Collective neutrino oscillations in core-collapse supernovae

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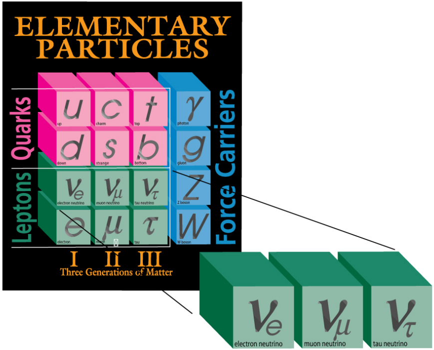
A. de Gouvêa and S. Shalgar, JCAP **1210**, 027 (2012) [arXiv:1207.0516 [astro-ph.HE]] and unpublished results

Introduction (Neutrinos in standard model)



Are neutrinos their own anti-particles?

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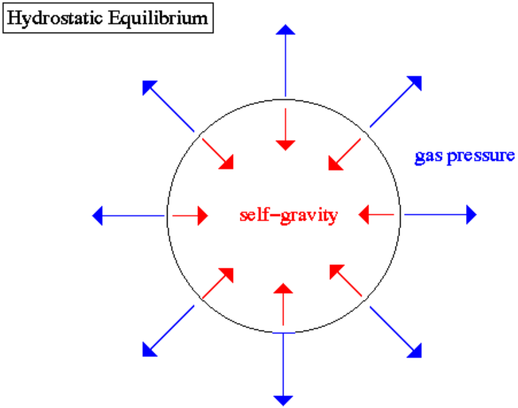
Introduction (Stellar equilibrium)



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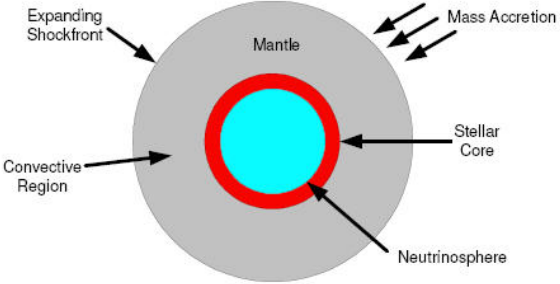
Introduction (Core collapse supernovae)



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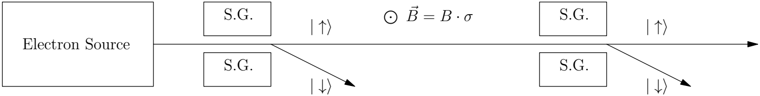
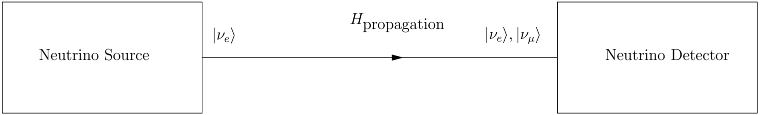


$$P(|\uparrow\rangle \rightarrow |\downarrow\rangle) \neq 0 \Rightarrow [S_Z, B \cdot \sigma] \neq 0$$



$$P(|\nu_e\rangle \rightarrow |\nu_\mu\rangle) \neq 0 \Rightarrow [H_{\text{weak}}, H_{\text{propagation}}] \neq 0$$

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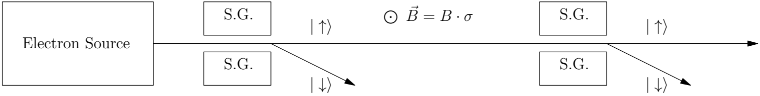
Equations of motion



$$\dot{\vec{S}} = g\vec{S} \times \vec{B}$$

We can express this equation in terms of density matrix

$$\rho = \frac{1}{2}\left(1 + \vec{\sigma} \cdot \vec{S}\right)$$

$$i\dot{\rho} = [H, \rho]$$

Equations of motion

$$\rho = \begin{pmatrix} \langle \psi^*_{\nu_e} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_e} \psi_{\nu_\mu} \rangle \\ \langle \psi^*_{\nu_\mu} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_\mu} \psi_{\nu_\mu} \rangle \end{pmatrix} \quad \rho^c = \begin{pmatrix} \langle \psi^{c*}_{\nu_e} \psi^c_{\nu_e} \rangle & \langle \psi^{c*}_{\nu_e} \psi^c_{\nu_\mu} \rangle \\ \langle \psi^{c*}_{\nu_\mu} \psi^c_{\nu_e} \rangle & \langle \psi^{c*}_{\nu_\mu} \psi^c_{\nu_\mu} \rangle \end{pmatrix}$$

