HVP Contribution to Muon g-2 from Lattice QCD: Theory Errors

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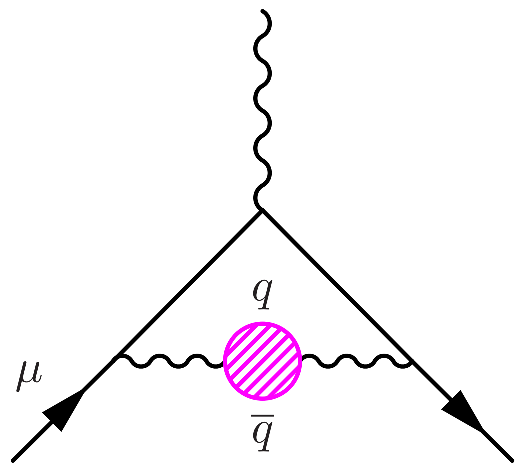
Hadronic Vacuum Polarization in g-2

- Dominant QCD correction to μ's g–2:



for $q = u,d$

- Best current theory from e^+e^- data: 0.7% ($\pm 4\times10^{-10}$).

- Need ≤0.25% errors to compete with new experiment ($\delta a_\mu \approx \pm 1.6\times10^{-10}$).

- Can LQCD meet this challenge? Focus here on theoretical errors in LQCD evaluation.

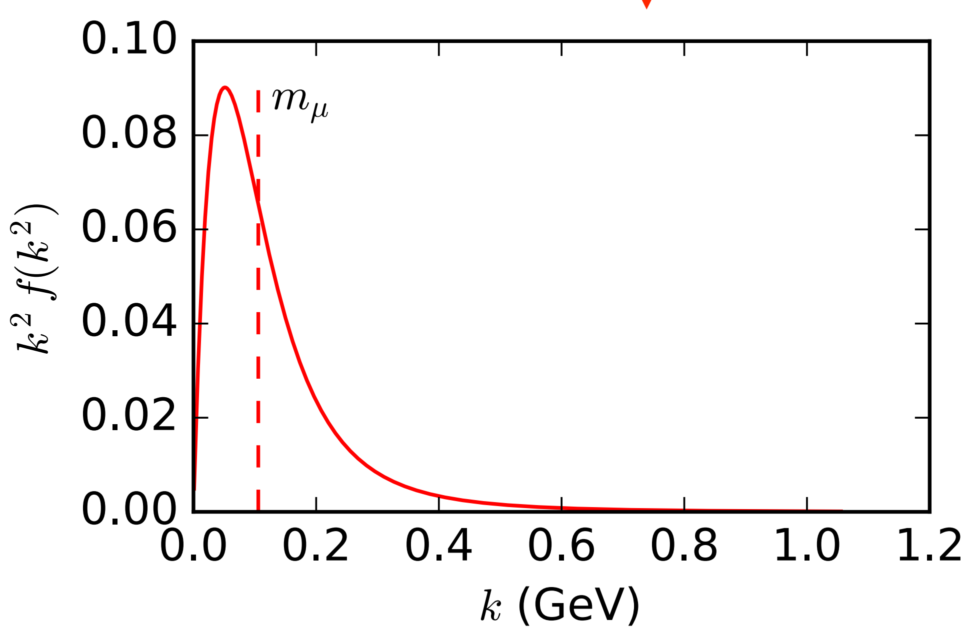
LQCD ⇒ $\langle j^i(x)\, j^i(0)\rangle$

- Gluon configurations from MILC.
  - HISQ quark discretization ⇒ very fast, highly improved.
- Lattice spacing a = 0.09, 0.12, 0.15 fm.
- $m_u = m_d = m_\ell$ with $m_\ell/m_s = 1/27.4$, 1/10, 1/20.
- Lattice volumes with $m_\pi L = 3.25 \rightarrow 5.4$.
- See (for HPQCD):
  - Chakraborty et al (HPQCD) 1403.1778 (2014) for $q = s, c$.
  - Colquhoun et al (HPQCD) 1408.5768 (2014) for $q = b$.
  - Chakraborty et al (HPQCD) 1601.03071 (2016) and 1512.03270 (2015) for $q = u,d$. ⟵ Focus on this

LQCD Approach

$$Q_q = \begin{cases} 2/3 & \text{for } u \\ -1/3 & \text{for } d \end{cases}$$

$$a_\mu^q(\mathrm{HVP,LO}) = 4Q_q^2\alpha_{\mathrm{QED}}^2 \int_0^\infty dk^2\, f(k^2)\,\hat{\Pi}_q(k^2)$$



$$k^2\,\Pi_q(k^2) \equiv a^4 \sum_t e^{ikt} \sum_{\vec{x}} Z_V^2 \,\langle j^i_{q\bar{q}}(\vec{x},t)\, j^i_{q\bar{q}}(0)\rangle$$

$$\hat{\Pi}_q(k^2) \equiv \Pi_q(k^2) - \Pi_q(0)$$

[Blum, Phys.Rev.Lett. 91, 052001 (2003).]

