Some open problems of neutrino masses and oscillations

S. Bilenky
JINR(Dubna)

November 12, 2012

A bit of history of neutrino oscillations

Uncertainty relations and neutrino oscillations

The seesaw mechanism and neutrinoless double β-decay

Soon after the discovery of the violation of parity and C in the weak interaction Landau, Lee and Yang and Salam proposed the theory of the two-component neutrino (1957)

For massless neutrino $\nu_L(x)$ and $\nu_R(x)$ satisfy two decoupled equations

$$i\gamma^{\alpha}\,\partial_{\alpha}\,\nu_L(x) = 0 \quad i\gamma^{\alpha}\,\partial_{\alpha}\,\nu_R(x) = 0$$

Basic assumptions made by Landau, Lee and Yang and Salam

1. Neutrino is massless particle
2. Neutrino field is $\nu_L(x)$ or $\nu_R(x)$

Under inversion

$$\nu'_{L(R)}(x') = \eta\gamma^0\nu_{R(L)}(x)$$

If neutrino field is $\nu_L(x)$ or $\nu_R(x)$
maximal violation of parity (in agreement with experimental data)

Important consequence of the theory

For massless neutrino

$$\gamma_5\ u^r(p) = r\ \ u^r(p), \quad \frac{1 \mp \gamma_5}{2}\, u^r(p) = \frac{1 \mp r}{2}\ u^r(p)$$

If neutrino field is $\nu_L(x)$ ($\nu_R(x)$), neutrino is a particle with helicity equal to -1 (+1)

Neutrino helicity was determined in spectacular Goldhaber et al experiment (1958) from the measurement of the circular polarization of γ's produced in the chain of reactions

$$e^- + ^{152}\mathrm{Eu} \to \nu + \ ^{152}\mathrm{Sm}^*$$
$$\downarrow$$
$$^{152}\mathrm{Sm} + \gamma.$$

The authors concluded "... our result is compatible with 100% negative helicity of neutrino emitted in orbital electron capture"

After this experiment during many years there was a general belief that the neutrino is a two-component massless particle

For the first time this opinion was challenged by B. Pontecorvo in 1957-58. He assumed that neutrinos have small masses, they are mixed and neutrino oscillations are possible

Pontecorvo believed in analogy between weak interaction of hadrons and leptons and looked for analogy of the famous $K^0 - \bar{K}^0$ oscillations in the lepton world. In such a way he came to an idea of neutrino oscillations

K^0 ($\bar{K}^0$) is a particle with $S = 1$ ($S = -1$)

They are produced in strong processes like $\pi^- + p \to K^0 + \Lambda$ etc.

$|K^0\rangle$ ($|\bar{K}^0\rangle$) states in the rest frame, $|\bar{K}^0\rangle = CP\, |K^0\rangle$

Weak interaction does not conserve S. Eigenstates of the total Hamiltonian are superpositions of $|K^0\rangle$ and $|\bar{K}^0\rangle$. We neglect small effects of the violation of CP

$$|K^0_1\rangle = \frac{1}{\sqrt{2}}\,(|K^0\rangle + |\bar{K}^0\rangle), \quad |K^0_2\rangle = \frac{1}{\sqrt{2}}\,(|K^0\rangle - |\bar{K}^0\rangle$$

are eigenstates of the total H with masses $m_{1,2}$ and total widths $\Gamma_{1,2}$.

$$|K^0\rangle = \frac{1}{\sqrt{2}}\ (|K_1^0\rangle + |K_2^0\rangle), \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\ (|K_1^0\rangle - |K_2^0\rangle)$$

$m_K \simeq 497.61$ MeV and mass difference $\Delta m = m_2 - m_1$ is extremely small $\Delta m \simeq 3.48 \cdot 10^{-12}$ MeV

Impossible to distinguish production (detection) of K_1^0 and K_2^0 in hadronic processes. Coherent superpositions are produced (detected)

If at $\tau = 0$ K^0 is produced at the proper time τ we have

$$|K^0\rangle_\tau = \frac{1}{\sqrt{2}}(e^{-i\lambda_1\tau}\ |K_1^0\rangle + e^{-i\lambda_2 2}\ |K_2^0\rangle) = g_+(\tau)|K^0\rangle + g_-(\tau)|\bar{K}^0\rangle$$

$$g_\pm(\tau) = \frac{1}{2}(e^{-i\lambda_1\tau} \pm e^{-i\lambda_2\tau}) \quad \lambda_{1,2} = m_{1,2} - i\frac{\Gamma_{1,2}}{2}$$

Because of coherence $|K^0\rangle_\tau$ is a superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$. $P(K^0 \to \bar{K}^0) = |g_-|^2$ is the transition probability

I. Coherent superpositions of eigenstates of the total H with very small mass difference are produced in the strong interaction.

