A Broadband/Resonant Experiment for Axion Dark Matter Detection

[arXiv:1602.01086]

Yoni Kahn
Princeton University

with Ben Safdi and Jesse Thaler @ MIT

TRIUMF Precision Workshop, 6/8/16

Axions and ALPS

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \widetilde{F}^{\mu\nu}$$

Axions: $m_a \propto g_{a\gamma\gamma}$

ALPs: $m_a,\ g_{a\gamma\gamma}$ independent

Axions: also couple to $G_{\mu\nu}\widetilde{G}^{\mu\nu}$

ALPs: not necessarily

Can be very light! Mass protected by a shift symmetry

$a \to a + \text{const.}$ because $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ is a total derivative

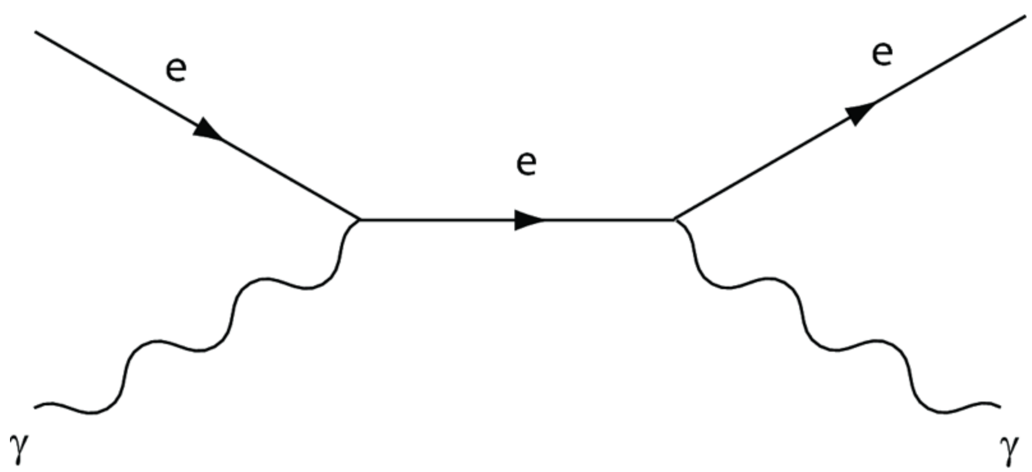
Motivations for light axions:

- Strong CP problem
- String theory

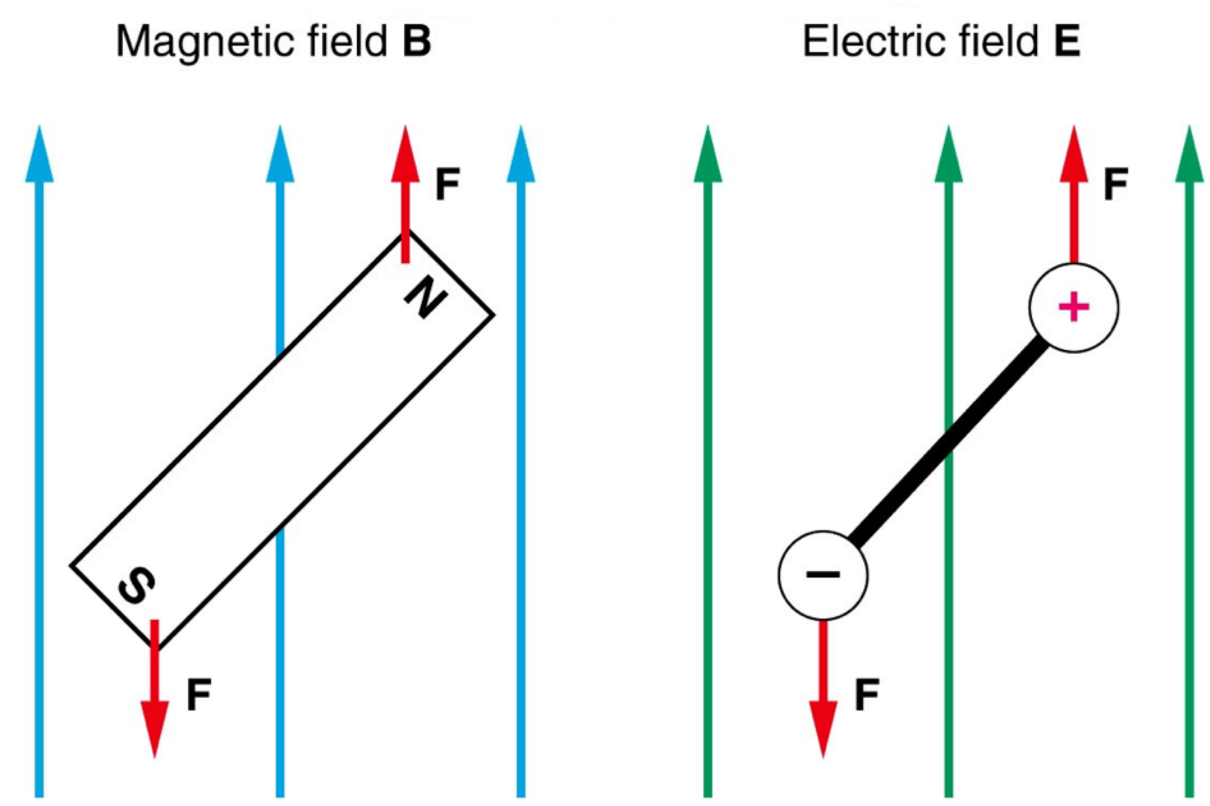
Can be DM either by thermal production (> eV)
or misalignment production (< eV)

ALP DM: field, not particle

Useful analogy:



vs.

