions that we have come across so far can be explained very consistently [1]. The Standard Mode (SM) is still *standing* although it is continuously going through the *acid tests* of rigorous experiments in different energy levels. This makes the SM the most important tool to both theoretical and experimental physicists. The SM is a $SU(3)\times SU(2)\times U(1)$ *gauge* theory which does not include the gravity. Like any other theory, SM has numerous predictions through which the theory could be tested. Physicists divide them in two basic categories depending on the energy range: the low energy tests and the high energy tests. In low energy area we focus our investigations in high precision measurements and look for any inconsistency with the SM predictions. Examples of such experiments are the nuclear beta decay, atomic parity violation, neutrino scattering etc. In the high energy regime attempts are made to *produce* the heavier particles that are parts of the SM, if we *see* the particles or their signatures, we conclude that the SM is correct. For example, SM says, we should have six qurks, and in different experiments the proof of their existence was proved. The only particle in the SM that we are waiting for to *show up* is the Higgs boson.

Because of all the extraordinary successes, we are very confident that, the new physics, in which ever form it appears is a small perturbative correction to the existing theory [2]. As mentioned earlier, the SM has a successful interpretation of all the fundamental interactions that we know of except the gravity (but being weakest of all known interactions, we have less concern about it, unless the question of completeness of the theory comes in), in this project I will focus on the electroweak interaction of the SM particles at low energy. When we say low energy we mean $E<<M_Z$ [3] where M_Z is the mass of Z boson that I will talk about later. The study of EW interactions are interesting and useful for a number of reasons. The force carrying particle of this interactions interact with almost all the SM particles except for the gluons and neutrinos. The tests ranges from pretty low (ultra-cold) to very high energy (LHC) limits. The variety and number of the experiments that were already performed and are underway are stunning. As expected, lot of them are related to another experiment. The advancement of technology opening the doors of new and and more precise experiments everyday. Choosing a particular one out of them is another challenge. After spending lot of time I decided to write on the pion (π) decay. In the next sections I will present some of the interesting features and experiment that supported the robustness of the SM.

Electroweak Interaction

As we all know, the electroweak interaction or the electroweak force is the unified version of the electromagnetic and weak forces and the unification takes place at an energy in the order of 100 GeV. The symmetry group of the electroweak interaction is $SU(2)\times U(1)$ and the carriers of the electroweak force are $W^{\pm}$ and Z bosons.

After spontaneous symmetry breaking the Lagrangian for the fermions is written as in Eq. (1) [4].

$$\begin{aligned}\mathcal{L}_{fermion} =& \sum_i \bar{\psi}_i \left(i\partial\!\!\!/ - m_i - \frac{g m_i H}{2M_W} \right) \psi_i - \frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i \gamma^\mu \left(1-\gamma^5\right) \left(T^+ W_\mu^+ + T^- W_\mu^-\right) \psi_i \\ & - e \sum_i q_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu - \frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i \gamma^\mu \left(g_V^i - g_A^i \gamma^5\right) \psi_i Z_\mu,\end{aligned} \tag{1}$$

where:

$$\begin{aligned}\theta_W &\equiv \tan^{-1}(g'/g), \\ e &= g\sin\theta_W, \\ A &\equiv B\cos\theta_W + W^3\sin\theta_W, \\ W^{\pm} &\equiv \tfrac{(W^+ \mp W^2)}{\sqrt{2}}\end{aligned}$$

Z bosons are neutral but the W bosons are charged, θ_W is the well known weak angle and A is the photon field. T^+ and T^- are the weak isospin raising and lowering operators. The g_V^i and g_A^i are the vector and vector and axial couplings defines as;

$$\begin{aligned}g_V^i &\equiv t_{3L}(i) - 2q_i \sin^2\theta_W, \\ g_V^i &\equiv t_{3L}(i)\end{aligned}$$

The second term in the Lagrangian gives the charged-current weak interaction [4]. We also remember from the lecture note that, the SM Lagrangian for the electroweak sector is completely specified by three parameters namely, $g, s_w \equiv \sin\theta_W$ and v. These three parameters are being determined by three independent experiments. At the end we are left with the following values [5, 6]:

$$\begin{aligned}v =& 174.105 \text{GeV}, \\ s_W^2 =& 0.234, \\ g =& 0.648419.\end{aligned}$$

After *Higgsing*, we get three *massive* bosons ($W^{\pm}$ and Z^0) one mass less boson (γ, the photon). The remaining symmetry is $U(1)_{em}$, i.e: $SU(2)_L \times U(1)_Y \to U(1)_{em}$. From this point, it's all about finding different parameter or quantity of interest that we want to measure to test the theory. In the next section I discuss some of interesting quantities that are measured to test the SM.