II. States with definite masses are evolved in time according to the Schrodinger equation.

This formalism was perfectly confirmed not only for $K^0 - \bar{K}^0$ but also for $B_d^0 - \bar{B}_d^0$ etc

In 1958 only one type of neutrino was known. B. Pontecorvo assumed that in addition to $\bar{\nu}_R$ produced in weak decays exist also sterile ν_R. By analogy with $K^0 - \bar{K}^0$ he assumed that

$$|\bar{\nu}_R\rangle = \frac{1}{\sqrt{2}}(|\nu_1\rangle + |\nu_2\rangle), \quad |\nu_R\rangle = \frac{1}{\sqrt{2}}(|\nu_1\rangle - |\nu_2\rangle)$$

$|\nu_{1,2}\rangle$ are states of Majorana neutrinos $\nu_{1,2}$ with small masses $m_{1,2}$ and momentum p

If at the time t $\bar{\nu}_R$ $(\bar{\nu}_e)$ was produced we have

$$|\bar{\nu}_R\rangle_t = \frac{1}{\sqrt{2}}(e^{-iE_1 t}|\nu_1\rangle + e^{-iE_2 t}|\nu_2\rangle) = \frac{1}{2}(g_+(t)|\bar{\nu}_R\rangle + g_-(t)|\nu_R\rangle)$$

$$g_\pm(t) = (e^{-iE_1 t} \pm e^{-iE_2 t}), \quad E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}$$

Survival probability

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}(1 - \cos\frac{\Delta m^2 L}{2E})$$

$$L \simeq t,\ \Delta m^2 = m_2^2 - m_2^1$$

The probability depends periodically on $\frac{L}{E}$ (neutrino oscillations)

If $\frac{\Delta m^2 L}{2E} \ll 1$ no effects. If $\frac{\Delta m^2 L}{2E} \gg 1$ cosine term will be averaged and $\frac{1}{2}$ suppression will be observed B. Pontecorvo in 1958 proposed to measure the flux of the reactor antineutrinos at different distances from a reactor.

Later in 1962 MNS considered ν_e and ν_μ as mixtures of neutrinos ν_1 and ν_2 with masses m_1 and m_2. They mentioned a possibility of "virtual transmutation" $\nu_\mu \to \nu_e$

NEUTRINO OSCILLATIONS TODAY
Basics

1. Exist three flavor neutrinos ν_e, ν_μ, ν_τ
2. The Standard Model CC and NC lepton interaction

$$\mathcal{L}_{\mathcal{I}}^{\mathcal{CC}}(x) = -\frac{g}{2\sqrt{2}} j_\alpha^{CC}(x) W^\alpha(x) + \mathrm{h.c.}$$

$$j_\alpha^{CC}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\, \gamma_\alpha\, l_L(x)$$

3. $\nu_{eL}(x), \nu_{\mu L}(x), \nu_{\tau L}(x)$ are "flavor mixed fields"

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\, \nu_{iL}(x)$$

$\nu_i(x)$ is the field of neutrino (Dirac or Majorana) with mass m_i and U is 3×3 unitary PMNS mixing matrix

Usually it is assumed that in weak processes flavor neutrinos ν_e, ν_μ, ν_τ which are described by coherent states

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^{*}\ |\nu_i\rangle$$

($|\nu_i\rangle$ is the state of the left-handed neutrino with mass m_i and momentum p) are produced

Let us consider in (lab. system) a decay

$$a \to b + l^{+} + \nu_i$$

a and b are some hadrons. Initial and final particles have definite momenta.

