Notes #2: Leptogenesis

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Leptogenesis is a class of mechanisms for baryogenesis in which a lepton asymmetry is created and then partially reprocessed into a baryon asymmetry by electroweak sphalerons [1, 2]. A very attractive feature of many leptogenesis mechanisms is that they connect the observed baryon asymmetry to the origin of neutrino masses and mixings. In these notes we provide a brief overview of the most simple realization of leptogenesis through the decay of the very heavy right-handed neutrinos in the Type-I seesaw model for the light neutrino masses. More detailed reviews of leptogenesis can be found in Refs. [3, 4, 5, 6, 7].

1 Neutrino Masses and Mixings

Neutrinos (and antineutrinos) in the minimal SM are predicted to be massless and to have one of three definite flavours corresponding to the e, μ, and τ charged leptons. There is no leptonic equivalent of the CKM matrix in the SM, and lepton flavour is predicted to be conserved. However, detailed measurements of neutrinos have detected the phenomenon of *neutrino oscillations*, in which a neutrino of one flavour transforms into another. These oscillations are definitive proof of new physics beyond the SM, and they imply further that at least some of the SM neutrinos have mass [8, 9, 10].

In contrast to the other SM fermions, neutrinos interact exclusively through the weak vector bosons. This makes them much harder to detect than the other SM fermions, and allows them to travel very long distances through matter without being scattered. The flavour of a neutrino when it is produced is deduced from the flavour of the charged lepton that is created (or decayed) along with it. Similarly, neutrinos are "detected" when they scatter with other matter, often in conjunction with a charged lepton. Neutrino oscillations are observed in a difference between the neutrino flavour at detection relative to production. This can be understood as a misalignment of the flavour eigenstates associated with the charged lepton at production and decay and the energy eigenstates with definite mass.

The relationship between the flavour and mass eigenstates is given by the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix,

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} . \tag{1}$$

It is conventional to decompose this matrix according to

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

This decomposition is useful because the various mixing angles have been measured in different systems [11]. The $\theta_{12} \simeq 35^\circ$ angle is determined best from measurements of neutrinos emitted by nuclear reactions in the sun, and is sometimes called the solar mixing angle. Neutrinos obtained from cosmic ray showers in the atmosphere gave the first good determination of $\theta_{23} \simeq 45^\circ$, and it is sometimes called the atmospheric mixing angle. Recent measurments of neutrinos produced in nuclear reactors have yielded $\theta_{13} \simeq 14^\circ$. Oscillation measurements also give values for the mass differences of neutrinos, with

$$\Delta m_{12}^2 \simeq 7.6 \times 10^{-5}\,\mathrm{eV}^2\ , \qquad |\Delta m_{23}^2| \simeq 2.4 \times 10^{-3}\,\mathrm{eV}^2\ . \tag{3}$$

There is also a limit on the sum of the SM neutrino masses from cosmological observations of $\sum_i m_i \lesssim 0.2\,\mathrm{eV}$ [16]. Compared to the other fermions of the SM, the neutrinos are orders of magnitude lighter, and their mixings are significantly larger than those of the CKM matrix.

Neutrino masses and mixings require new physics beyond the SM (as we have defined it). The easiest way to generate them is to add three gauge-singlet right-handed neutrinos $N_{Ri} = (1,1,0)$, $i = 1,2,3$, with the Yukawa couplings

$$-\mathscr{L}\ \supset\ -\lambda_{Ai}\,\overline{N}_{Ri}\,H\!\cdot\! L_A + (h.c.)\ , \tag{4}$$

where $H = (H^+, H^0)^t$, $L_A = (\nu_{L\,A}, e_{L\,A})^t$, and $A\!\cdot\! B = \epsilon_{ab}A^aB^b$ with $\epsilon_{12} = +1$ for any pair of $SU(2)_L$ doublets. After electroweak symmetry breaking, $H \to (0, v + h/\sqrt{2})^t$, this generates a neutrino mass matrix with entries

$$(m_\nu)_{AB} = (\lambda\, v)_{AB}\ . \tag{5}$$

In the end, we get three massive Dirac neutrinos and a mixing matrix connecting them to the charged leptons via the W boson. However, given the extreme smallness of the observed neutrino masses, many consider this solution unsatisfying on its own.