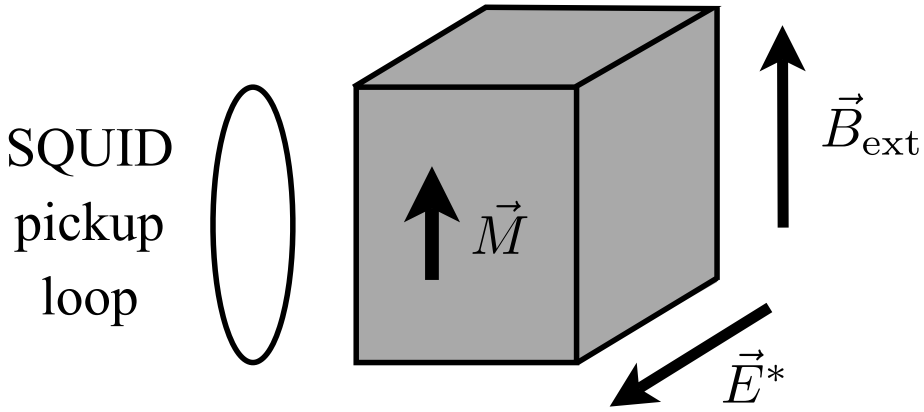


Light bosonic DM behaves collectively:
think in terms of charges and currents,
not Feynman diagrams

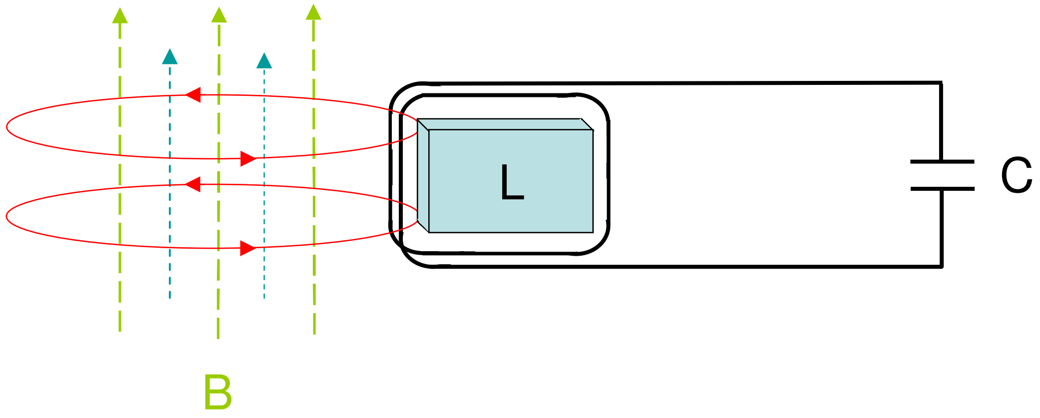
Strategies for ALP DM detection

Resonant cavity (ADMX)

Nuclear EDMs (CASPEr)



LC circuit (Thomas/Cabrera/Sikivie)



Signal is a weak, oscillating magnetic field

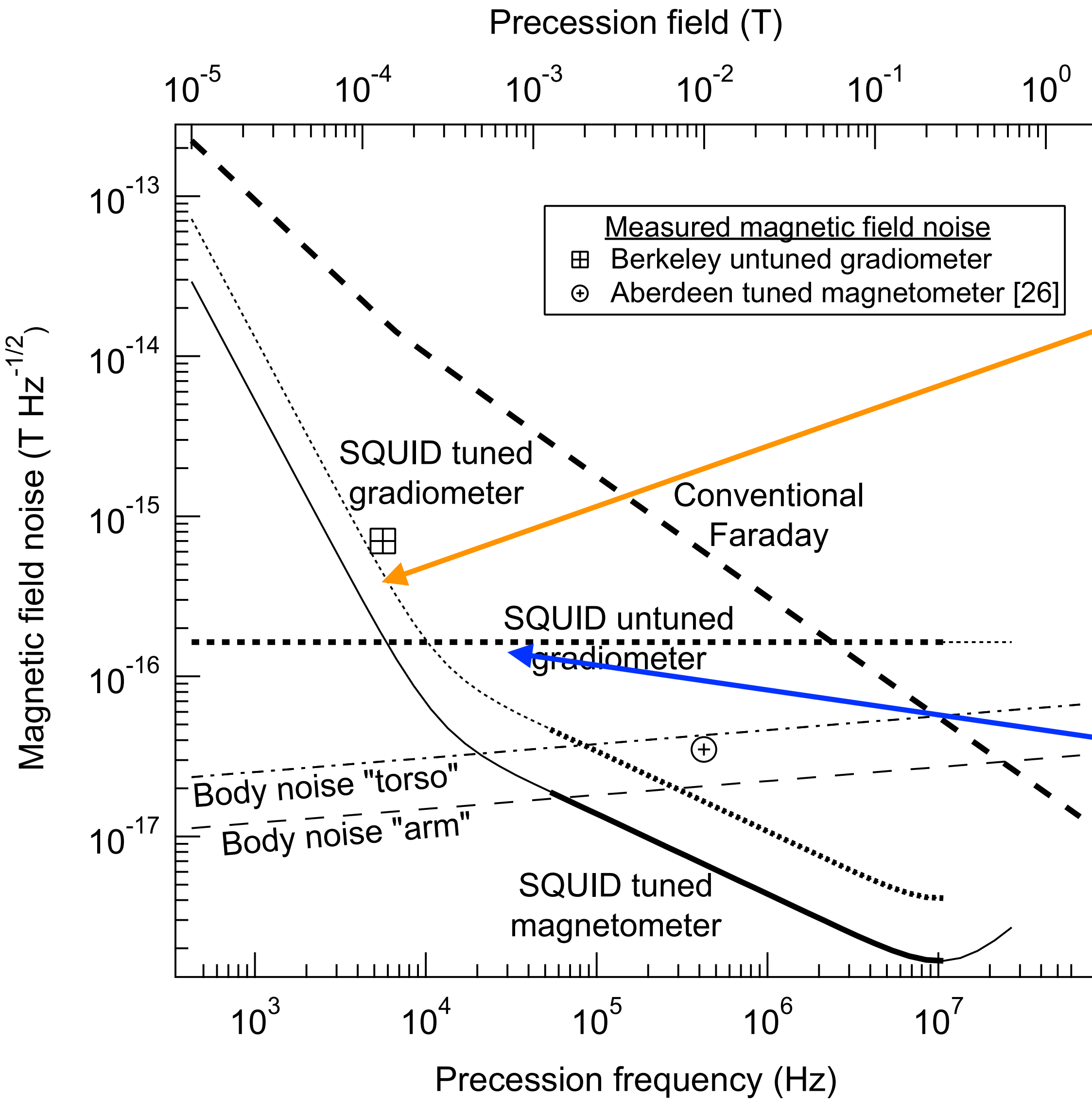
All require resonance, scanning over frequencies