$$\rho(t) = e^{-iHt} \rho(0) e^{iHt}$$

$$H = \begin{pmatrix} -\omega \cos(2\theta) & \omega \sin(2\theta) \\ \omega \sin(2\theta) & \omega \cos(2\theta) \end{pmatrix} \quad \omega = \frac{m_2^2 - m_1^2}{2E}$$

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta) sin^2 \left(\frac{\omega}{2} L \right)$$

P independent of sign of ω

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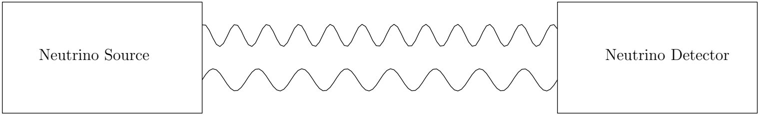
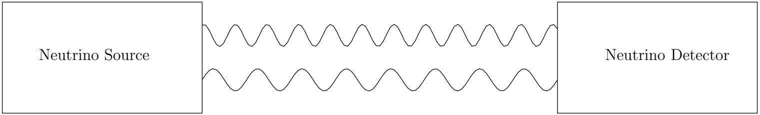
Physical picture

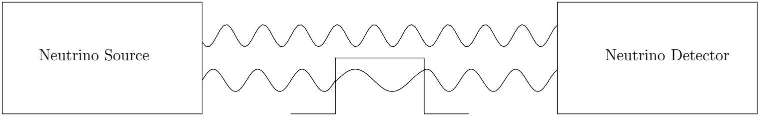


Can external factors affect the relative phase?

Physical picture



Can external factors affect the relative phase?



Matter effect is sensitive to sign of ω

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Matter modified Hamiltonian

$$H = \begin{pmatrix} -\omega\cos(2\theta) & \omega\sin(2\theta) \\ \omega\sin(2\theta) & \omega\cos(2\theta) \end{pmatrix} + \begin{pmatrix} \pm\sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix}$$

Equations of motion for are dependent on the sign of ω

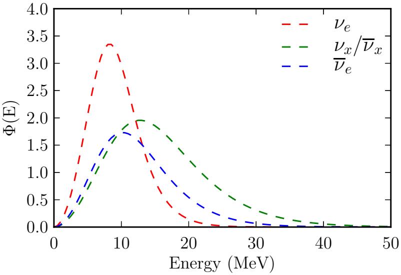
Self Interactions

$$H^{\rho}_{self} = \sqrt{2} G_F n_\nu \int dE \; (\rho(E) - \rho(E)^{c*}) + \mathrm{Tr}\left((\rho(E) - \rho(E)^{c*})\right)$$

If we know the flux (temperature and chemical potential) of the neutrinos we can calculate the flux that will be seen on the earth (sort of)

Including self-interactions makes the equation of motion very sensitive to the sign of ω.

Initial flux



M. T. .Keil, G. G. Raffelt and H. -T. Janka, Astrophys. J. **590**, 971 (2003) [astro-ph/0208035]

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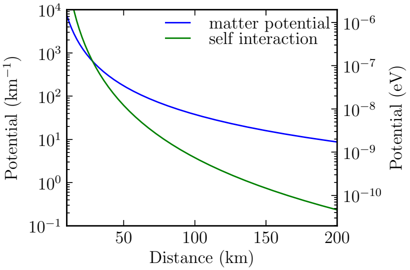
Matter and self-interaction potential