The Tests

In this section I will try to give a quick overview of what physical quantities are measured in different attempts to test the SM. Once the three parameters (g, s_w and v) are determined, we have a number of experiments that we can do to test the SM, some of the very well knows branches are as follows: 1. Neutrino physics (neutrino scattering), 2. Search for permanent electric dipole moment, 3. Neutral current, 4. weak decay, 5. Muon physics, 6. Supersymmetry, 7. Atomic parity violation, and 8. More and more. Even a very short description of few of them will take lot of space (I tried my best to include at least five but could not). So, here I will give a very brief idea of what is measured in two very interesting experiments and then move to the experiments that I discuss in little more details.

Neutrino physics is interesting for various reasons, first of all, the neutrinos are very difficult to detect, we know about them only by observing the other particles that are their decay partners. Secondly, any anomaly in the observation related to neutrino will give an indication about the new physics! When we talk about the test of the SM via neutrino physics at low energy, we basically mean the scattering of the neutrinos that are considered mass-less. One of the quantity that is measured to test the SM in the scattering process is the ratio of the cross sections of neutral and charged current *deep-inelastic* ν_μ-nucleus cross section which is denoted by R_ν [7].

$$R_\nu = \frac{\sigma(\nu N \to \nu X)}{\sigma(\nu N \to l^- X)} \tag{2}$$

NuTeV is a collaboration that measured this quantity (subject to some controversy) with:

$$\delta R_\nu = R_\nu^{exp} - R_\nu^{SM} = -0.0033 \pm 0.0007.$$

Another extremely interesting test of the SM is via the Atomic parity violation or Parity Non-Conserving (PNC). PNC is a very powerful tool for testing the EW sector of the SM. The weak nuclear charge Q_W quantifies the strength of the EW interaction between atomic electrons and the quarks in the nucleus [8]. The relation between the PNC amplitude E_{PNC} and Q_W is given by:

$$E_{PNC} = kQ_W, \tag{3}$$

here, k is an atomic structure factor.

In the SM the weak charge (Q_W) has a very sharply defined value. In the Jefferson Lab, an attempt is underway to measure this value using the running of the weak mixing angle $sin^2\theta_W$ [9] from the Z^0 pole down to the low energies. This is an experiment that will start running in the high energy and end on the low energy. If the result is found consistent with the prediction of the SM it will strengthen the SM however, inconsistency will indicate towards new physics beyond the SM as any other tests.

These were just two examples of the test of the SM with extreme brief description for motivation. In the next section I will present another very important test of the SM but in a little more details.

Pion Decay

In the low energy regime, one of the most important branch of EW interaction study is the decay of pions ($\pi^{\pm}$andπ^0). Although pions are bosons, they are in fact under the group of hadrons, so they are composite particles and the constituent particles are quarks. Pions decay into almost all the leptons, and this is a very interesting and important feature that facilitates the test of the SM. The decay of pions (charged ones) is dominated by the decay mode $\pi^- \to \mu^- \bar{\nu}_\mu$. However, this dominant mode is of little help as the computation of this mode at high precision is restricted by the non-perturbative strong interactions that involve the light quarks [10]. But it is possible to calculate the value of the f_π (see class notes and assignment, in assignment we do the calculation at tree level, but for a better result it is necessary to take into account the radiative correction [10] and [**?**]) which is useful on investigating the chiral dynamics of the strong interactions. Another very important aspect of the pion decay process is that, by means of including the QCD corrections, this process could be of help on investigating the physics beyond standard model [11]! The other decay mode $\pi^+ \to \pi^0 e^+ \nu_e$ (or equivalently $\pi^- \to \pi^0 e^- \bar{\nu}_e$), that is called *pion β decay* and their radiative counterparts are also insensitive to the strong interaction uncertainties that makes them important and interesting for the test of SM. In this section we will see how we can predict the electron-muon universality in the SM. The Feynman diagram for the pion (π^-) is shown in Fig.(1).