A popular variation on the simple picture above is called the Type-I neutrino seesaw. Since the N_R are gauge singlets, we can also add (diagonal) Majorana masses for them of the form

$$-\mathscr{L}\ \supset\ \frac{1}{2}M_i\,\overline{(N_{Ri}^c)}N_{Ri} + (h.c.)\ , \tag{6}$$

where $N_R^c = -i\gamma^2\gamma^0\overline{N}_R^t$. Combined with the Dirac mass term of Eq. (4), the full neutrino mass matrix takes the schematic form

$$M_\nu = \begin{pmatrix} 0 & \lambda v \\ \lambda v & M_N \end{pmatrix}\ . \tag{7}$$

For $M_N \gg y_N v$, the mass eigenstates then consist of six Majorana fermions with mass eigenvalues of the form

$$m_\nu \simeq \frac{(\lambda v)^2}{M_N}\ , \quad M_N\ . \tag{8}$$

The three light states are identified with the SM neutrinos, while the three heavy neutrinos are mostly singlets and very difficult to detect. For $\lambda \sim 1$, the SM-like neutrinos have sub-eV masses for $M_N \sim 10^{13}$ GeV.

It is also illuminating to look at the EFT obtained by integrating out the very massive right-handed neutrinos. The leading operator generated from doing so is

$$-\mathscr{L}_{EFT} \supset \sum_i \frac{\lambda_{Ai}\lambda_{Bi}}{2M_i}(\overline{L^c_A}\cdot H)(L_B\cdot H) + (h.c.) \ . \tag{9}$$

This is the lowest-dimensional non-renormalizable operator that can be built out of SM fields alone. After electroweak symmetry breaking, it generates neutrino masses on the order of $m_\nu \sim \lambda^2 v^2/M_N$, as expected from the neutrino seesaw.

2 Decay Asymmetries

The most popular model of leptogenesis relies on the decays of the heavy right-handed neutrinos in the Type-I seesaw model to create a lepton asymmetry in the early universe. Creating such an asymmetry requires both C and CP violation. In this section we investigate the decay asymmetry due to the lightest of the heavy neutrinos, N_i, and we show its connection to C and CP.

The partial asymmetry created by N_i decays in lepton flavour A is defined to be

$$\varepsilon_{1A} = \frac{\Gamma(N_i \to \ell_A + H) - \Gamma(N_i \to \bar{\ell}_A H^\dagger)}{\sum_B \left[\Gamma(N_i \to \ell_B + H) + \Gamma(N_i \to \bar{\ell}_B H^\dagger)\right]} \ . \tag{10}$$

The asymmetry can be computed in terms of Feynman diagrams in an expansion in the (assumed to be small) Yukawa couplings λ_{iA}. In Fig. 1 we show the tree-level and one-loop diagrams for $N_i \to \ell_A H$ (taken from Ref. [7]). These quantities can all be computed by brute force and used to find the decay asymmetry explicitly. However, it is instructive to delve into their general form.

Consider the matrix elements correponding to the diagrams in Fig. 1 as well as for the conjugate process. These can be written in the schematic form [12][1]

$$\begin{aligned} \mathcal{M}(N_i \to \ell_A H) &= c_0 + c_1\mathcal{F}_1 \\ \mathcal{M}(N_i \to \bar{\ell}_A H^\dagger) &= c_0^* + c_1^*\mathcal{F}_1 \ , \end{aligned} \tag{11}$$

where the indices refer to the tree-level and one-loop pieces, and $c_0 \propto \lambda$ and $c_1 \propto \lambda^3$ contain all the factors of the couplings. This form implies

$$|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2 = -4\,\mathrm{Im}(c_0 c_1^*)\,\mathrm{Im}(\mathcal{F}_1) \ , \tag{12}$$

and thus a non-zero asymmetry requires interference between tree- and loop-level contributions. The first factor contains phases in the couplings that violate C and CP, as expected

[1] The non-schematic form has external spinors that give the same result when summed and squared.

on general grounds. However, we also see that a CP conserving phase is needed in the one-loop $\mathcal{F}_1$ factor above.