Simplify k^2 Dependence

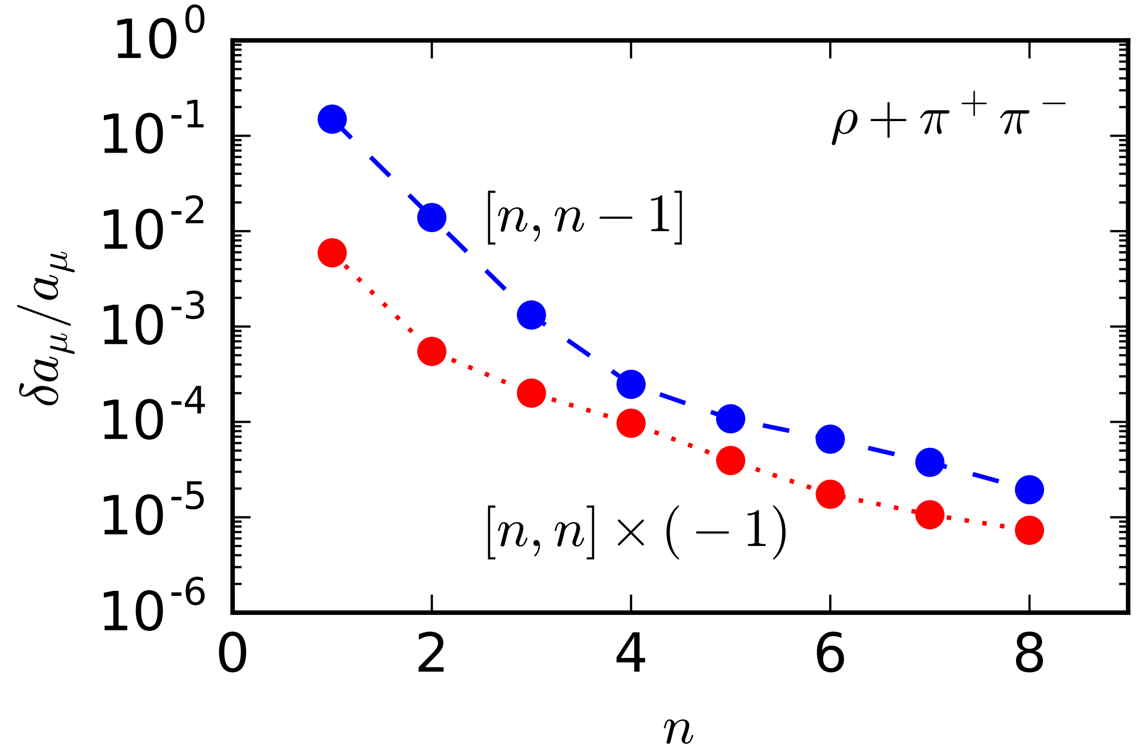
- $k \approx m_\mu$ dominates $\Rightarrow$

$$\hat{\Pi}(k^2) = \sum_{j=1}^{N} k^{2j}\,\Pi_j$$

$$\Pi_j \rightarrow \frac{(-1)^{j+1}}{(2j+2)!}\sum_t \sum_{\vec{x}} t^{2j+2} Z_V^2 \langle j^i(\vec{x},t)\, j^i(0)\rangle$$

$\rightarrow$ $[n,n]$ Padé approx. for any k

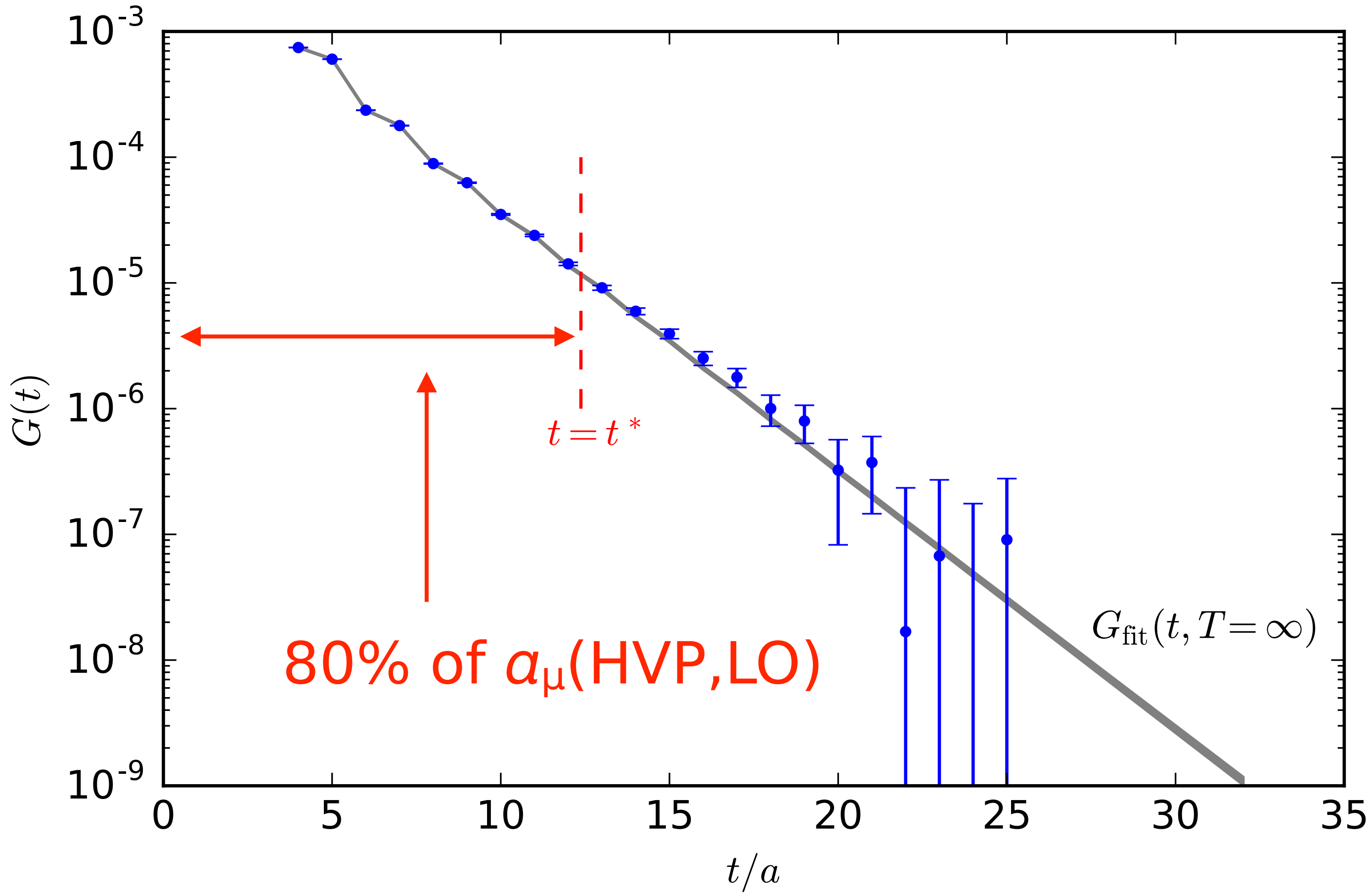
- Parameterize HVP to better than 0.1% with N=4 moments (for [2,2] Padé):