Broadband + Resonant

Does axion DM detection require resonant enhancement?

Hints from precision magnetometry:

[Myers et al 2007]



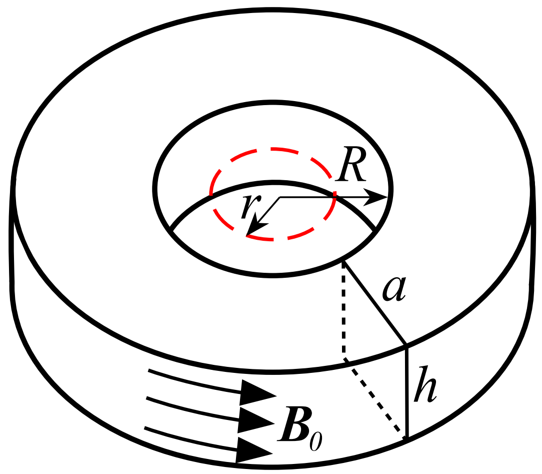
resonant ("tuned") circuit: thermal noise

broadband ("untuned") circuit: magnetometer noise

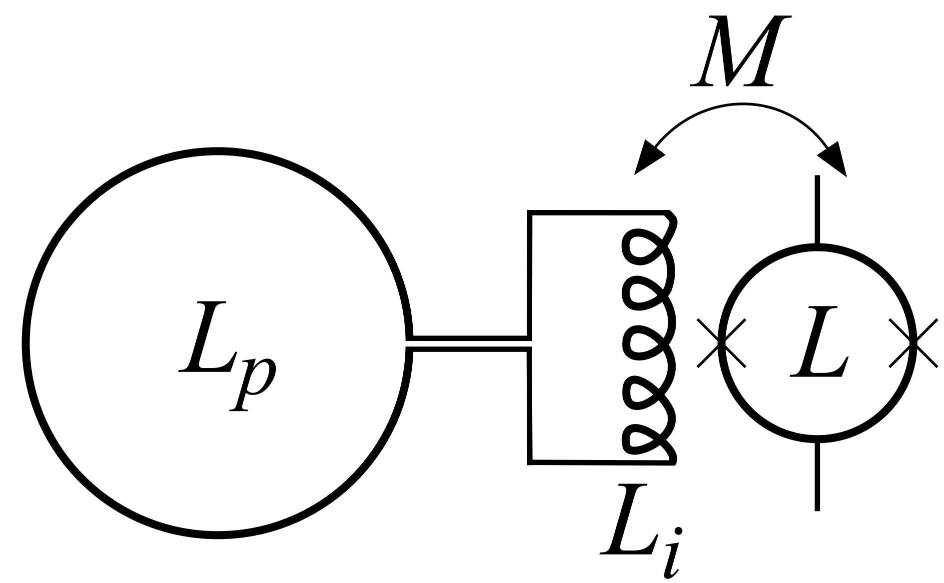
Requires superconducting pickup: zero-field detection