The origin of the phase in the $\mathcal{F}_1$ can be understood from the unitarity of the S matrix [13]. Recall that for asymptotic *in* and *out* states $|i\rangle$ and $\langle f|$ it is defined by

$$\langle f|\, S\, |i\rangle \equiv S_{fi} \ . \tag{13}$$

It is conventional to write $S = \mathbb{I} - iT$, in which case we identify

$$\langle f|\, T\, |i\rangle \equiv T_{fi} = \mathcal{M}(i \to f) \ , \tag{14}$$

where $\mathcal{M}(i \to f)$ is the matrix element for the $i \to f$ process. Unitarity of S implies

$$i(T - T^\dagger) = T^\dagger T \ . \tag{15}$$

Sandwiching this relation between arbitrary initial and final states, one finds

$$i\left[\mathcal{M}(i \to f) - \mathcal{M}^*(f \to i)\right] = \sum_{\{n\}} \langle f|\, T^\dagger\, |n\rangle \, \langle n|\, T\, |i\rangle \ \ , \tag{16}$$

where the sum runs over all possible intermediate states. In a weakly-coupled QFT, these intermediate states correspond primarily to on-shell intermediate particles. In the present case, we are interested in the decay of N_i with $|i\rangle = |N_i\rangle$ and $\langle f| = \langle \ell_A, H|$.[2] In the limit of real couplings, we can identify the left-hand side of Eq. (16) with $\mathrm{Im}(\mathcal{F}_1)$ at leading non-zero order. The right-hand side of this equation then shows that the phase in $\mathcal{F}_1$ arises from intermediate states within the loops going on-shell, corresponding to $|n\rangle = |\ell_B, H\rangle$ and $|\bar{\ell}_B, H^\dagger\rangle$, and this phase does not require CP violation.

A full calculation of the asymmetry gives the leading result [6, 7]

$$\begin{aligned} \varepsilon_{iA} \ &= \ \frac{1}{8\pi}\frac{1}{(\lambda^\dagger\lambda)_{ii}}\left(\sum_j \mathrm{Im}\left[\lambda^*_{Ai}(\lambda^\dagger\lambda)_{ij}\lambda_{Aj}\right] g(x_{ji})\right. \\ &\qquad\qquad \left. + \ \sum_j \mathrm{Im}\left[\lambda^*_{Ai}(\lambda^\dagger\lambda)_{ji}\lambda_{Aj}\right] \frac{1}{1-x_{ji}}\right) \ , \end{aligned} \tag{17}$$

where $x_{ji} = M_j^2/M_i^2$ and

$$g(x) = -\sqrt{x}\left[\frac{2}{x-1} + \ln(1-1/x)\right] \ . \tag{18}$$

This result has a number important features. First, the combinations of couplings are genuinely CP violating in that they cannot be removed by field redefinitions. Second, both coupling combinations vanish for $i = j$. Third, the second term above vanishes when summed over A and is said to be L conserving but flavor violating.

[2] There is a slight subtlety here in that N_i is unstable and therefore not a true asymptotic state [6].

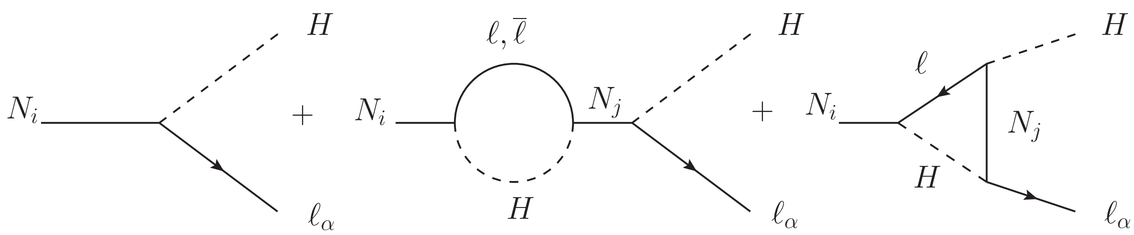

Figure 1: Leading diagrams contribution to the decay asymmetry of the heavy right-handed neutrino state N_i (from Ref. [7]).