G. L. Fogli, E. Lisi, A. Marrone and A. Mirizzi, JCAP **0712**, 010 (2007) [arXiv:0707.1998 [hep-ph]]

Final flux

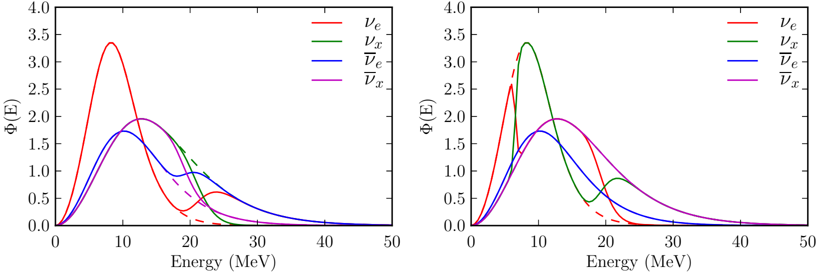


Figure: Initial and final fluxes for normal (left) and inverted hierarchy (right). The initial(final) flux spectra are denoted by dashed(solid) lines.

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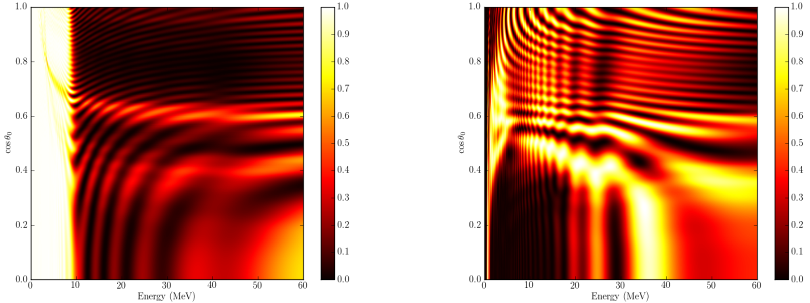
Multiangle plots



Reproduction of results from H. Duan, G. M. Fuller, J. Carlson and Y. -Z. Qian, Phys. Rev. Lett. **97**, 241101 (2006) [astro-ph/0608050]

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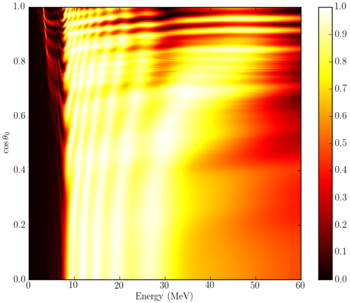
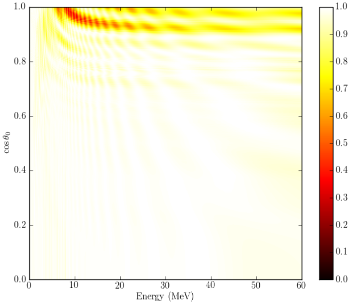
Multiangle plots





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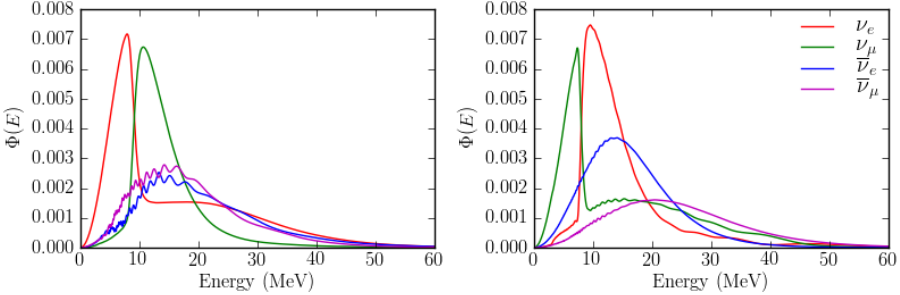
Multiangle plots



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“Switch on” effect

- matter effect (electron) $\propto$ mixing angle θ
- self interactions: very sensitive to sign of ω for even a very small value of θ
- Is this the case only for θ_{13}?
- Does it matter whether neutrinos are their own anti-particles

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Magnetic moment

It determines the rate of $\nu_{iL} \rightarrow \nu_{jR}$ due to interaction with electromagnetic fields

$$\left.\begin{array}{c}\mu^{\rm D}_{ij} \\ \epsilon^{\rm D}_{ij}\end{array}\right\} = \frac{eG_F}{8\sqrt{2}\pi^2}\left(m_i \pm m_j\right)\sum_{l=e,\mu,\tau} f\left(\frac{m_l^2}{m_W^2}\right) U^*_{li} U_{lj}$$

$$\mu^{\rm D}_{ii} \simeq 3.2 \times 10^{-19}\left(\frac{m_i}{\rm eV}\right)\mu_B$$