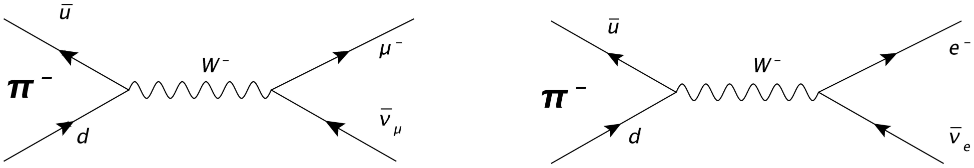


Figure 1: Pion decay: The dominant mode is the one on the left.

Now, the most widely accepted pion decay rate is given by [12]:

$$\begin{aligned}\Gamma\left[\pi^- \to l^- \bar{\nu}_l(\gamma)\right] =& \frac{G_\mu^2 |V_{ud}|^2}{4\pi} f_\pi^2 m_\pi m_l^2 \left[1 - \frac{m_l^2}{m_\pi^2}\right] \left[1 + \frac{2\alpha}{\pi} \ln \frac{M_Z}{\mu}\right] \\ & \left[1 - \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\frac{\mu}{m_\mu} + \bar{C}_1(\mu) + \bar{C}_2(\mu)\frac{m_l^2}{\Lambda_\chi^2}\ln\frac{\mu^2}{m_l^2} + \bar{C}_3(\mu)\frac{m_l^2}{\Lambda_\chi^2} + ...\right\}\right]\left[1 + \frac{\alpha}{\pi}F(x)\right]\end{aligned} \tag{4}$$

We have seen up to the first squire bracket in our assignment and we know how to calculate f_π from using these information. The quantities proportional to α arise from the radiative corrections which includes both EW and QED.These additional contributions contain the constants $\bar{C}_i$s that are unknowns and parameterizes the non-perturbative QCD effects, $\Lambda_\chi =$

$4\pi f_\pi$ is the chiral scale, and $x = m_l^2/m_\pi^2$. The terms of the form $\left[\frac{\alpha}{\pi}\ln\frac{M_Z}{\mu}\right]^n$ are summed in [12] using Renormalization Group (RG) to yield a better estimate for the short distance correction factor $S_W(\mu, M_Z)$ and this replaces $1+\frac{2\alpha}{\pi}\ln\frac{M_Z}{\mu}$. If we choose $\mu = m_\rho$, one gets $S_{EW}(\mu, M_Z) = 1.0232$. In reference [12] the uncertainty in $\Gamma\left[\pi \to \mu\nu(\gamma)\right]$ is also estimated to be ±0.56%. After the process specified in the Ref.[10] and [**?**] the result provided for $f_\pi$is:

$$f_\pi = 92.4 \pm 0.07 \pm 0.25\text{MeV},$$

where the first uncertainty is from the experimental uncertainty in V_{ud} and the second uncertainty is from $\bar{C}_1$. Compared to the decay amplitude of π^- decay, the ratio of the electronic to muonic decay is less sensitive to the strong interactions uncertainties and can provide the interpretable test of the electron-muon universality. So finally we have:

$$\begin{aligned} R_{e/\mu} =& \frac{\Gamma\left[\pi^- \to e^-\bar{\nu}_e(\gamma)\right]}{\Gamma\left[\pi^- \to \mu^-\bar{\nu}_\mu(\gamma)\right]} \\ =& \frac{m_e^2}{m_\mu^2}\left[\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right]^2 \left\{1 + \frac{\alpha}{\pi}\left[F\left(\frac{m_e}{m_\pi}\right) - F\left(\frac{m_\mu}{m_\pi}\right) + \frac{m_\mu^2}{\Lambda_\chi^2}\left(\bar{C}_2\ln\frac{m_\mu^2}{\Lambda_\chi^2} + \bar{C}_3\right)\right]\right\}, \end{aligned} \tag{5}$$

where the terms proportional to αm_e^2 were dropped. After considering all the corrections the SM (theoretical) value for the ratio $R_{e/\mu}$ was calculated to be [12]:

$$R_{e/\mu}^{SM} = (1.2352 \pm 0.0005) \times 10^{-4} \tag{6}$$

. In the next section we will see how the experiment was performed at TRIUMF, what result they got and to what extent the experimental finding matches to the theoretical prediction made in this section.