3 Cosmological Lepton and Baryon Production

With decay asymmetries in hand, we turn next to the cosmological processes that create and destroy lepton and baryon asymmetries in the Type-I seesaw model. Production of L occurs through the CP-violating decays of the heavy right-handed neutrinos, which is then converted partially to B by sphaleron transitions. At the same time, inverse decay and scattering processes tend to erase the L charge created in the decays, and a departure from thermodynamic equilibrium is needed to avoid this. In this section we examine the redistribution of B and L charges by electroweak sphalerons in full equilibrium, and we investigate the cosmological time evolution of these and other charges away from equilibrium.

3.1 Sphaleron Redistribution

As discussed previously, electroweak sphalerons transitions violate $(B+L)$. Recall that in the SM, these are active with an effective rate greater than Hubble for $T \in [130,\, 10^{12}]$ GeV. This range covers most of the temperature range relevant for leptogenesis, and the sphaleron transitions play an essential role in transforming the L charge created into a B charge. We study how this works in full equilibrium here.

Recall that at high temperatures $T \gg m_i$, the number density asymmetry of a particle species ψ in equilibrium is given by

$$n_\psi - n_{\bar{\psi}} \;\simeq\; \left\{ \begin{array}{lll} g_\psi \mu_\psi T^2/6 & ; & \psi = \text{fermion} \\ g_\psi \mu_\psi T^2/3 & ; & \psi = \text{boson} \end{array} \right. , \tag{19}$$

where g_ψ the number of internal degrees of freedom (like spins and colours). It follows that we can keep track of particle asymmetries through their chemical potentials. Now, thermodynamic equilibrium also implies that the chemical potentials are constrained by the set of fast reactions that can occur. This can be used to compute the final B and L charges resulting from an instantaneous injection of L (or other) charge [4, 14].

At the temperatures relevant for leptogenesis, the SM gauge interactions are in equilibrium with $\mu_Y = \mu_W = \mu_g = 0$ for the corresponding vector bosons. This implies that all the components of a given gauge multiplet have the same chemical potential. Thus, we will write

$$\mu_{Q_i} = \mu_{u_{L\,i}} = \mu_{d_{L\,i}} \ , \qquad \mu_{\ell_i} = \mu_{\nu_{L\,i}} = \mu_{e_{L\,i}} \ , \tag{20}$$

where i labels the generation number. Applying this to the SM Yukawa interactions,[3]

$$0 \ = \ \mu_{Q\,i} + \mu_H - \mu_{u\,j} \ = \ \mu_{Q\,i} - \mu_H - \mu_{d\,j} \ = \ \mu_{L\,i} - \mu_H - \mu_{e\,i} \tag{21}$$

where μ_{u_i}, μ_{d_i}, and μ_{e_i} refer to the right-handed fermions. Hypercharge neutrality of the universe (corresponding to electric charge neutrality) implies

$$0 \ = \ \sum_i \left[6\left(\frac{1}{6}\right)\mu_{Q_i} + 3\left(\frac{2}{3}\right)\mu_{u_i} - 3\left(\frac{1}{3}\right)\mu_{d_i} - 2\left(\frac{1}{2}\right)\mu_{\ell_i} - \mu_{e_i} + \frac{2{\cdot}2}{n_g}\left(\frac{1}{2}\right)\mu_H\right] \ , \tag{22}$$

where $n_g = 3$ is the number of SM generations and the extra factor of $2/n_g$ for the Higgs comes from the fact there is only one Higgs copy and the factor of two for bosons in Eq. (19). Finally (and most importantly), there are additional relations implied by the sphaleron transitions. For the $SU(2)_L$ sphalerons, we have

$$0 \ = \ \sum_i (3\mu_{Q_i} + \mu_{\ell_i}) \tag{23}$$

while for the strong $SU(3)_c$ sphalerons we get

$$0 \ = \ \sum_i (2\mu_{Q_i} - \mu_{u_i} - \mu_{d_i}) \ . \tag{24}$$

These relations can be used to relate B and L in equilibrium.