State of the final particles

$$|f\rangle = \sum_i |l^{+}\rangle|\nu_i\rangle|b\ \rangle\langle l^{+}\nu_i b|S|a\rangle$$

If neutrino masses are equal $p_i = p$ Thus, we have $p_i \simeq p + a\frac{m_i^2}{2E}$ p is the momentum of the lightest neutrino, a is a constant of the order of one

$$|\Delta p_{ik}| - |p_k - p_i| = |a|\frac{|\Delta m^2_{ik}|}{2E} = \frac{2\pi}{L^{\rm osc}_{ik}}$$

$L^{\rm osc}_{ik}$ is the oscillation length

From the Heisenberg uncertainty relation

$$(\Delta p)_{QM} \simeq \frac{1}{d}$$

d is QM microscopic size of a source

For the atmospheric neutrinos $L^{\rm osc}_{23} \simeq 1000$ km, for reactor neutrinos $L^{\rm osc}_{12} \simeq 100$ km

$|\Delta p_{ik}| \ll (\Delta p)_{QM}$ We can not distinguish production of neutrinos with different masses for neutrinos with energies relevant for neutrino oscillation experiments

$\frac{m_i^2}{E^2} \leq 10^{-12}$ neutrino masses can be safely neglected in the matrix element

$$\langle l^+ \nu_i b|S|a\rangle \simeq U^*_{li}\ \langle l^+ \nu_l b|S|a\rangle_{SM}$$

$\langle l^+ \nu_l b|S|a\rangle_{SM}$ is the SM matrix element of the process

$a \to b + l^+ + \nu_l$

For the final state we find

$$|f\rangle = |l^+\rangle|\nu_l\rangle|b\ \rangle\langle l^+\nu_l b|S|a\rangle_{SM}$$

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^*\ |\nu_i\rangle$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i and momentum p

Together with l^+ flavor neutrino ν_l (analog of $K^0, \bar{K}^0$), which is described by the mixed coherent state $|\nu_l\rangle$, is produced in a weak decay

1. The flavor state does not depend on the production process
2. In neutrino production and detection processes neutrino mass differences can be neglected. Flavor lepton numbers L_e, L_μ, L_τ are effectively conserved

EVOLUTION OF THE FLAVOR NEUTRINO STATES

Evolution equation for states in QFT is the Schrodinger equation

If at $t = 0$ flavor neutrino ν_l was produced at the time t we have

$$|\nu_l\rangle_t = e^{-iH_0 t} \sum_i |\nu_i\rangle \ U_{li}^* = \sum_i |\nu_i\rangle e^{-iE_i t} \ U_{li}^*$$

Thus, at the time t the neutrino state is a superposition of states with different energies (nonstationary state)

Neutrinos are detected via observation of weak processes. We have

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle (\sum_i U_{l_1 i} \ e^{-iE_i t} \ U_{li}^*)$$

The probability of $\nu_l \to \nu_{l'}$ transition is given

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l_1 i} \ e^{-i(E_i - E_p)t} \ U_{li}^*|^2$$

p is fixed index (common phase)

Transitions are due to coherence(mixing) and phase differences

$$(E_i - E_p)t \simeq \frac{\Delta m^2_{pi} L}{2E}$$

L is source-detector distance

The transition take place if

$$|E_i - E_p| \; t \geq 1$$

This is time-energy uncertainty relation

It was shown by Mandelstam and Tamm that in any quantum theory

$$\Delta E \; \Delta t \geq 1$$

The time-energy uncertainty relation is a consequence of

$$i \frac{\partial O(t)}{\partial t} = [O(t), H]$$

($O(t)$ is an Heisenberg operator and H is the total Hamiltonian) and Couchy inequality. Δt is time interval during which state of the system is significantly changed, ΔE is uncertainty in energy

In many papers evolution in space and time is considered Neutrino state at the point $x - (x^o, \vec{x})$

$$|\nu_l\rangle_x = e^{-iPx}|\nu_l\rangle_0 = \sum_i e^{-ip_i x} U^*_{li}|\nu_i\rangle = \sum_{l'} |\nu_{l'}\rangle \left(\sum_i U_{l'i} e^{-ip_i x} U^*_{li} \right)$$

Evolution operator e^{-iPx}, P is the operator of the total momentum

Transition probability

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i} e^{-i(E_i t - p_i L)} U^*_{li}|^2$$