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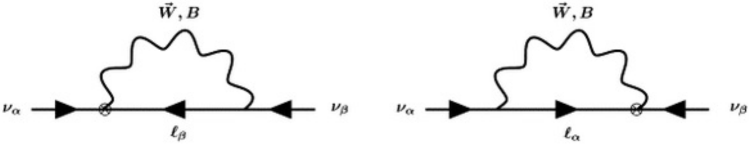
Transition magnetic moment



S. Davidson, M. Gorbahn and A. Santamaria, Phys. Lett. B **626**, 151 (2005)

$$\left.\begin{matrix}\mu_{ij}^{\mathrm{D}} \\ \epsilon_{ij}^{\mathrm{D}}\end{matrix}\right\} \simeq -4 \times 10^{-23} \left(\frac{m_i \pm m_j}{\mathrm{eV}}\right) \sum_{l=e,\mu,\tau} \left(\frac{m_l}{m_\tau}\right)^2 U_{li}^* U_{lj} \; \mu_B$$

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Majorana transition magnetic moment

If ν_i and ν_j have same $\mathcal{CP}$ phase

$$\mu_{ij}^M = 0 \qquad \epsilon_{ij}^M = 2\epsilon_{ij}^D$$

If ν_i and ν_j have opposite $\mathcal{CP}$ phases

$$\mu_{ij}^M = 2\mu_{ij}^D \qquad \epsilon_{ij}^M = 0$$

C. Broggini, C. Giunti and A. Studenikin, Adv. High Energy Phys. **2012**, 459526 (2012) [arXiv:1207.3980 [hep-ph]]

$$\text{Present Limits}: \begin{array}{lcl} \mu_{ii}^D & \lesssim & 10^{-12}\mu_B \\ \mu_{ij}^M & \lesssim & 10^{-10}\mu_B \end{array}$$

Particle Data Group, K. Nakamura et al., J.Phys.G G**37**, 075021 (2010)

Vacuum Hamiltonian

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} & \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho_{xe} & \rho_{xx} & \rho_{x\bar{e}} & \rho_{x\bar{x}} \\ \rho_{\bar{e}e} & \rho_{\bar{e}x} & \rho_{\bar{e}\bar{e}} & \rho_{\bar{e}\bar{x}} \\ \rho_{\bar{x}e} & \rho_{\bar{x}x} & \rho_{\bar{x}\bar{e}} & \rho_{\bar{x}\bar{x}} \end{pmatrix}$$

$$H_{vac} = \begin{pmatrix} -\omega \cos 2\theta & \omega \sin 2\theta & 0 & \mu B_T \\ \omega \sin 2\theta & \omega \cos 2\theta & -\mu B_T & 0 \\ 0 & -\mu B_T & -\omega \cos 2\theta & \omega \sin 2\theta \\ \mu B_T & 0 & \omega \sin 2\theta & \omega \cos 2\theta \end{pmatrix}$$

$$H_{self} = \sqrt{2} G_F n_\nu \int dE \, \left(\rho(E) - \rho(E)^{c*}\right) + \frac{1}{2} \mathrm{Tr}\left(\left(\rho(E) - \rho(E)^{c*}\right)\right)$$

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What is the magnetic field?

The core collapse causes the magnetic flux to be compressed in to a very small volume ($B \sim 10^{12}$ gauss)
We use the following magnetic field

$$B(r) = \left(\frac{50}{r(km)}\right)^2 10^{12} \; gauss$$

There is no way of telling the direction of the magnetic field. What happens if we assume it to be in the transverse direction?