Let us now define baryon and lepton chemical potentials by[4]

$$n_B \ = \ \mu_B T^2/6 \ , \qquad n_L \ = \ \mu_L T^2/6 \ , \tag{25}$$

where n_B and n_L refer to the charge density *asymmetries*. It follows that

$$\mu_B = \sum_i (2\mu_{Q_i} + \mu_{u_i} + \mu_{d_i}) \ , \qquad \mu_L = \sum_i (2\mu_{\ell_i} + \mu_{e_i}) \ . \tag{26}$$

Combining these definitions with the relations above and solving, the relationship between charges in full SM equilibrium is

$$\mu_B \ = \ c_s(\mu_B - \mu_L) \tag{27}$$

$$\mu_L \ = \ (c_s - 1)(\mu_B - \mu_L) \ , \tag{28}$$

with $c_s = (8n_g + 4)/(22n_g + 13) \stackrel{n_g \to 3}{\longrightarrow} 28/79$. Combined with Eq. (25), this result implies that that final B density after equilibration is proportional to the total $(B - L)$ charge For leptogenesis, suppose the instanteneous decay of a heavy neutrino creates a lepton density asymmetry yield of $Y_L(t_i)$. Equilibration by sphaleron and other processes after the decay will then produce a final baryon asymmetry yield today ($t = t_0$) equal to

$$Y_B(t_0) \simeq -c_s Y_L(t_i) \ . \tag{29}$$

Let us also point out that SM interations do not change $(\mu_B - \mu_L)$

[3] The equilibration of the light fermion Yukawas is not guaranteed during leptogenesis and can interesting effects, but we neglect this here.

[4] The notation $B \equiv \mu_B$ and $L \equiv \mu_L$ is often used.

3.2 Cosmological Density Evolution

Successful baryogenesis requires a departure from thermodynamic equilbrium, and for leptogenesis this enters in the production and decay of the heavy neutrinos. This can be described by a set of semi-classical Boltzmann equations for the evolution of the N_i and $(B-L)$ number densities. To simplify the presentation, we focus on the scenario where $M_1 \ll M_2$, M_3 and N_1 gives the dominant contribution to the lepton asymmetry.

The approximate Boltzmann equations for the system are [3, 4, 5, 6, 7]

$$\frac{dY_{N_1}}{dz} = -(D+S)(Y_{N_1} - Y_{N_1}^{eq}) \,, \tag{30}$$

$$\frac{dY_{B-L}}{dz} = -\varepsilon_1 D(Y_{N_1} - Y_{N_1}^{eq}) - W Y_{B-L} \,, \tag{31}$$

where $z = M_1/T$, $Y_i = n_i/s$, $s = 2\pi^2 g_{*S}T^3/45$, $\varepsilon_1 = \sum_A \varepsilon_{1A}$ is the total decay asymmetry, and $Y_{N_1}^{eq}(z)$ is the equilibrium yield of N_1. The D term in these equations is an effective decay rate of N_1, S corresponds to $\Delta L = 1$ scattering processes, and W term is the rate of lepton number washout by inverse decays and scattering processes.