Outline

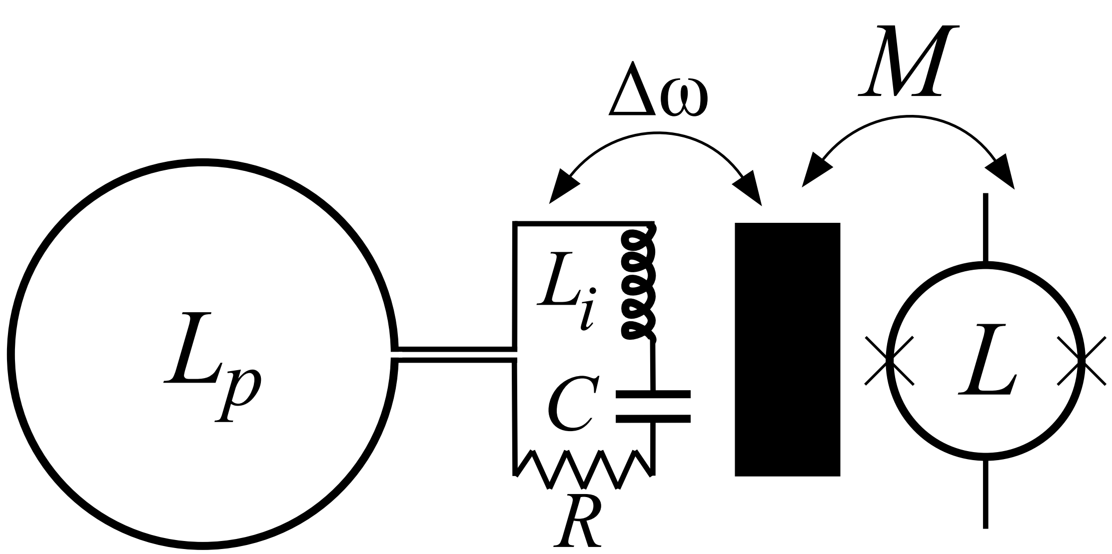
I. Experimental design



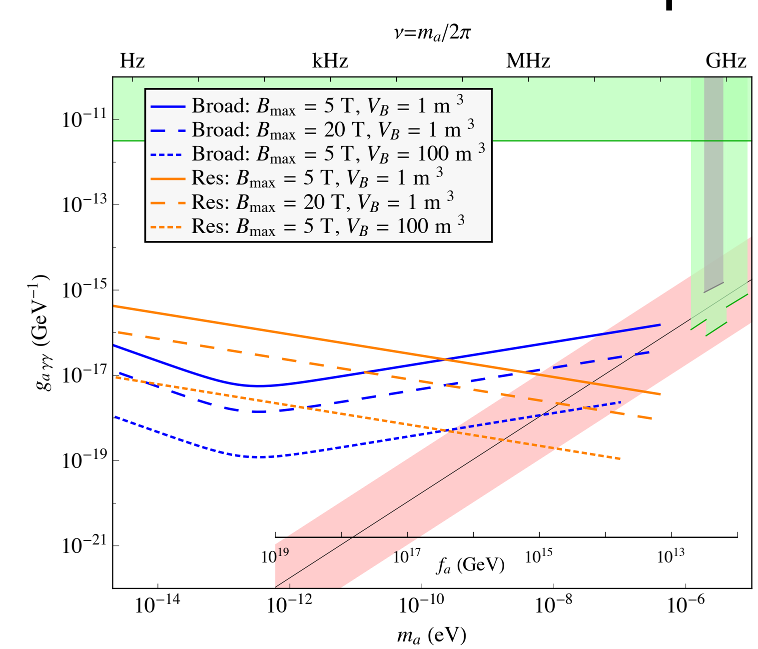
II. Broadband readout



III. Resonant readout

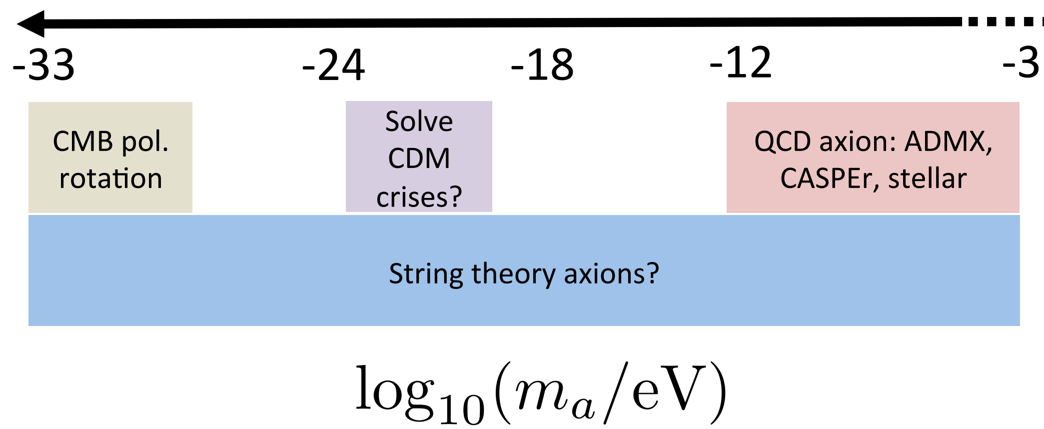


IV. Reach and comparison



Axion-like-particle (ALP) parameter space

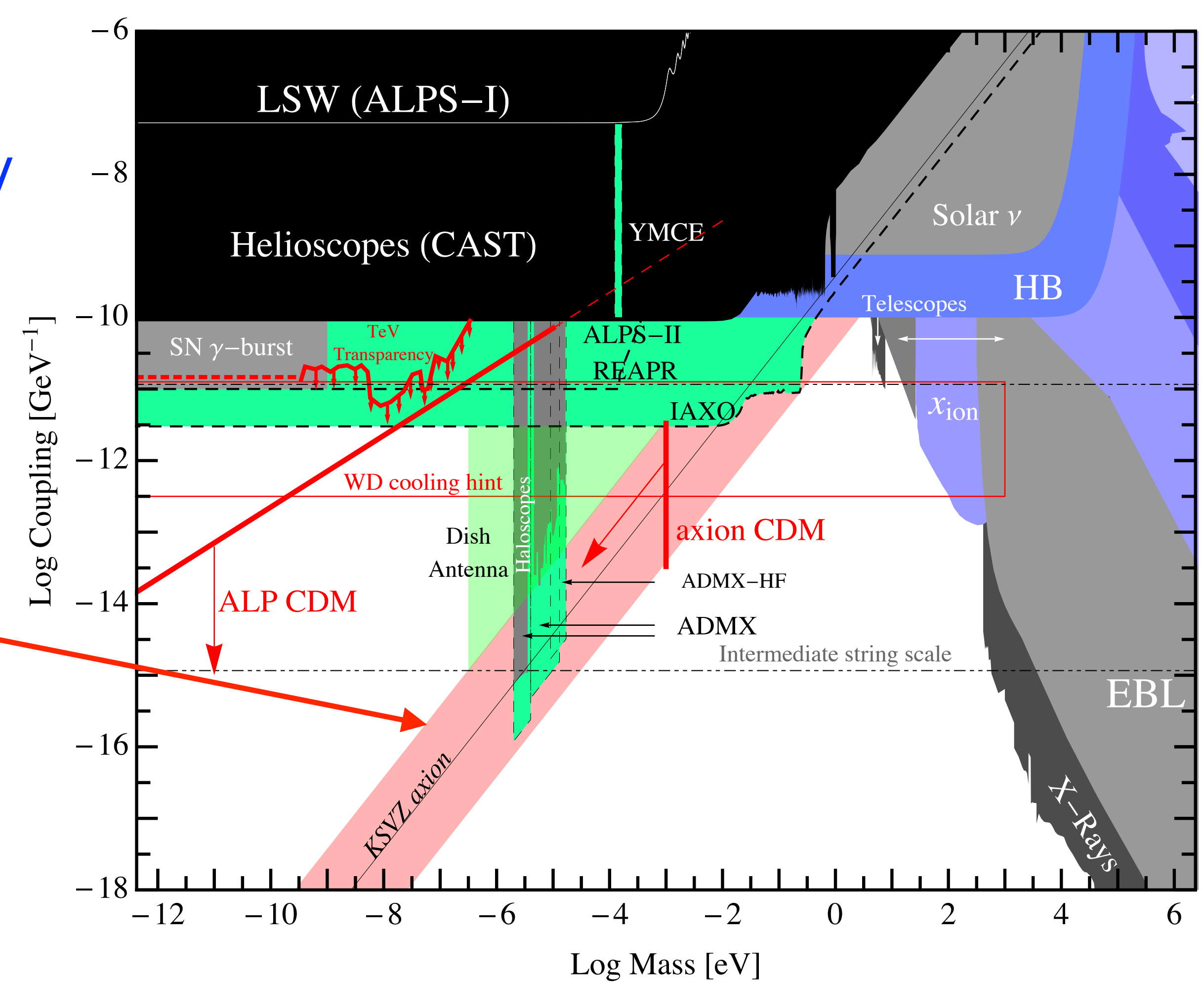
Enormous mass range:



[D. Marsh, 1510.07633]

Couples very weakly to photons:

QCD axion: mass $\propto$ coupling



[Snowmass, 1311.0029]

ALP DM: Properties today

Focus on mass range $m_a \ll 1\text{eV}$

Bosonic DM + macroscopic occupation # = classical field:

$$a(t) = a_0 \sin(m_a t) = \frac{\sqrt{2\rho_{\rm DM}}}{m_a} \sin(m_a t)$$

ALP DM: Properties today

Focus on mass range $m_a \ll 1\text{eV}$

Bosonic DM + macroscopic occupation # = classical field:

$$a(t) = a_0 \sin(m_a t) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \sin(m_a t)$$

Spatially and temporally coherent on macroscopic scales:

$$\lambda \sim \frac{2\pi}{m_a v_{DM}} \approx 100 \text{ km} \frac{10^{-8} \text{ eV}}{m_a}$$

$$\tau \sim \frac{2\pi}{m_a v_{DM}^2} \approx 0.4 \text{ s} \frac{10^{-8} \text{ eV}}{m_a}$$

ALP DM: Properties today

Focus on mass range $m_a \ll 1\mathrm{eV}$

Bosonic DM + macroscopic occupation # = classical field:

$$a(t) = a_0 \sin(m_a t) = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_a} \sin(m_a t)$$

In presence of background EM fields,
induces oscillating response fields:

ALP DM: Properties today

Focus on mass range $m_a \ll 1\mathrm{eV}$

Bosonic DM + macroscopic occupation # = classical field:

$$a(t) = a_0 \sin(m_a t) = \frac{\sqrt{2\rho_{\rm DM}}}{m_a}\sin(m_a t)$$

In presence of background EM fields,
induces oscillating response fields:

$$\nabla \times \mathbf{B}_r = \frac{\partial \mathbf{E}_r}{\partial t} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \nabla a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

$$\nabla \cdot \mathbf{E}_r = -g_{a\gamma\gamma}\mathbf{B}_0 \cdot \nabla a$$

ALP DM: Properties today

Focus on mass range $m_a \ll 1\mathrm{eV}$

Bosonic DM + macroscopic occupation # = classical field:

$$a(t) = a_0 \sin(m_a t) = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_a} \sin(m_a t)$$

In presence of background EM fields,
induces oscillating response fields:

$$\nabla \times \mathbf{B}_r = \frac{\partial \mathbf{E}_r}{\partial t} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \cancel{\nabla} a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

$$\nabla \cdot \mathbf{E}_r = -g_{a\gamma\gamma} \mathbf{B}_0 \cdot \cancel{\nabla} a$$