[Chakraborty et al (HPQCD) 1403.1778 (2014)]

Improve Signal vs Noise

- Noisy correlator:



- Use:

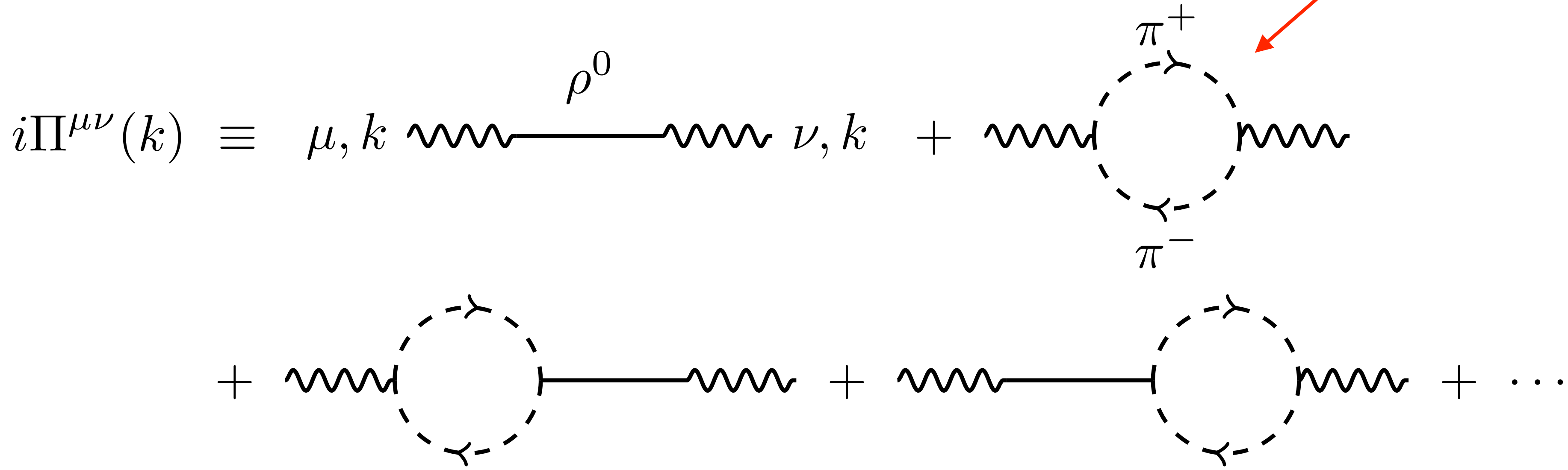
$$G(t) = \begin{cases} G_{\rm data}(t) & \text{for } t \leq t^* \\ G_{\rm fit}(t, T = \infty) & \text{for } t > t^* \end{cases}$$

- Vary t^* from 0.5 fm to 1.5 fm to estimate errors: ±0.5%

Correct Finite-Volume/Stagg.-Pions

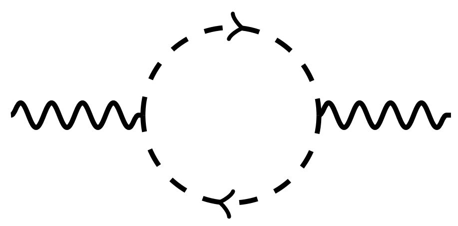
- Effective field theory for IR behavior: χPTh + explicit ρ.
  - ρ pole gives 70–80% of HVP contribution.
  - Dominant contributions:

70–80% of IR sensitivity

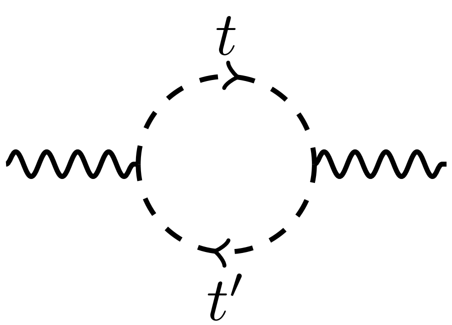


  - Parameters (m_π, e_π, m_ρ, f_ρ, $f_{\rho\pi\pi}$) all well known.

- Use effective theory to correct for two issues:
  - Finite lattice volume ($m_\pi L = 3.25 \rightarrow 5.4$).
  - Mass splittings ($\Delta m^2 \propto \alpha_s a^2$) between pions of different taste:



$$\rightarrow \frac{1}{\#\ \text{terms}} \sum_{\text{tastes } tt'}$$

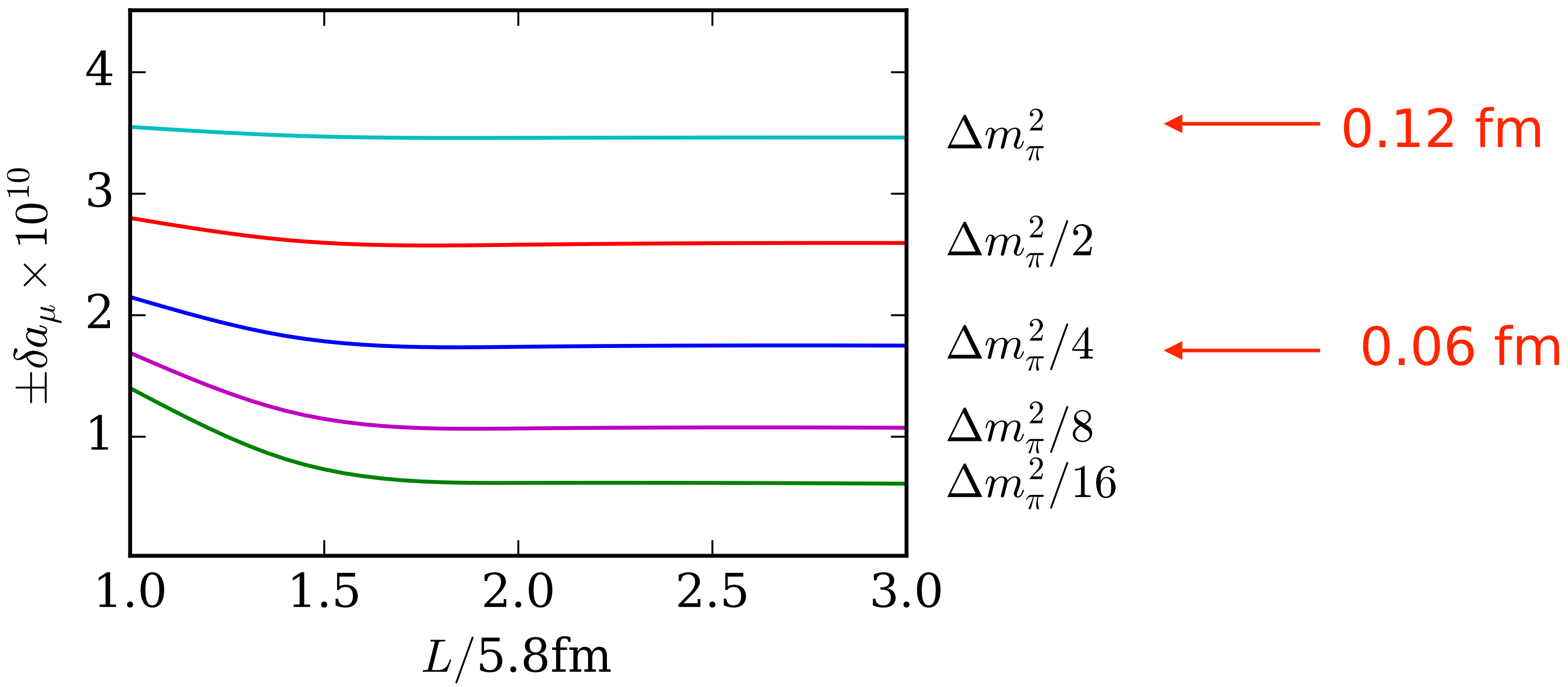


  - Correction is +7.0(7)% for physical u,d mass; 10x smaller for $m_{u,d}/m_s = 0.2$.

[Chakraborty et al (HPQCD) 1512.03270 (2015);

see also C. Aubin et al 1512.07555 (2015).]

- Staggered pion effects dominate finite volume errors, so need smaller lattice spacings to reduce this error:

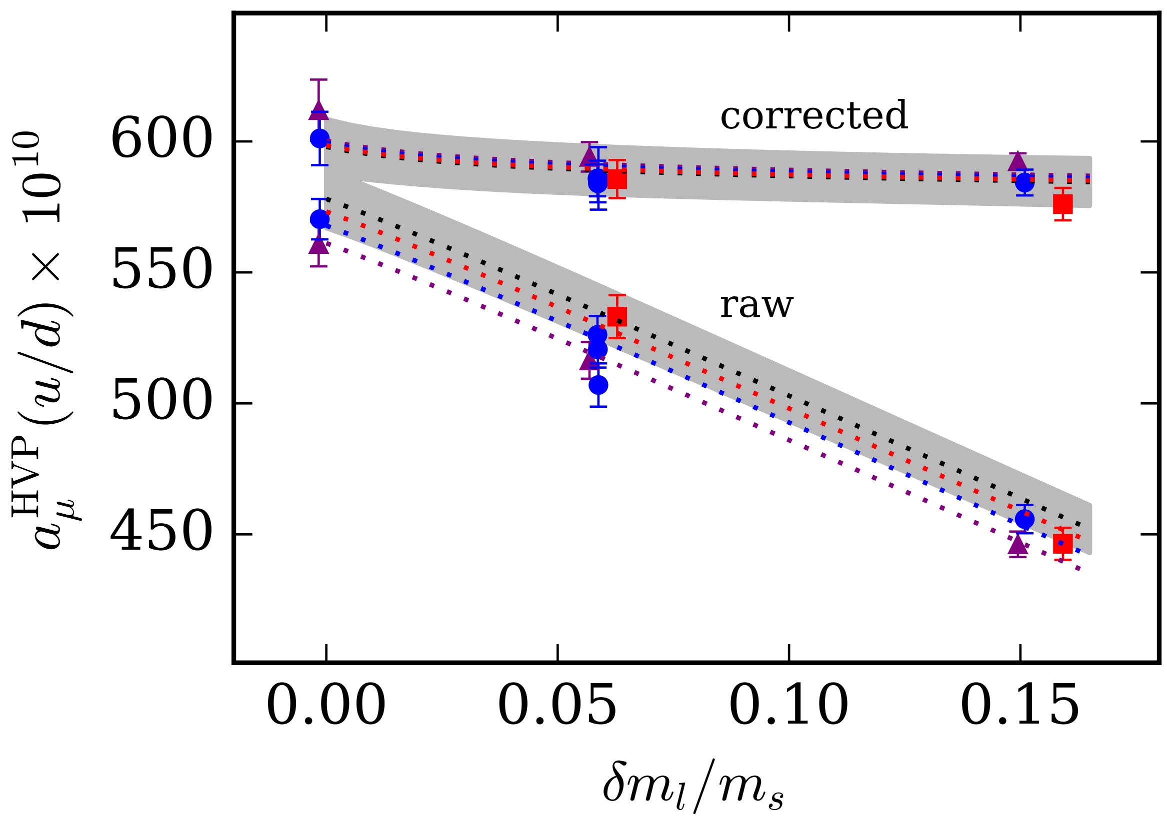


- And/or larger *u,d* masses (e.g., 0.1 m_s or 0.2 m_s).