Experimental Setup

The schematic diagram of the experimental setup is show in Fig.(2). Here I will give a very brief description of the experimental setup that was used in the experiment (for details please check [13]). The TRIUMF cyclotron's energetic proton beam (500 MeV, 100 μA) hits a graphite target (1cm thick) that produces a π^+ beam (with momentum 83±1 MeV). This beam is rich in pions (80% pions, 10% positrons, and 10% muons). Each species (pions, muons and positron) are distinguishable by their characteristic TOFs. The incoming beam is detected with plastic scintillation counter $B1$ and transversely defined with VL and VR. Pions were stopped in the target counter assembly which included plastic scintillator $B3$, $B2$, $B4$, LL, LR counters. The whole assembly was tilted by an angle of 45^0 with respect to the beam. Particles that were able to go through the assembly were detected at $V0$. Muons that are the decay product of π^+ (stopped) were fully confined within the target counter array. Positrons from the decays of stopped pions in the target were detected at 90^0 to the beam, passing through the B_4 counter two planer wire chambers for positron measurement and a telescope four plastic counters $T1 - T4$ of different thickness. The positron energy was analyzed by crystal TINA that also detected additional charged particles entering into it. Robust calculations and smart tricks were applied to extract the information from the data collected including the idea of "binning" [13]. Fig. 3 shows the energy spectra of the

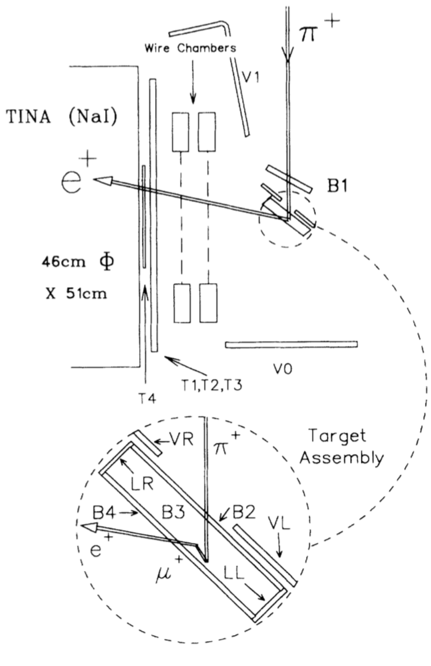


Figure 2: Experimental setup (taken from [13]).

positron. To eliminate the background events, cut was applied without causing dependence on the positron energy.

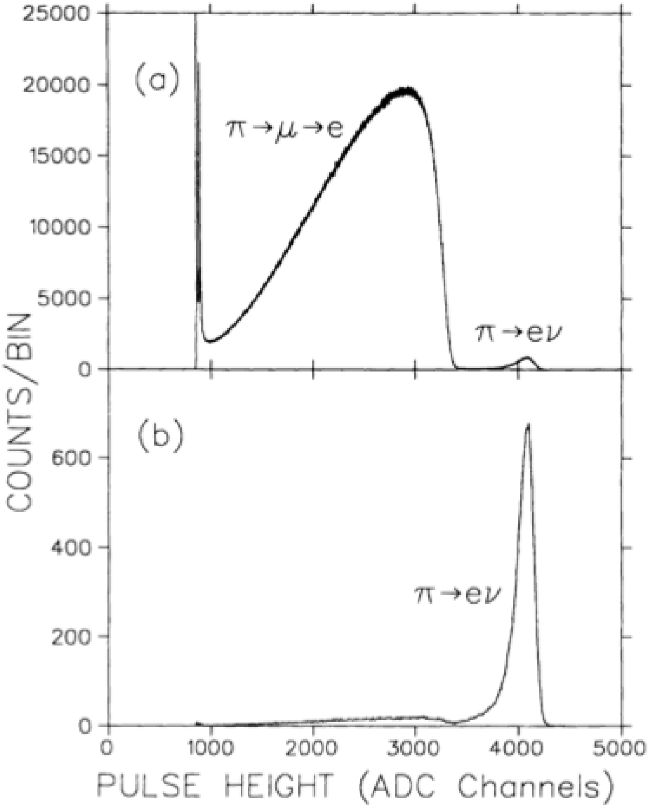
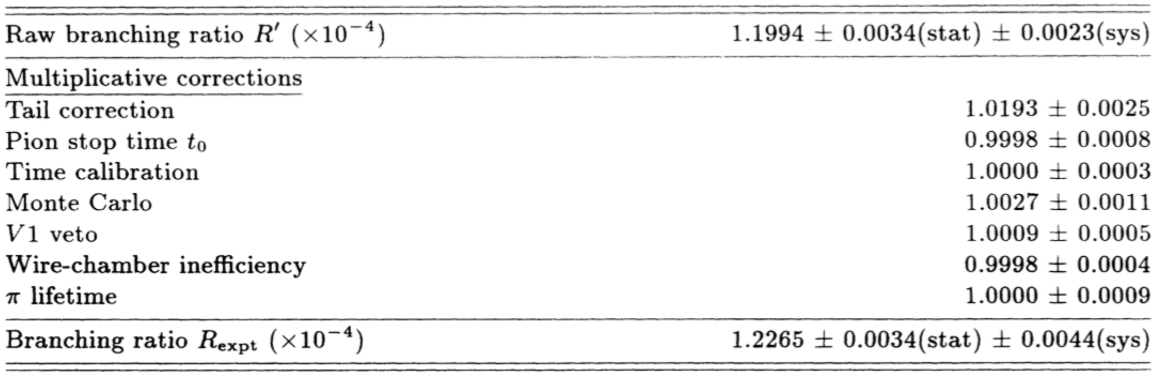