gradients suppressed by $v_{DM} \sim 10^{-3}$

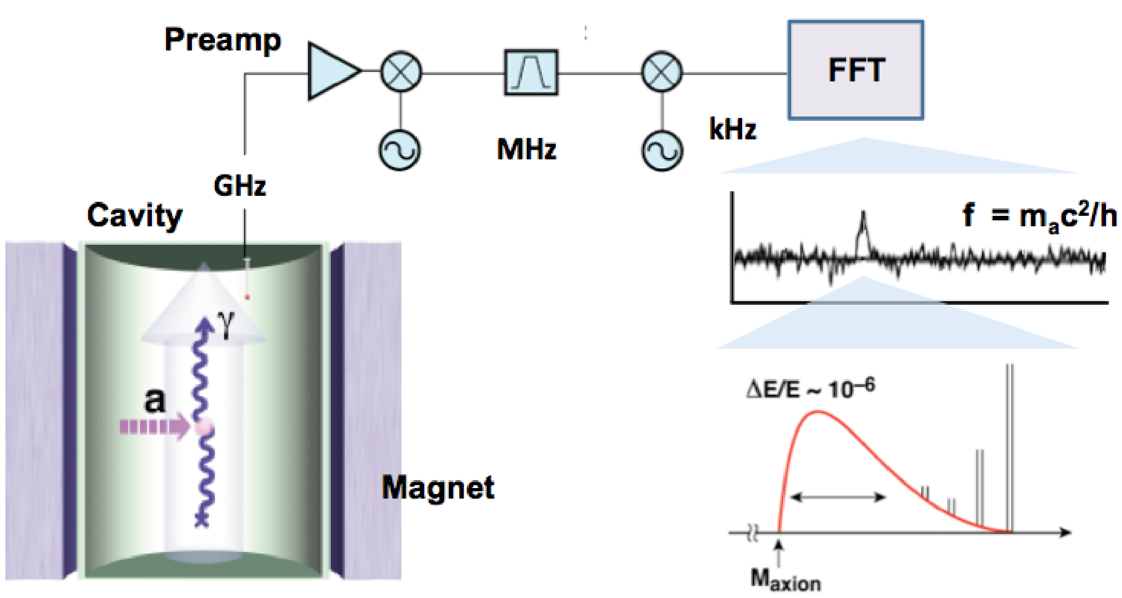
ADMX: resonant cavity detection

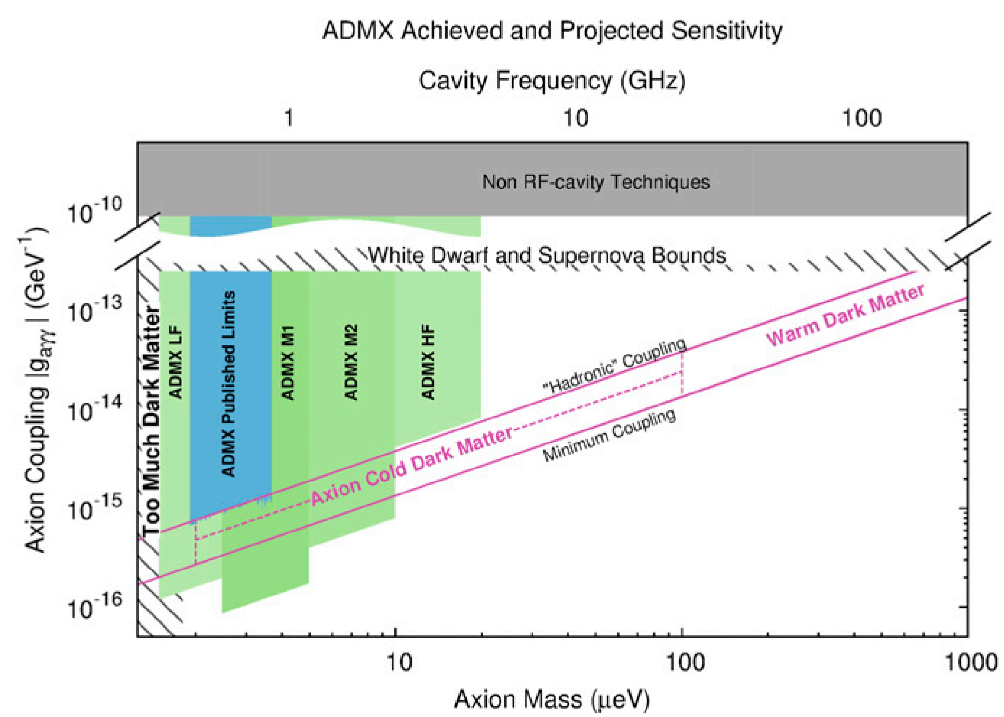
[see G. Rybka's talk for more details]

$$\mathcal{L} \supset a\mathbf{E} \cdot \langle \mathbf{B} \rangle$$

static B-field

$$P \sim g_{a\gamma\gamma}^2 \frac{\rho_{\rm DM}}{m_a} B_0^2 V Q$$





- Measures coupling to $F_{\mu\nu}\widetilde{F}^{\mu\nu}$
- Measurement taken in external B field
- Cavity b.c. fix mass range to cavity size