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Final flux (with magnetic moment)

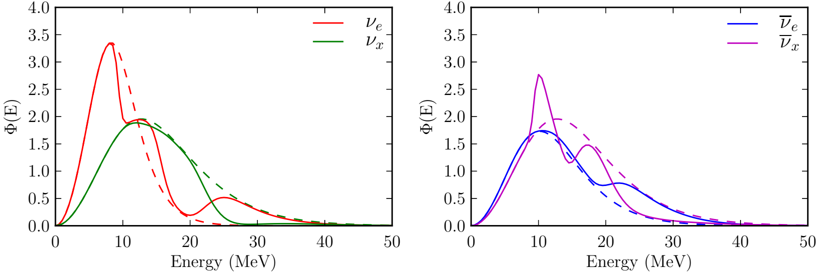

Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with normal hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-2}(\mu_{\nu_D} B)_{sm}$

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Final flux (with magnetic moment)

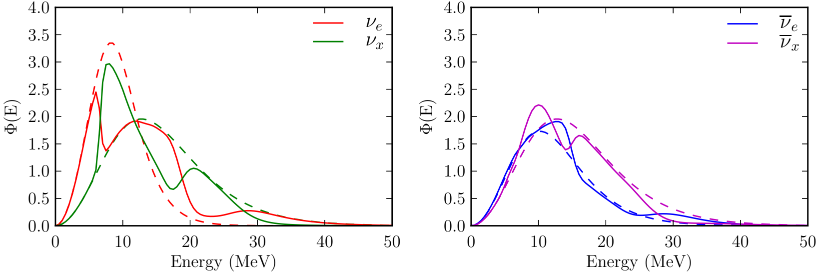

Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-2}(\mu_{\nu_D} B)_{sm}$

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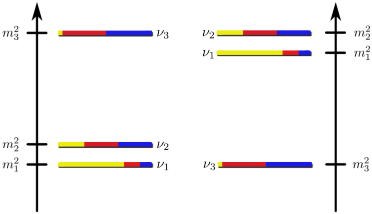
Three flavor Hamiltonian



$$H_{vac} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{12}^2 & 0 \\ 0 & 0 & \Delta m_{13}^2 \end{pmatrix} U^\dagger$$

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Three flavor Hamiltonian

$$H_{vac} = \begin{pmatrix} H_\theta & H_{\mu B} \\ -H_{\mu B} & H_\theta \end{pmatrix}$$

where, H_θ and $H_{\mu B}$ are given below

$$H_\theta = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{12} & 0 \\ 0 & 0 & \Delta m^2_{13} \end{pmatrix} U^\dagger$$

$$H_{\mu B} = \begin{pmatrix} 0 & \mu_{e\mu} B & \mu_{e\tau} B \\ -\mu_{e\mu} B & 0 & \mu_{\mu\tau} B \\ -\mu_{e\tau} B & -\mu_{\mu\tau} B & 0 \end{pmatrix}$$

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Three flavor calculations

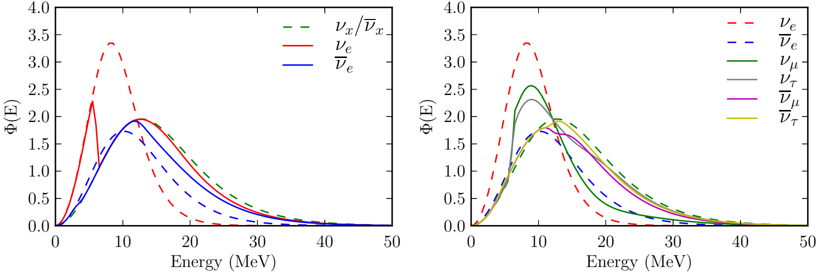

Figure: Initial and final flux spectra without including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$

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Three flavor calculations

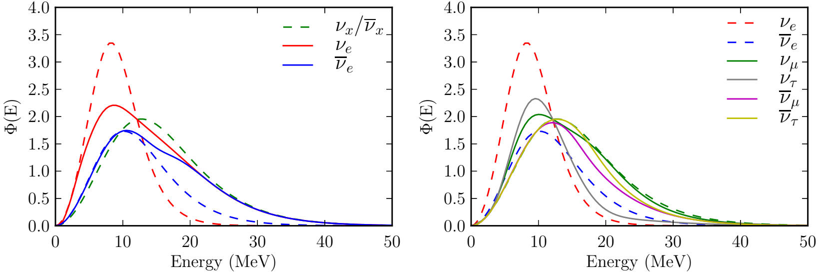

Figure: Initial and final flux spectra without including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with normal hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$