For the ultrarelativistic neutrino $t \simeq L$

Compare two approaches

I (t). The flavor neutrino state with definite momentum is produced osc. phase: $E_i t \simeq (p + \frac{m_i^2}{2E})L$

II(x). Different energies and momenta . Osc. phase:

$$(E_i t - p_i L) \simeq (E_i - p_i)L \simeq \frac{m_i^2}{2E} L$$

Common phase factor e^{-pL} can not be observed There is no way to distinguish two approaches in usual neutrino oscillation experiments

Comments to space-time approach

1. e^{-iPx} is not operator of the evolution of states. It determines the evolution of operators in QFT $O(x) = e^{iPx} O(0) e^{-iPx}$
2. States of neutrino with definite momentum can not depend on x (be localized) $|\nu_i\rangle = c^{\dagger}(p_i)|0\rangle$

$$\sum_i U^*_{li}\ e^{i(\vec{p}_i\vec{x}-E_i t)}$$

can be considered as a superpositions of solutions of the Dirac equation. In this QM approach

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i} e^{i(p_i L - E_i T)}\ U^*_{li}|^2$$

$p_i L - E_i t$ is the change of the phase of the plane wave which describes ν_i at the distance L after the time t

Different approaches can be distinguished if special Mossbauer neutrino experiment will be done

$$^3H \leftrightarrows ^3 He(\mathrm{atom}) + \bar{\nu}_e$$

^3H and ^3He in lattice

Monochromatic antineutrinos with energy 18.6 KeV are produced and detected

resonance cross section $\sigma_R = 5 \cdot 10^{-32}\ cm^2$

Estimated energy uncertainty of $\bar{\nu}_e$ is $\Delta E \simeq 8.6 \cdot 10^{-12}$ eV

Much smaller than $E_3 - E_2 \simeq \frac{\Delta m^2_{23}}{2E} \simeq 6.7 \cdot 10^{-8}$ eV

Thus, in such an experiment $E_i \simeq E$

In the case of the approach based on the Schrodinger evolution equation oscillations will not take place (in accordance with the time-energy uncertainty relation)

In the case of the space-time approach oscillations will take place due to different neutrino momenta (the time-energy uncertainty relation is not satisfied in this case)

Seesaw mechanism and $0\nu\beta\beta$-decay

Why discovery of neutrino oscillations is a signature of a beyond the SM physics?

Main reason: neutrino masses are much smaller than masses of leptons and quarks

For example, for the third generation

$$m_t \simeq 173.5 \cdot 10^2 \text{ GeV}, \quad m_b \simeq 4.65 \text{ GeV}$$

$$m_3 \leq 2.3\ 10^{-9} \text{ GeV}, \quad m_\tau \simeq 1.77 \text{ GeV}$$

It is very unlikely that neutrino masses are of the same SM origin as masses of the leptons and quarks

Different mechanisms of small neutrino mass generation were proposed. The most plausible (popular) is the seesaw mechanism

The most general realization of the seesaw idea is based on the effective Lagrangian approach (Weinberg).

Basic assumptions

1. In SM neutrino are massless particles
2. Neutrino masses and mixing are generated by a beyond the SM interaction

Taking into account effective interactions generated by a beyond the SM physics we have

$$\mathcal{L}(x) = \mathcal{L}_{SM}(x) + \sum_{n \geq 1} \frac{1}{\Lambda^n} \mathcal{L}_{4+n}(x)$$

$\mathcal{L}_{SM}(x)$ is the SM Lagrangian; the second term is the effective nonrenormalizable Lagrangian. Λ has dimension of mass and characterizes a scale of a new physics. The operator $\mathcal{L}_{4+n}(x)$ has dimension M^{4+n}. It is built from the SM fields and satisfies $SU(2) \times U(1)$ invariance of the SM

We are interested in the generation of a neutrino mass term

Neutrino fields ν_{lL} enters into the lepton doublets

$$L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix} \quad l = e, \mu, \tau$$

$(\bar{L}_{lL}\tilde{H})$ is $SU(2) \times U(1)$ invariant which has the lowest dimension $(M^{5/2})$