At leading order in the couplings, the D term is given by [7]

$$D(z) = z\left\langle\frac{1}{\gamma}\right\rangle \frac{\Gamma_1}{H(z=1)} \tag{32}$$

where $\langle 1/\gamma\rangle = \langle M_1/E_1\rangle$ is a thermal average of the time dilation factor for the decay with numerical value

$$\left\langle\frac{1}{\gamma}\right\rangle \simeq \frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)} \simeq \left\{\begin{array}{ccc} 1 & ; & z \gg 1 \\ z/2 & ; & z \ll 1 \end{array}\right. \,, \tag{33}$$

where $\mathcal{K}_\alpha(z)$ is the modified Bessel function of order α. In Eq. (30), the D term describes the change in number density of N_1 from decays. Note that if $Y_{N_1} < Y_{N_1}^{eq}$, inverse decays of the form $\ell + H \to N_1$ and $\bar{\ell} + H^\dagger \to N_1$ dominate over regular decays and tend to increase the N_1 density. A similar expression weighted by ε_1 appears in Eq. (31) since N_1 decays create a net lepton number by their decay asymmetry. We will neglect the scattering S term in the discussion to follow.

For the washout term W, we will focus exclusively on the leading contribution (in the couplings) from inverse decays, although it can also receive significant contributions from $\Delta L = 1$, 2 scattering processes. The explicit expression for W is then [7]

$$W(z) = \frac{1}{2} D(z)\,\frac{Y_{N_1}^{eq}(z)}{Y_\ell^{eq}} \,. \tag{34}$$

Washout of lepton number from inverse decays occurs because a positive asymmetry in L implies there are more ℓH states than $\bar{\ell}H^\dagger$ states in the plasma, and thus the process $\ell + H \to N_1$ is more likely to occur than $\bar{\ell} + H^\dagger \to N_1$.

A formal solution for the $(B-L)$ density is [4, 7]

$$Y_{B-L}(z) \;=\; Y_{B-L}(z_i)\, e^{-\int_{z_i}^{z} dz'\, W(z')} - \int_{z_i}^{z} dz'\, \varepsilon_1 D(Y_{N_1} - Y_{N_i}^{eq})\, e^{-\int_{z'}^{z} dz''\, W(z'')} \;, \tag{35}$$

where z_i is where we start the evolution. The first term describes the washout of an initial asymmetry, while the second gives the asymmetry production from N_1 decays away from equilibrium. Let us now set $z_i \to 0$, and take $Y_{B-L}(z_i \to)$ since we expect zero initial asymmetry. Turning to the second term, simple approximate solutions can be found in the *weak washout* ($K \ll 1$) and *strong washout* ($K \gg 1$), where the parameter K is defined by

$$K = \frac{\Gamma_{N_1}}{H(z=1)} \;. \tag{36}$$

It is instructive to look at both limits.

For weak washout, we have slow decays at $T = M_1$ implying $\exp \int W \to 1$. The integrand of the second term of Eq. (35) then turns into a total derivative via Eq. (30), giving [4, 7]

$$Y_{B-L}(z \to \infty) \;\simeq\; \varepsilon_1 Y_{N_1}(0) \;. \tag{37}$$

Note that this requires an initial density of $Y_{N_1}(0)$, and is therefore sensitive to initial conditions. However, for $Y_{N_1}(0) \to 0$ there is still a subleading contribution (beyond our level of approximation) to the asymmetry from N_1 created by inverse decays, given by [4, 7]

$$Y_{B-L}(z \to \infty) \;\simeq\; \varepsilon_1 K^2 Y_{N_1}^{eq}(0) \qquad (\text{for } Y_{N_1}(0) \to 0) \;. \tag{38}$$

In the regime of strong washout, the relation $K > 1$ implies that N_1 decays are fast compared to the Hubble rate at $T = M_1$. This implies that the N_1 density will track its equilibrium value closely until the inverse decays turn off due to kinematics at some value of $z = z_f > 1$, corresponding to $W(z_f) \to 1$. Taking Eq. (30) and solving in an expansion in $\Delta Y_{N_1} = Y_{N_1} - Y_{N_1}^{eq}$, the leading term in an expansion in $(1/K)$ is [4, 7]

$$\Delta Y_{N_1}(z) \;\simeq\; \frac{1}{D}\frac{dY_{N_1}^{eq}}{dz} \;, \tag{39}$$

where we have equated $Y_{N_1} \to Y_{N_1}^{eq}$ in the derivative term since it is not enhanced by a factor of K. Going to Eq. (31) and similarly setting $Y_{B-L} \to Y_{B-L}^{eq} = 0$ in the derivative, the result is

$$\begin{aligned} Y_{B-L}(z \to \infty) \;&\simeq\; -\varepsilon \frac{1}{W}\frac{dY_{N_1}^{eq}}{dz} \\ &\simeq\; -\frac{\pi^2}{6 z_f K}\varepsilon_1 Y_{N_1}^{eq}(0) \;. \end{aligned} \tag{40}$$