Reduce Dependence on $m_\ell \equiv m_u = m_d$

Reduce dependence in 3 steps (see also ETMC 1308.4327):

- Remove $\pi^+\pi^-$ contribution from Π_j (m_π = lattice value).
- Rescale Π_j by $(m_\rho^{\mathrm{latt}} / m_\rho^{\mathrm{phys}})^{2j}$.
- Reintroduce $\pi^+\pi^-$ contribution but with physical m_π.



- Reduces $m_{u,d}$ dependence.
- Reduces a^2 dependence.
- Tests finite-volume + staggered-pion correction.

Answer: Connected HVP

- Fit corrected lattice data with:

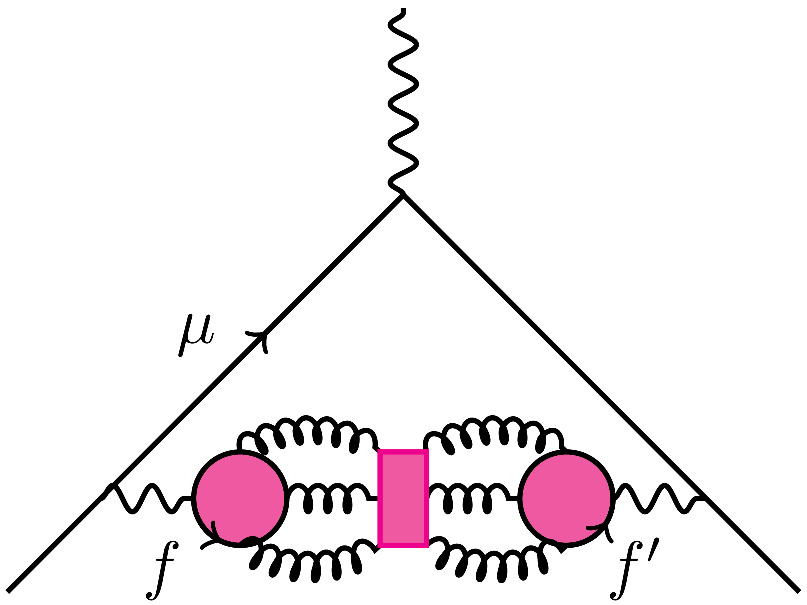
$$a_\mu(\mathrm{HVP,LO})\left(1 + c_\ell \frac{\delta m_\ell}{\Lambda} + c_s \frac{\delta m_s}{\Lambda} + \tilde{c}_\ell \frac{\delta m_\ell}{m_\ell} + c_{a^2} \frac{(a\Lambda)^2}{\pi^2}\right)$$

- Final *u,d* HVP from connected diagrams: 598(6)(8)x10^{-10}

- Error budget:

	$a_\mu^{\mathrm{HVP,LO}}(u/d)$
QED corrections:	1.0 %
Isospin breaking corrections:	1.0 %
Staggered pions, finite volume:	0.7 %
Noise reduction (t^*):	0.5 %
Valence m_ℓ extrapolation:	0.4 %
Monte Carlo statistics:	0.4 %
Padé approximants:	0.4 %
$a^2 \to 0$ extrapolation:	0.3 %
Z_V uncertainty:	0.4 %
Correlator fits:	0.2 %
Tuning sea-quark masses:	0.2 %
Lattice spacing uncertainty:	< 0.05 %
Total:	1.9 %

Answer: Disconnected HVP



- HPQCD + Hadspec 1512.03270:
  - phenom. : –1.5% x connected.
  - simulation: –0.2% x connected.
  - final estimate: 0 ± 1.5%
- RBC/UKQCD 1512.09054:
  - simulation: –1.6(7)%
- Need more precision but progress straightforward.

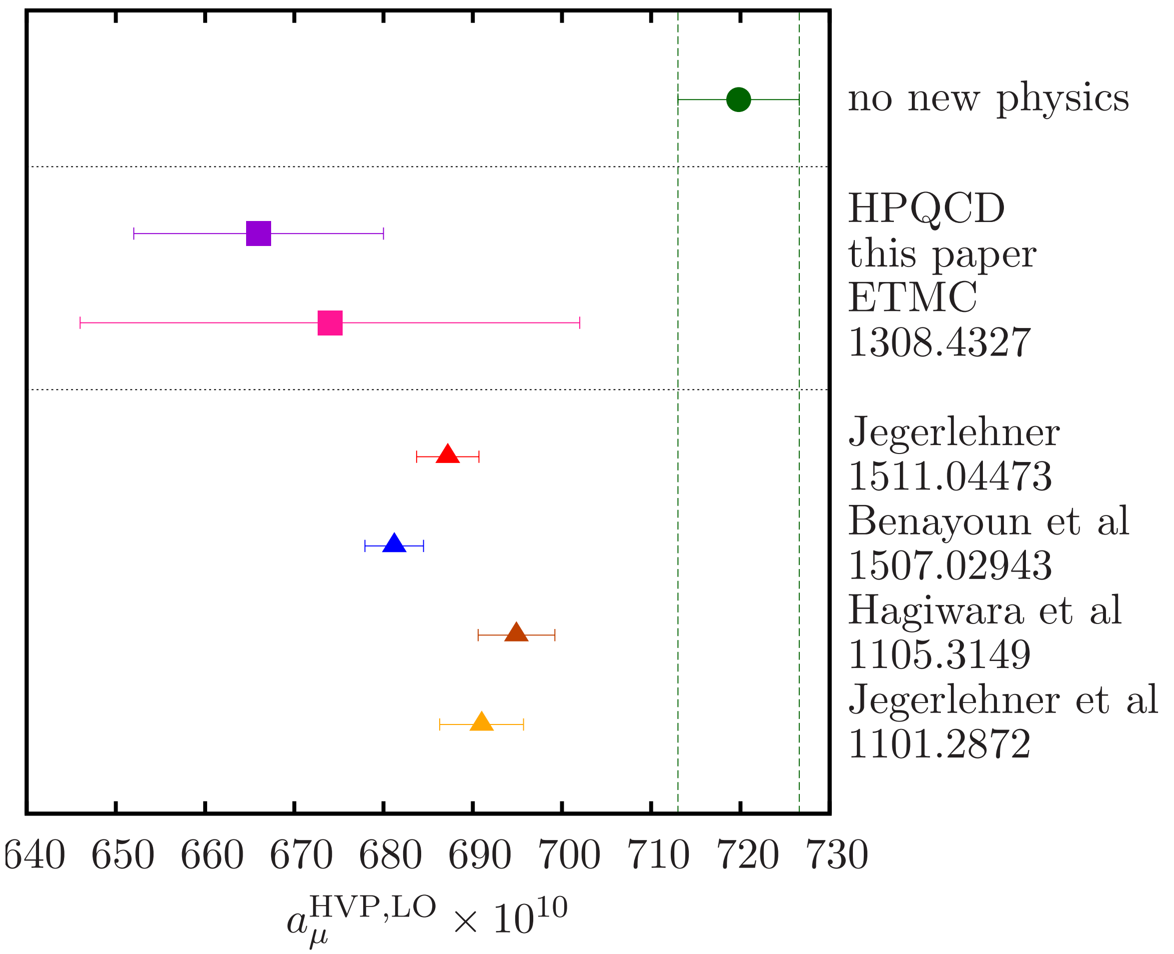
Answer: Total HVP Summary (HPQCD)