Here $\tilde{H} = i\tau_2 H^*$, H is the Higgs doublet

In order to built a Lorenz-scalar, quadratic in neutrino fields, we need to use (right-handed) conjugated doublets $(L_{lL})^c = C\bar{L}_{lL}^T$

We come to the following dimension five, invariant Lagrangian

$$\mathcal{L}_5^{\rm eff} = \frac{1}{\Lambda} \sum_{l',l} \bar{L}_{l'L} \tilde{H} Y_{l'l} \tilde{H}^T (L_{lL})^c + \rm{h.c.}$$

Here $Y_{l'l} = Y_{ll'}$ are dimensionless constant. $\mathcal{L}^{eff}$ is the only dimension 5 effective Lagrangian which can be build from the SM fields

Let us stress that

1. The Lagrangian $\mathcal{L}_5^{\rm eff}$ violate L
2. It can be build only if Higgs particle exists
3. Λ characterizes scale of a new physics which violate L

After the spontaneous violation of the symmetry

$$\tilde{H} = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}, \quad v = (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV}$$

we come to the Majorana mass term

$$\mathcal{L}^M = -\frac{1}{2}\sum_{l'l} \bar{\nu}_{l'L} M_{l'l} (\nu_{lL})^c + \text{h.c.}$$

Here $M_{l'l} = \frac{v^2}{\Lambda} Y_{l'l}$

After the standard diagonalization of the symmetrical matrix M $(M = UmU^T)$

$$\mathcal{L}^M = -\frac{1}{2}\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i$$

$\nu_i = \nu_i^c = C\bar{\nu}_i^T$ is the field of neutrino Majorana with mass m_i

Majorana neutrino mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}$$

Neutrino mass

$$m_i = y_i v \, \frac{v}{\Lambda}$$

$y_i v$ is of the order of the SM mass

$\frac{v}{\Lambda} = \frac{\text{SM scale}}{\text{scale of a new physics}}$ is the suppression factor. Assuming $y_i \simeq 1$, $\Lambda \simeq 10^{15}$ GeV

Summarizing, if neutrino masses are of the standard seesaw origin

1. Neutrino masses are small (if $\Lambda \gg v$)
2. Neutrinos ν_i are Majorana particles.
3. The number of neutrinos ν_i is equal to the number of flavor neutrinos (three).

Nature of neutrinos with definite masses can be revealed via investigation of $0\nu\beta\beta$-decay

If $0\nu\beta\beta$-decay will be observed, can we check that only three Majorana neutrino masses contribute?

The lepton part of the process is determined

$$\sum_i \frac{1-\gamma_5}{2} U_{ei} \frac{\gamma \cdot p + m_i}{p^2 - m_i^2} U_{ei} \frac{1-\gamma_5}{2} \simeq m_{\beta\beta} \frac{1-\gamma_5}{2} \frac{1}{p^2}$$

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

is the effective Majorana mass We took into account $p^2 \gg m_i^2$. The nuclear matrix elements do not depend on m_i. Half-life of the $0\nu\beta\beta$-decay

$$\frac{1}{T_{1/2}^{0\,\nu}(A,Z)} = |m_{\beta\beta}|^2 \, |M(A,Z)|^2 \, G^{0\,\nu}(E_0,Z)$$

$M(A,Z)$ is NME, $G^{0\,\nu}(E_0,Z)$ is a known phase-space factor

Calculation of NME is a challenging nuclear problem. 5 different methods are used at present