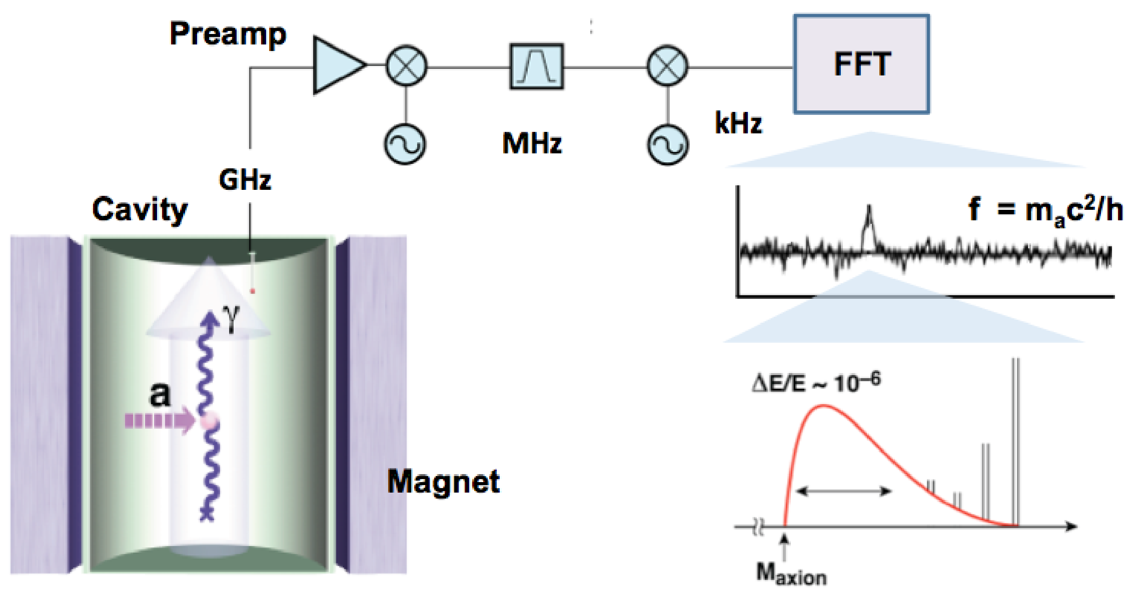
ADMX: resonant cavity detection

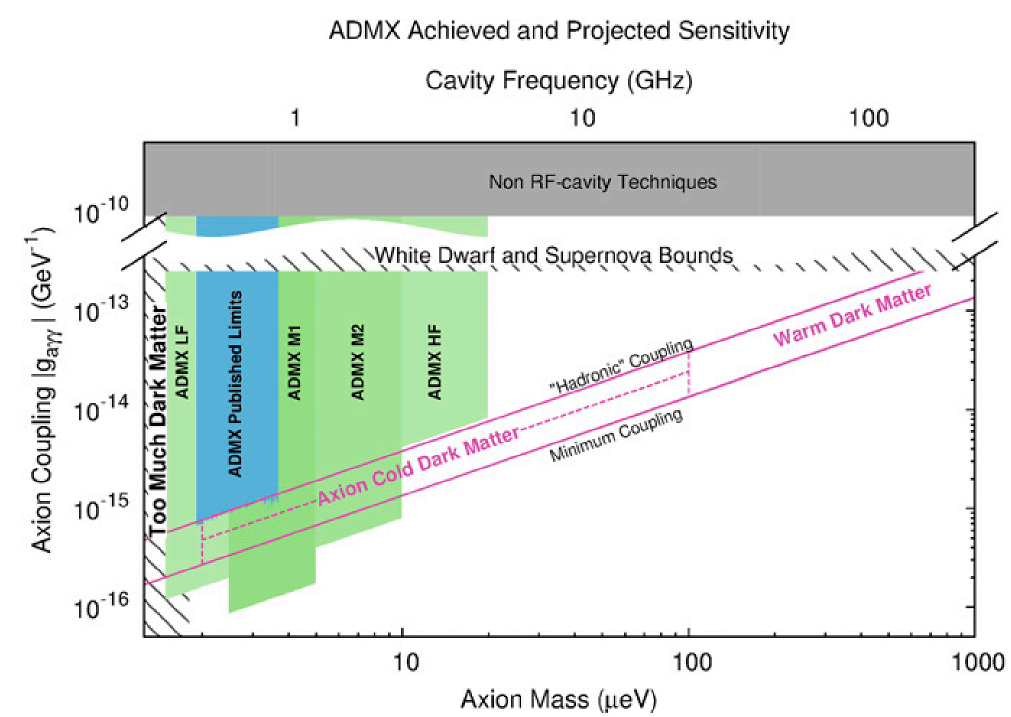
[see G. Rybka's talk for more details]