$$a_\mu(\mathrm{HVP,LO}) \times 10^{10} = \begin{cases} 598(11) & \text{from } u/d \\ 53.4(6) & \text{from } s \\ 14.4(4) & \text{from } c \\ 0.27(4) & \text{from } b \\ 0(9) & \text{from disc.} \end{cases}$$

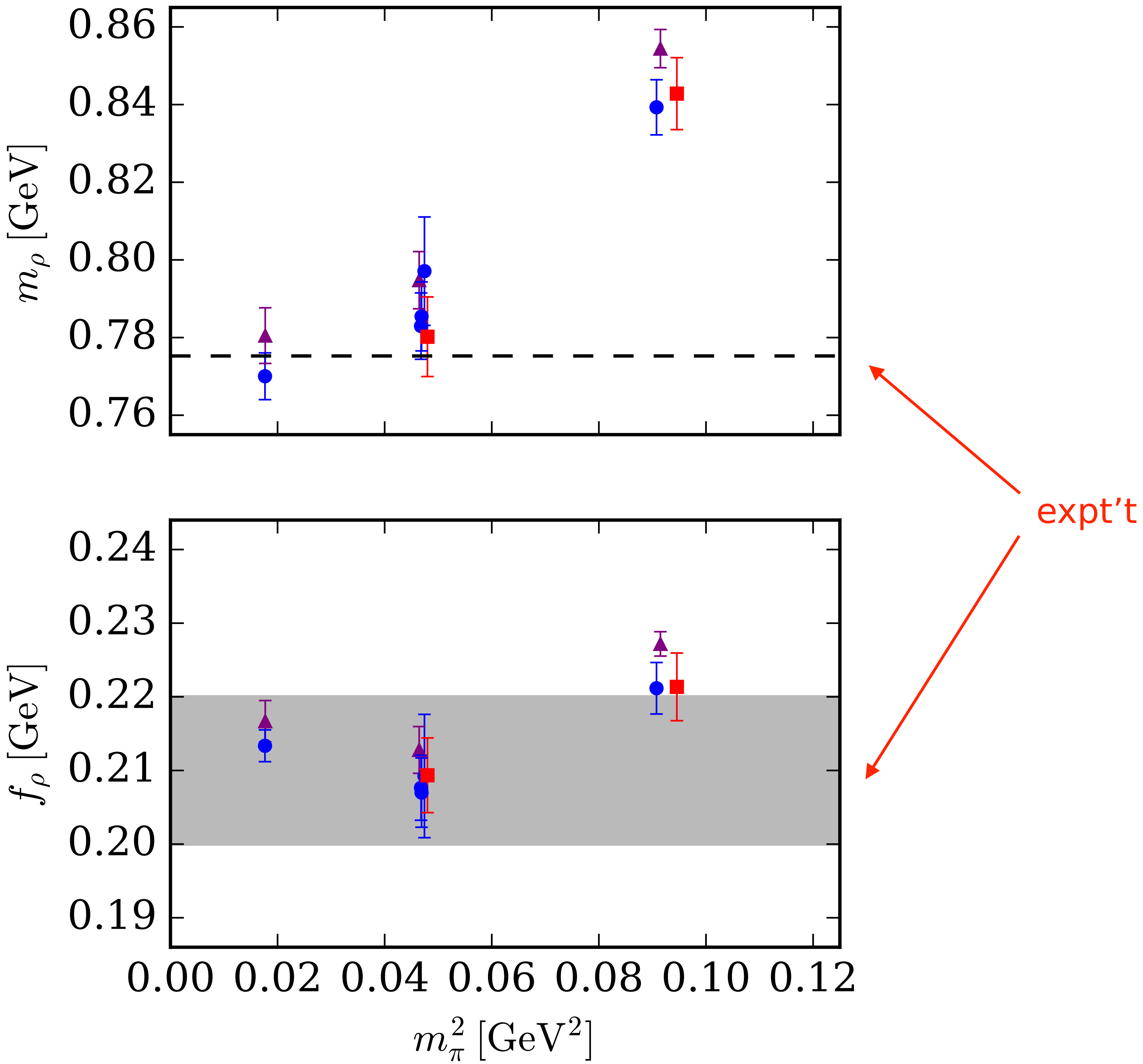
$$= \color{red}{666(6)(12)}$$

Lattice

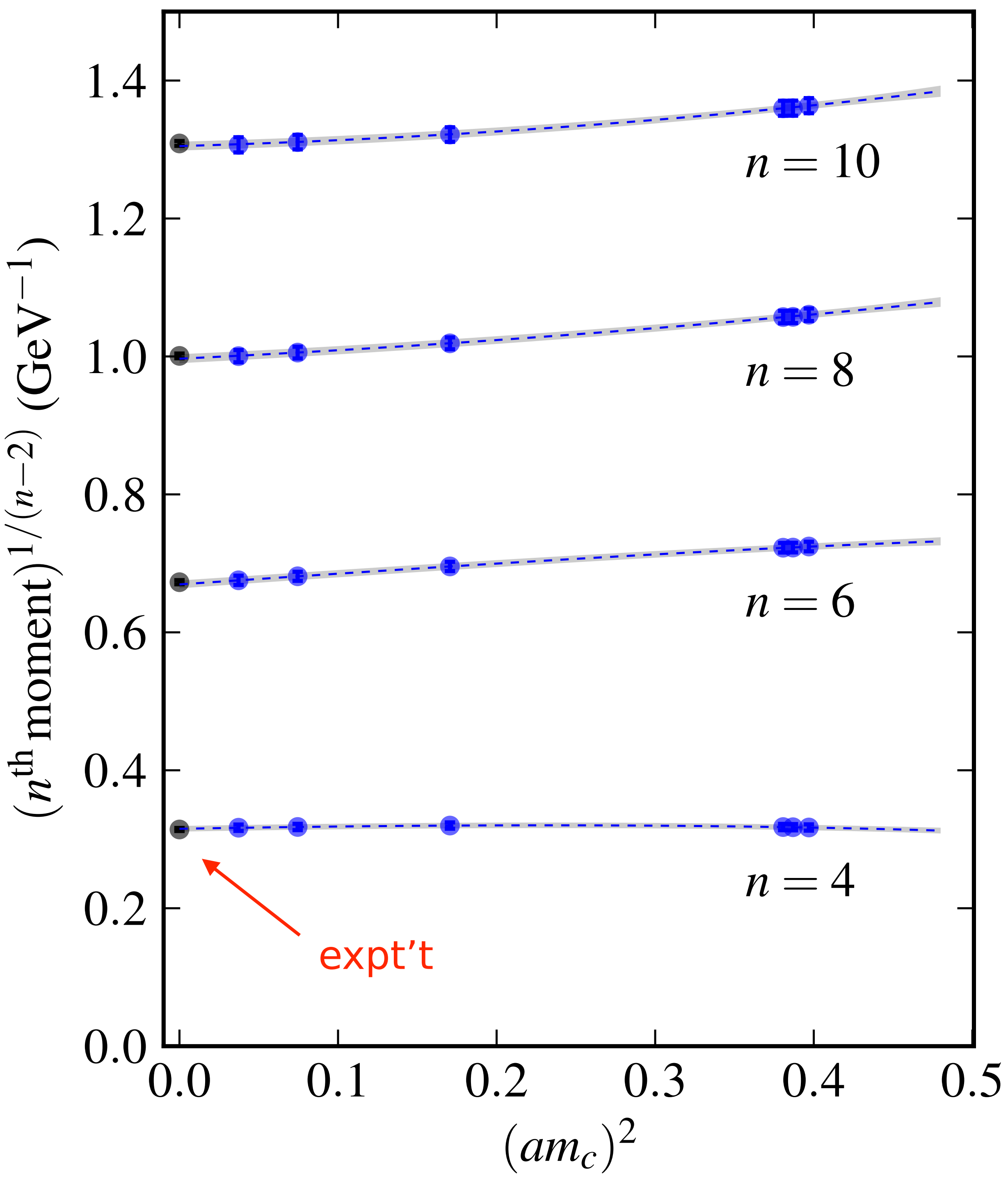
QED + Isospin



Sanity Checks: ρ Physics