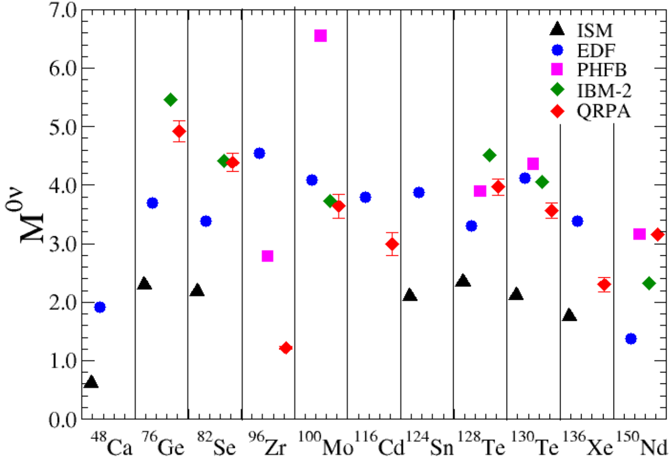

Figure: NME in different models

Transition	$M(A,Z)$				
	ISM	QRPA	IBM-2	PHFB	EDF
$^{48}Ca \rightarrow {}^{48}Ti$	0.61				1.91
$^{76}Ge \rightarrow {}^{72}Se$	2.30	4.92	5.47		3.70
$^{82}Se \rightarrow {}^{82}Kr$	2.18	4.39	4.41		3.39
$^{96}Zr \rightarrow {}^{96}Mo$		1.22		2.78	4.54
$^{100}Mo \rightarrow {}^{100}Ru$		3.64	3.73	6.55	4.08
$^{116}Cd \rightarrow {}^{116}Sn$		2.99			3.80
$^{124}Sn \rightarrow {}^{124}Te$	2.10				3.87
$^{128}Te \rightarrow {}^{128}Xe$	2.34	3.97	4.52	3.89	3.30
$^{130}Te \rightarrow {}^{130}Xe$	2.12	3.56	4.06	4.36	4.12
$^{136}Xe \rightarrow {}^{136}Ba$	1.76	2.30			3.38
$^{150}Nd \rightarrow {}^{150}Sm$		3.16	2.32	3.16	1.37

From today's experiments no evidence for the $0\nu\beta\beta$-decay was obtained. The following bounds were inferred

$$\begin{aligned} |m_{\beta\beta}| &< (0.20-0.32)\ eV \quad ({}^{76}\mathrm{Ge}), \\ &< (0.30-0.71)\ eV \quad ({}^{130}\mathrm{Te}), \\ &< (0.50-0.96)\ eV \quad ({}^{130}\mathrm{Mo}), \\ &< (0.14-0.38)\ eV \quad ({}^{136}\mathrm{Xe}) \end{aligned} \tag{1}$$

In future experiments, CUORE, EXO, MAJORANA, SuperNEMO , SNO$+$, Kamland-ZEN and others a sensitivity

$$|m_{\beta\beta}| \simeq \text{a few } 10^{-2}\ \mathrm{eV}$$

is planned to be reached

We consider three-neutrino mixing. The value of $|m_{\beta\beta}|$ strongly depends on the type of neutrino mass spectrum

1. Normal spectrum (NS)

$$m_1 < m_2 < m_3, \quad \Delta m^2_{12} \ll \Delta m^2_{23},$$

2. Inverted spectrum (IS)

$$m_3 < m_1 < m_2, \quad \Delta m^2_{12} \ll |\Delta m^2_{13}|.$$

$$\Delta m^2_S = \Delta m^2_{12}(NS) = \Delta m^2_{12}(IS) \quad \Delta m^2_A = \Delta m^2_{23}(NS) = |\Delta m^2_{13}|(IS)$$

$m_0 = m_1(NS) = m_3(IS)$ lightest mass

Δm^2_S and Δm^2_A do not depend on spectrum

Let us consider the case of small m_0

I. Normal mass hierarchy $m_1 \ll m_2 \ll m_1$

$$m_1 \ll \sqrt{\Delta m^2_S}, m_2 \simeq \sqrt{\Delta m^2_S}, m_3 \simeq \sqrt{\Delta m^2_A}, , |m_{\beta\beta}| \leq 6 \cdot 10^{-3} \mathrm{eV}$$

(too small to be reached in future experiments)

II. Inverted mass hierarchy $m_3 \ll m_1 \ll m_2$

$$m_3 \ll \sqrt{\Delta m^2_A}, m_1 \simeq \sqrt{\Delta m^2_A}, m_2 \simeq \sqrt{\Delta m^2_A}$$

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_A} \; (1 - \sin^2 2\,\theta_{12} \sin^2 \alpha_{12})^{\frac{1}{2}}$$

$\alpha_{12} = \alpha_2 - \alpha_1$ ($U_{ei} = |U_{e1}| e^{i\alpha_i}$ (i=1,2)). α_{12} is unknown. Only upper and lower bounds can be predicted

$$\sqrt{\Delta m^2_A} \; \cos 2\,\theta_{12} \leq |m_{\beta\beta}| \leq \sqrt{\Delta m^2_A}$$