In the above, we have set $Y_{B-L}(z \to \infty) \simeq Y_{B-L}(z_f)$, the point at which inverse decays cease and the N_1 density deviates from equilibrium. Note that this result is largely independent of initial conditions.

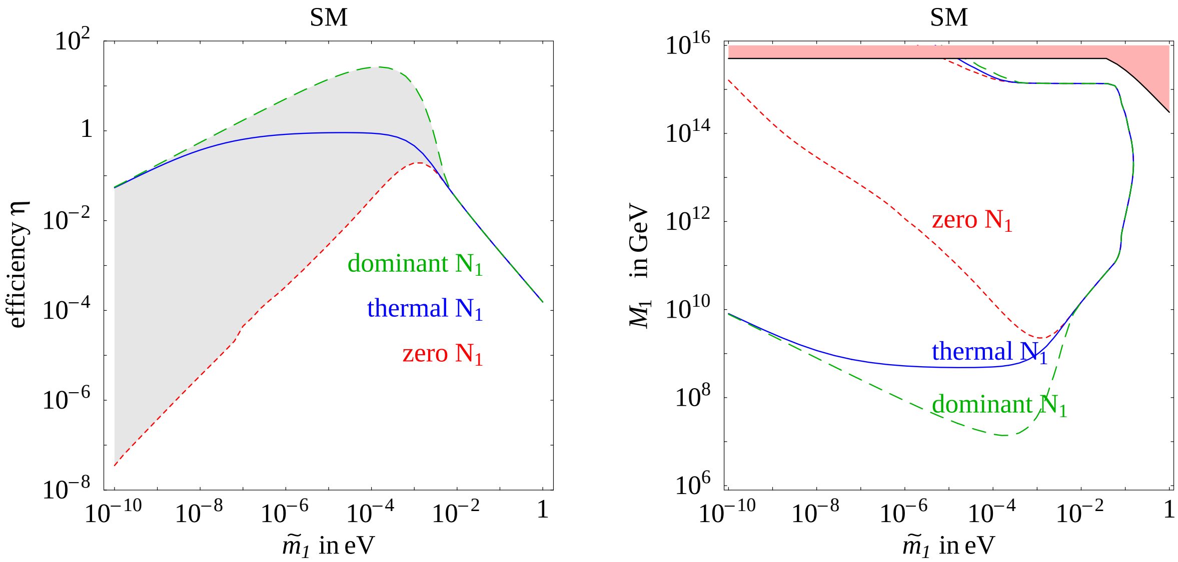

Figure 2: **Left:** Efficiency factor $\kappa = \eta$ for standard leptogenesis in the Type-I seesaw model as a function of $\widetilde{m}_1/\mathrm{eV} = (2.3 \times 10^{-3})\,K$ assuming either a dominant, thermal, or zero initial N_1 density (from Ref. [10]). **Right:** Ranges of $\widetilde{m}_1/\mathrm{eV}$ and M_1 (within the boundary curves) consistent with the observed baryon asymmetry from leptogenesis (from Ref. [10]).

Once the yield $Y_{B-L}(z \to \infty)$ has been computed, the baryon yield can be obtained from the result of Eq. (27). Converting to a fraction relative to photons, the baryon asymmetry today from leptogenesis is often written in the form

$$\eta \equiv \frac{n_B}{n_\gamma} \simeq -\kappa\varepsilon_1 \; , \tag{41}$$

where κ is called the *efficiency factor*. The parameter ε_1 corresponds to C and CP violation, and κ measures the degree of departure from equilibrium. We show a plot of κ (labelled as η) in the left panel of Fig. 2 (from Ref. [10]) as a function of $\tilde{m}_1 \equiv (2.3 \times 10^{-3}\,\mathrm{eV}) \times K$. The shaded grey band indicates the range the efficiency factor can take in the weak washout regime.