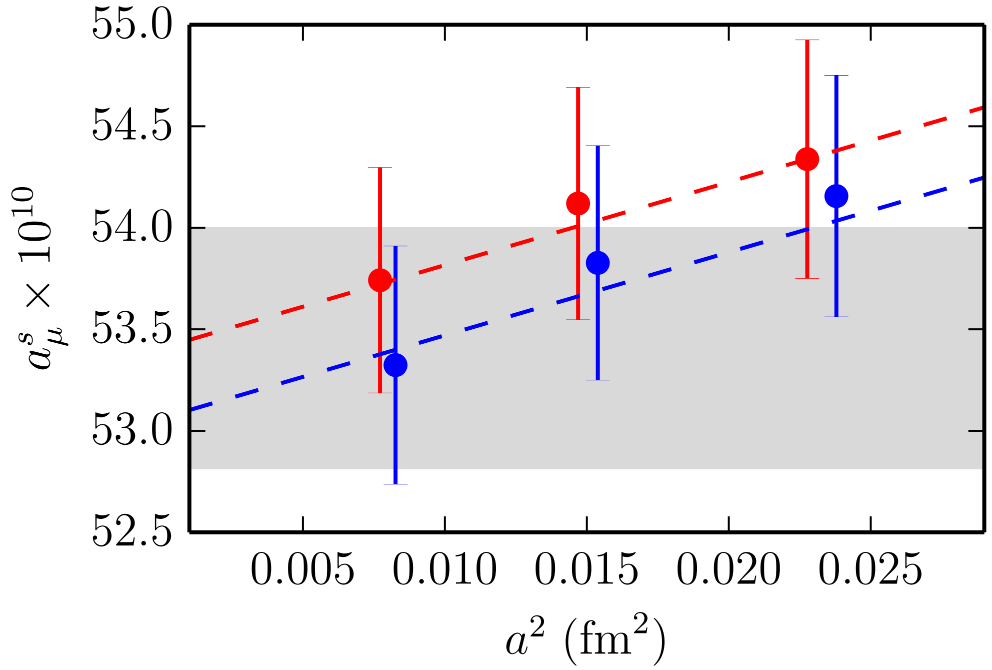
Sanity Checks: *c* Moments



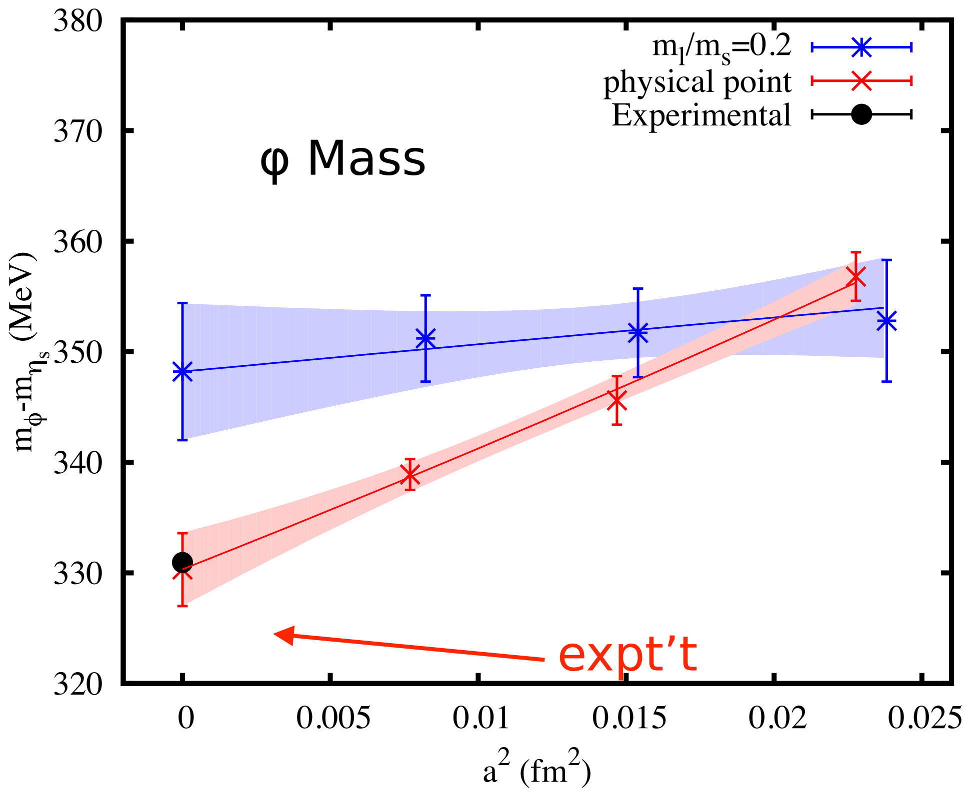
[HPQCD 1208.2855, 1004..4285]

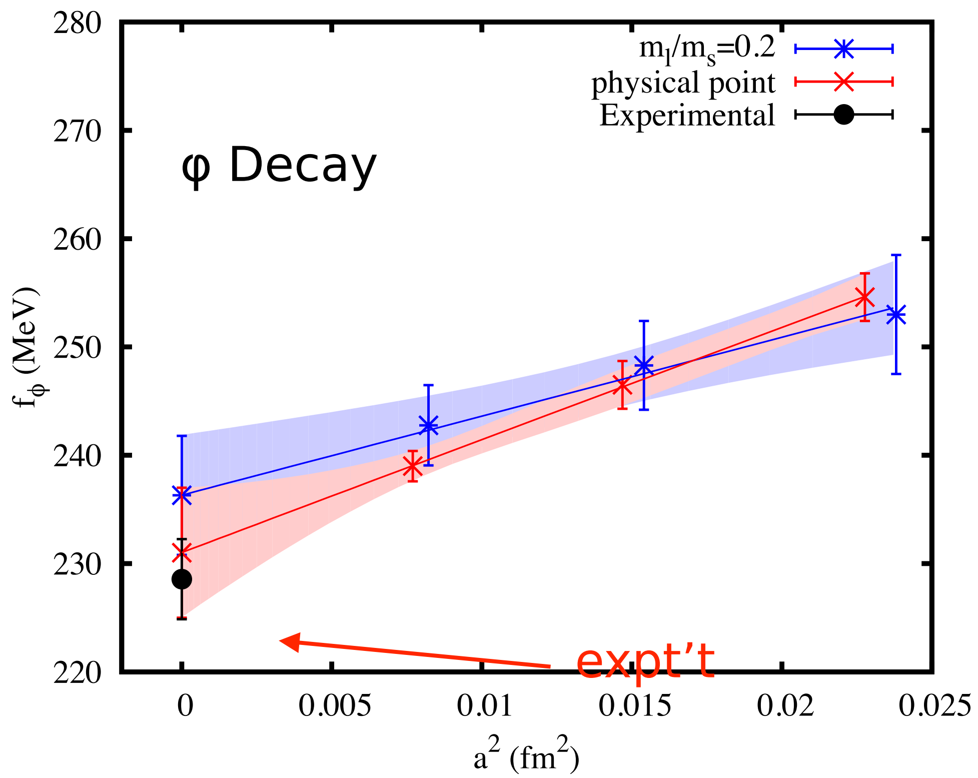
Sanity Checks: *s* Physics

[HPQCD 1403.1178]



m_ℓ 5x too big





Other Sanity Checks

- Lots of other sub-1% LQCD results: see tomorrow's colloquium.

Future: ±2% → ±0.2% ?

- Need QED and isospin breaking in simulations (1.4%).
- Need improved analysis of disconnected HVP (0.7%).
- Need smaller lattice spacings (0.06 fm).
  - Remove staggered-pion effects (0.7%).
- Need more statistics (10x).
  - Reduces statistical errors.
  - Reduces systematic errors from Padés, quark-mass and a^2 extrapolations, noise reduction (t^*) … (0.2–0.4% each).
- Merge LQCD with continuum e^+e^- analyses? Moments?