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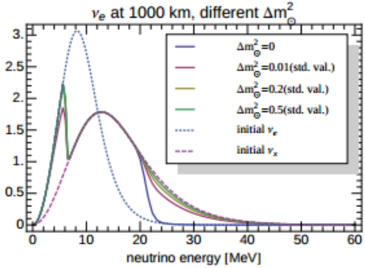
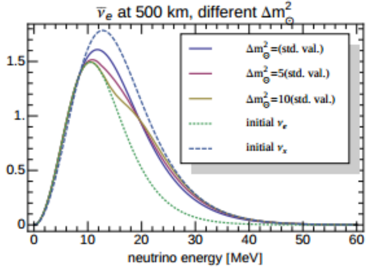
why do we need three flavor calculations?





Two flavor approximation can lead to 'fake' instabilities

A. Friedland, Phys. Rev. Lett. **104**, 191102 (2010) [arXiv:1001.0996 [hep-ph]]

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Three flavor calculations

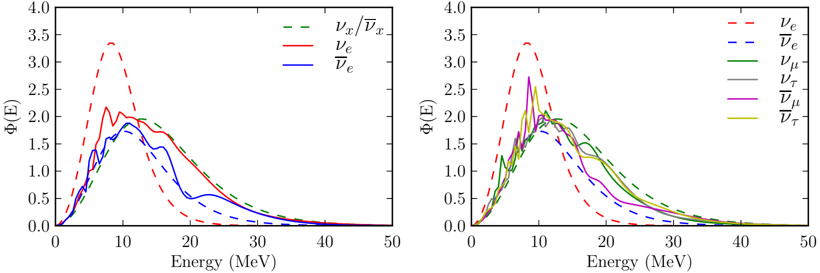

Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$

Understanding $\mathcal{CP}$ phase dependence

$$\phi_e^f = \phi_e^i P_{ee} + \phi_\mu^i P_{\mu e} + \phi_\tau^i P_{\tau e}$$
$$\xrightarrow{\phi_\mu = \phi_\tau} \phi_e^i P_{ee} + \phi_\mu^i (1 - P_{ee})$$

Remember, P_{ee} is independent of δ. This result holds in the case of collective oscillations. (J. Gava and C. Volpe, Phys. Rev. D **78**, 083007 (2008) [arXiv:0807.3418 [astro-ph]])

This $\mu - \tau$ symmetry can be broken by transition magnetic moment.

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Three flavor calculations

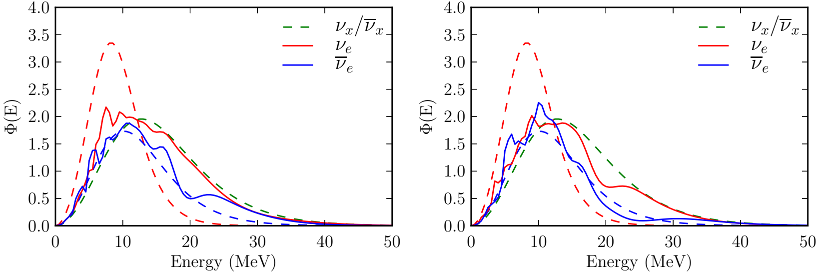

Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos with $\delta = 0°$ and $\delta = 180°$ with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$

Three flavor calculations

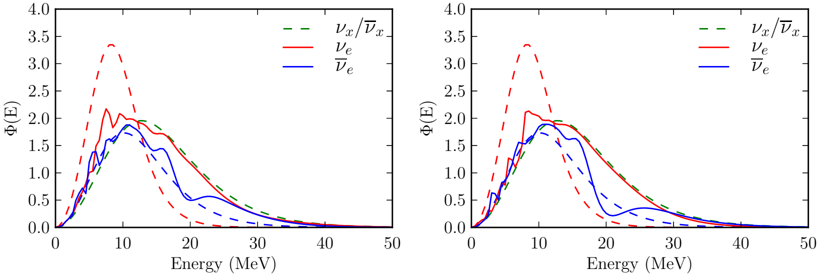

Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos with and with inverted hierarchy.
$\mu_{e\mu}B(r) = 10^{-4}(\mu_{\nu_D}B)_{sm}$ (left) and $\mu_{\mu\tau}B(r) = 10^{-4}(\mu_{\nu_D}B)_{sm}$ (right)