Let us also emphasize that our treatment of leptogenesis has concentrated on the basic story. Many other effects that we have neglected can modify the final baryon density (typically by an order unity amount). These include thermal corrections, flavor effects, resonance effects, and much more [3, 4, 5, 6, 7].

3.3 Connection to Neutrino Observables

Leptogenesis is a very attractive mechanism for baryogenesis because it connects the source of the baryon asymmetry to the new physics underlying neutrino oscillations. Despite this connection, the new right-handed neutrinos responsible for standard leptogenesis must typically be very heavy if they are to generate the observed baryon density. Furthermore, low-energy measurements of neutrino properties at lower energies, such as masses and mixings via neutrino oscillations and other observables, are not enough to fully characterize the massive neutrino theory.

A counting of independent parameters in the standard Type-I seesaw model (with three heavy neutrinos) gives 21 independent parameters in the lepton sector [6, 10]. In contrast, there are twelve independent parameters in the lepton sector of the SM effective theory valid well below the heavy neutrino masses. These can be taken to be [6]

$$m_e,\, m_\mu,\, m_\tau,\, m_1,\, m_2,\, m_3,\, s_{12},\, s_{13},\, s_{23},\, \delta_{CP},\, \alpha_1,\, \alpha_2\ . \tag{42}$$

In fact, not all of the low-energy parameters have been measured yet! Many different parametrizations of the full 21 lepton-sector parameters exist. In general, it can be shown that a full knowledge of the low-energy parameters is not enough to completely fix the predictions of the theory for leptogenesis.

Even though we are not able to make unambigous predictions for leptogenesis in the Type-I seesaw theory with only low-energy data, it is possible to place a lower bound on the mass scale of the heavy neutrinos if they are to create the full baryon asymmetry (assuming it is dominated by N_1 with much heavier non-degenerate N_2 and N_3) [15]. Comparing the expression for light neutrino masses with that for the asymmetry due to N_1, the *Davidson-Ibarra* bound can be derived,

$$|\varepsilon_1| \;\leq\; \frac{3}{16\pi}\frac{M_1}{v^2}(m_{max}-m_{max}) \;=\; \frac{3}{16\pi}\frac{M_1}{v^2}\left(\frac{|\Delta m^2_{23}|}{m_{max}+m_{min}}\right) \tag{43}$$

where m_{max} and m_{min} are the largest and smallest of the SM-like neutrino masses and $|\Delta m^2_{23}|^2$ is the largest of the observed mass differences. Given the cosmological upper bound on the sum of light neutrino masses, $\sum_i m_i \lesssim 0.2\,\mathrm{eV}$ [16], this relation implies a lower bound on the heavy mass M_1 if leptogenesis is to be large enough to create the full baryon asymmetry [15]. The precise bound depends on the efficiency factor defined in Eq. (41), but is typically $M_1 \gtrsim 10^8$ GeV. This relation can also be used to derive an upper bound on the combination of neutrino parameters

$$\widetilde{m}_1 \;\equiv\; \frac{v^2}{M_1}\sum_A |\lambda_{A1}|^2 \;=\; 8\pi\frac{v^2}{M_1}\Gamma_{N_1}\ . \tag{44}$$

It can be shown that this quantity is greater than the smallest SM-like neutrino mass, $\widetilde{m} \geq m_{min}$[15]. The range of $\widetilde{m}$ and M_1 consistent with standard Type-I seesaw leptogenesis is shown in the right panel of Fig. 2 (from Ref. [10]). Let us emphasize that these limits apply to the specific scenario we have presented, and can be weakened in other realizations of leptogenesis.

4 Other Realizations of Leptogenesis

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References

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