We have

$$1.5 \cdot 10^{-2}\ \mathrm{eV} \leq |m_{\beta\beta}| \leq 5.0 \cdot 10^{-2}\ \mathrm{eV}$$

Such values of the effective Majorana mass are planned to be reached in future $0\nu\beta\beta$-experiments

Let us discuss a possibility to check three Majorana neutrino mass mechanism in the case if the $0\nu\beta\beta$-decay will be observed in experiments sensitive to 10^{-2} eV range

$|m_{\beta\beta}|$ as a function of m_0, calculated in the framework of the tree-neutrino mixing, is presented in Fig. 1

In order to establish that we are in the allowed region we need to have an information not only about $|m_{\beta\beta}|$ but also about m_0

Such information can be obtained from cosmological data, sensitive to $\sum_i m_i$. From existing data $\sum_i m_i \lesssim 0.5$ eV. Future measurements (galaxy distributions, gravitational lensing etc) will be sensitive to $\sum_i m_i \simeq (10^{-1} - 10^{-2})$ eV

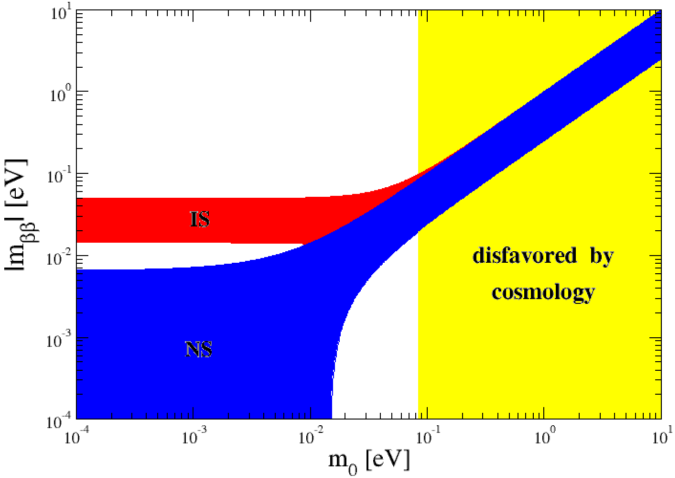

Figure: $m_{\beta\beta}$ as function of m_0.

Upper band in Fig. corresponds to the inequality (see Fig 2)

$$\frac{1}{\Delta m_A^2 |M|^2 G^{0\nu}(E_0, Z)} \leq T_{1/2}^{0\nu} \leq \frac{1}{\Delta m_A^2 \cos^2 2\theta_{12} |M|^2 G^{0\nu}(E_0, Z)}$$

If measured half-lives are in this range it will be an evidence in favor of the three-neutrino mechanism

1. There are no reasons to expect that other possible mechanisms of $0\nu\beta\beta$-decay (SUSY with R-parity violation etc) give contributions to $|m_{\beta\beta}|$ which are close to the $3-\nu$ contribution (different parameters, different NME etc)
2. No doubts that at the time when $0\nu\beta\beta$-decay will be observed NME will be known much better than today

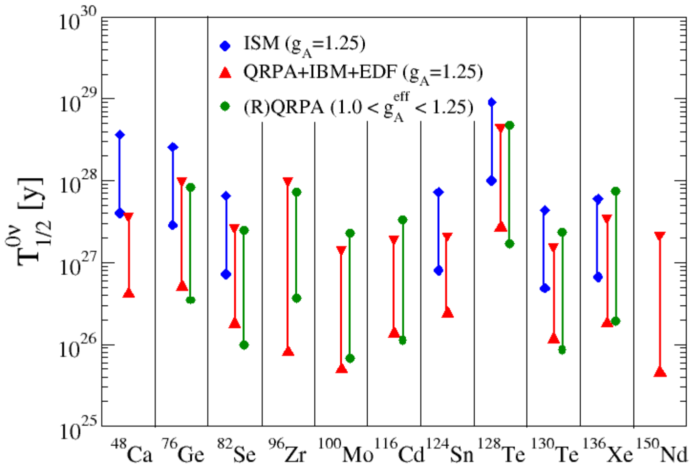

Figure: $3-\nu$ allowed values of $T^{0\,\nu}_{1/2}$