MSW effect

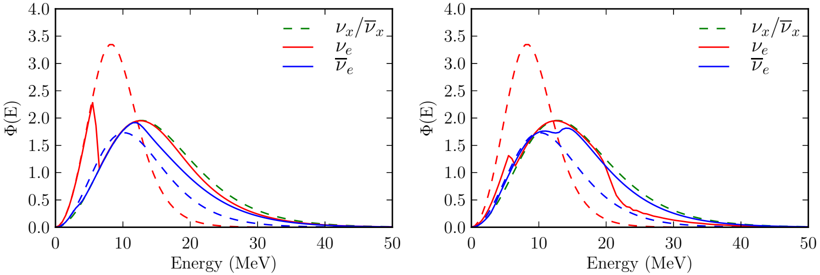

Figure: Flux before and after the standard MSW effect for inverted hierarchy.

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MSW effect

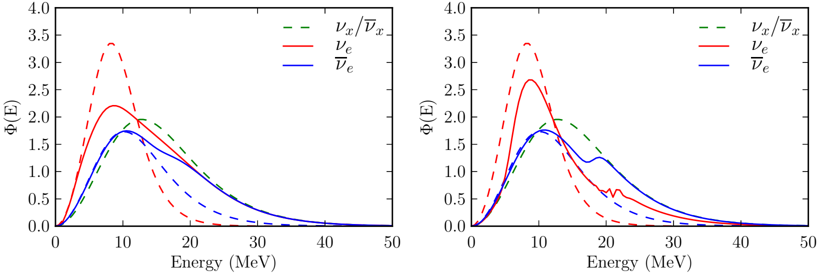

Figure: Flux before and after (Flux averaged over 24000-25000 km) the standard MSW effect for normal hierarchy.

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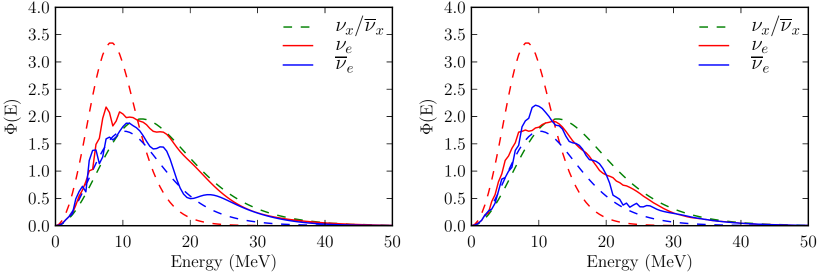
MSW effect



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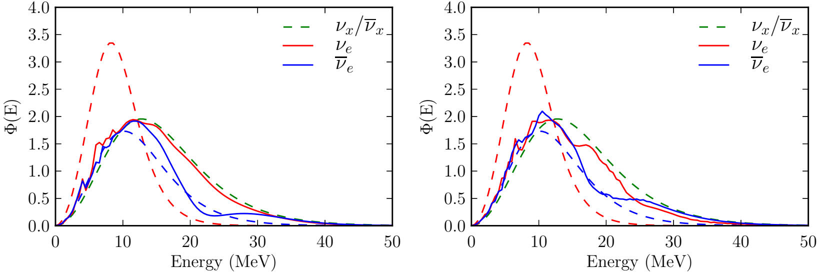
MSW effect



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Conclusions

- The neutrino flux spectra for Majorana neutrinos is significantly different than Dirac neutrinos
- Transition magnetic moment of neutrinos can have a "switch-on" effect on $\nu_L \to \nu_R$ oscillations, just like θ_{13} for $\nu_e \to \nu_x$
- Neutrinos from galactic supernova(e) is the only phenomenon for which transition magnetic moments of the order predicted by standard model can have significant impact. This is the only known way of detecting non-zero transition magnetic moments.
- Multi-angle calculations with transition magnetic moment needed for solid conclusions to be drawn.

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