Figure 3: Energy spectrum: (a) Positron energy spectrum or early time (¡30 ns). The two peaks at low energy are due to the zero-energy pedestal and the 0.511 MeV annihilation γ rays from low energy positrons which satisfy the trigger requirement but do not penetrate TINA. The two counts for 0.7% of the $\pi \to \mu \to e$ counts. (b) Positron spectrum after suppression technique described in [13]. (taken from [14]).

From the decay $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu \to e^+$ one can calculate the branching ratio R_{exp}. According to the Ref.[**?**], this has some advantages over the previously done tests, one can remain free of any worry about different normalization factors, such as the solid angle of position detection and the number of incident pion stops, cancel in the first order and only small energy dependent effects like those due to multiple Coulomb scattering and positron annihilation need to be corrected for. Simultaneous fitting of the time distributions of low and high energy regions provides the yields of $\pi^+ \to \mu^+ \to e^+$ and $\pi^+ \to e^+\nu$. By low-energy decay we mean the decay $\pi \to \mu \to e$ and by high-energy decay we mean $\pi \to e\nu$. In this experiment, the group was able to suppress the events with additional energy in stopping counter due to π → μ which was a problem in the previous experiments. The PIENU experiment [14] at TRIUMF measured:



Raw branching ratio R' ($\times 10^{-4}$)	1.1994 ± 0.0034(stat) ± 0.0023(sys)
Multiplicative corrections	
Tail correction	1.0193 ± 0.0025
Pion stop time t_0	0.9998 ± 0.0008
Time calibration	1.0000 ± 0.0003
Monte Carlo	1.0027 ± 0.0011
$V1$ veto	1.0009 ± 0.0005
Wire-chamber inefficiency	0.9998 ± 0.0004
π lifetime	1.0000 ± 0.0009
Branching ratio R_{expt} ($\times 10^{-4}$)	**1.2265 ± 0.0034(stat) ± 0.0044(sys)**

Figure 4: Summery of the $\pi \to e\nu$ branching ratio (taken from [13]).

$$R^{exp}_{e/\mu} = 1.2265 \pm 0.0034 \pm 0.0044, \tag{7}$$

where the first uncertainty is statistical and the second uncertainty is systematic. After the taking the ratio of the the experimental and the theoretical values we get:

$$\frac{R^{exp}_{e/\mu}}{R^{SM}_{e/\mu}} = 0.9966 \pm 0.0030 \pm 0.0004, \tag{8}$$

where the first uncertainty is experimental and the second is theoretical.

Equation (8) shows the excellent agreement of the experiment with the theoretical prediction made by the SM.

Conclusion

In this project I have tried to show the link between what we learned in the class and how the tests are performed in an experiment. Although we saw that the SM is in excellent agreement with the experimental finding. However, it is possible to go for more precise calculation in the theory and do more accurate tests to check if the theory breaks down at a certain level of accuracy. Despite all the success and being simple, SM has some limitations that fuels the searches of any physics beyond the SM, instability of the EW scale, the absence of gravity, violation of discrete symmetry are some of them. However, as mentioned in the text, we can say, the SM is at least a subset of the true theory of particle physics but hopefully not wrong.

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