$$\mathcal{L} \supset a\mathbf{E} \cdot \langle \mathbf{B} \rangle$$

static B-field

$$\boxed{P \sim g_{a\gamma\gamma}^2 \frac{\rho_{\mathrm{DM}}}{m_a} B_0^2 V Q}$$





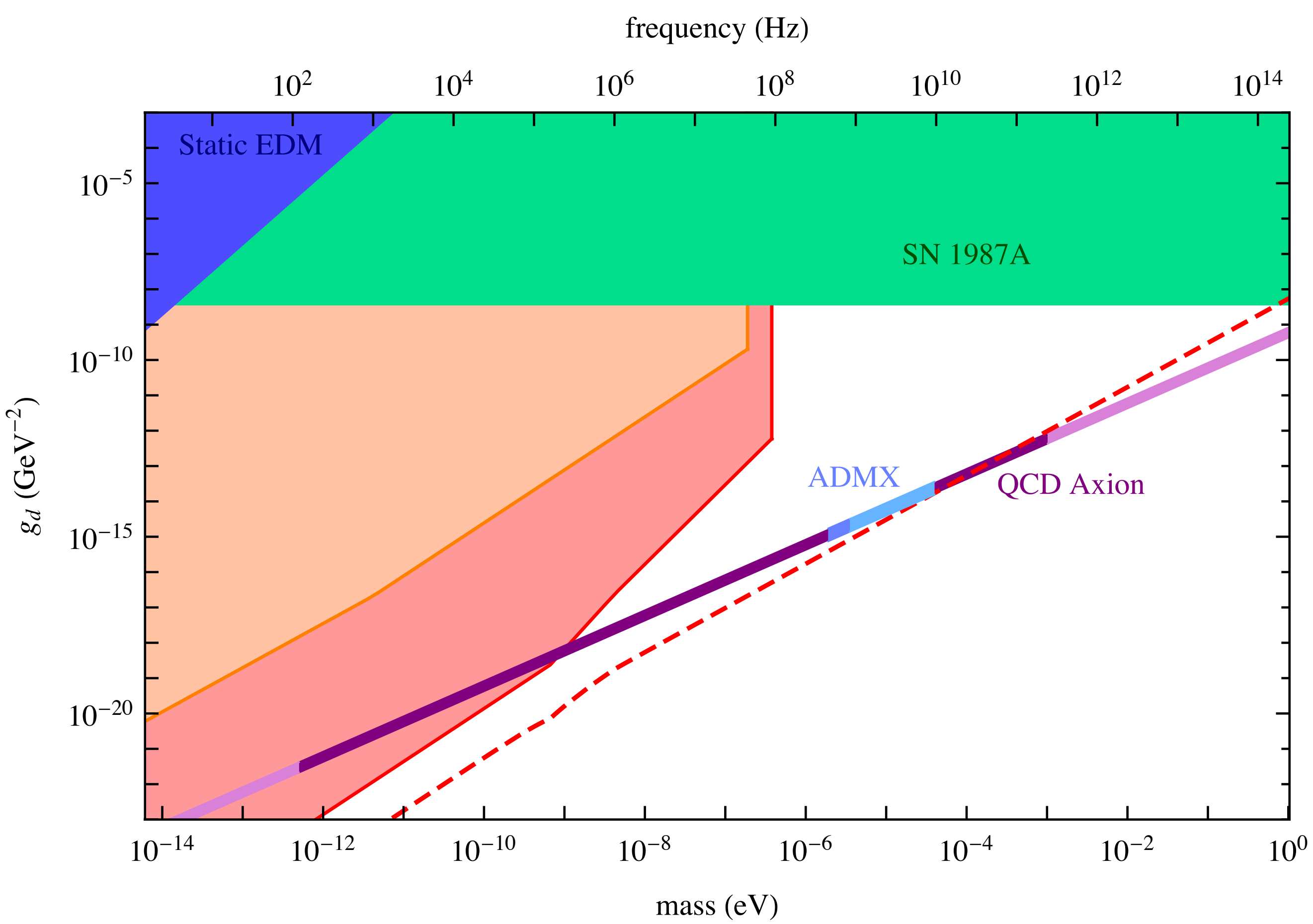
- Measures coupling to $F_{\mu\nu}\widetilde{F}^{\mu\nu}$
- Measurement taken in external B field
- Cavity b.c. fix mass range to cavity size

CASPEr: NMR detection

[Budker et al., 1306.6089]

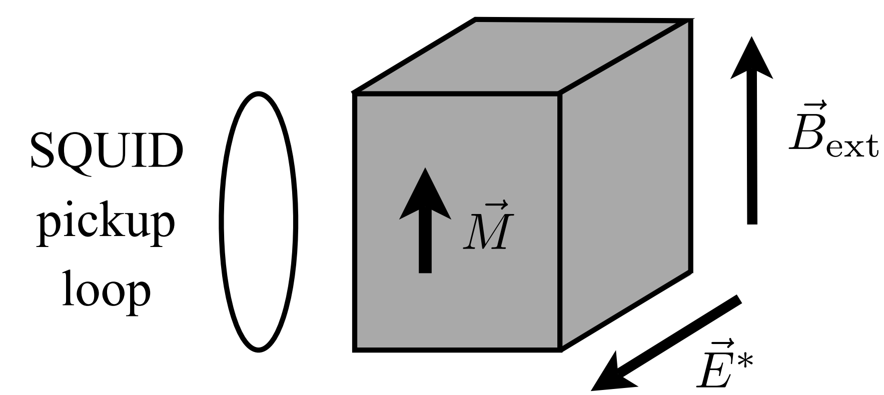
$$\mathcal{L} \supset -\frac{i}{2}\underbrace{g_d}_{d_n \approx 2.4\times 10^{-16}\frac{a}{f_a}e\cdot\mathrm{cm}\ \text{for QCD}} a\overline{N}\sigma_{\mu\nu}\gamma_5 N F^{\mu\nu}$$

$$d_n = g_d \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_a}\cos(m_a t)$$



resonance

$$M(t) \approx np\mu E^*\epsilon_S d_n \frac{\sin\left[(2\mu B_{\mathrm{ext}} - m_a)t\right]}{2\mu B_{\mathrm{ext}} - m_a}\sin(2\mu B_{\mathrm{ext}} t)$$



- Measures coupling to $G_{\mu\nu}\widetilde{G}^{\mu\nu}$ through nucleon spin
- Measurement taken in external B (and E) field
- Sensitivity to much lower masses! Win by volume

Axion-sourced current

$$\nabla \times \mathbf{B}_r = \frac{\partial \mathbf{E}_r}{\partial t} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \nabla a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

Axion-sourced current

$$v_{DM} \ll 1$$

$$\nabla \times \mathbf{B}_r = \frac{\partial \mathbf{E}_r}{\partial t} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \cancel{\nabla} a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

Axion-sourced current

$$v_{DM} \ll 1$$

$$\nabla \times \mathbf{B}_r = \cancel{\frac{\partial \mathbf{E}_r}{\partial t}} - g_{a\gamma\gamma} \left(\mathbf{E}_0 \times \cancel{\nabla} a - \mathbf{B}_0 \frac{\partial a}{\partial t} \right)$$

(quasistatic approximation)

Axion-sourced current

$$v_{DM} \ll 1$$

$$\nabla \times \mathbf{B}_r = \cancel{\frac{\partial \mathbf{E}_r}{\partial t}} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \cancel{\nabla} a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

(quasistatic approximation)

$$\implies \mathbf{J}_{\text{eff}} = g_{a\gamma\gamma}\sqrt{2\rho_{\text{DM}}}\cos(m_a t)\mathbf{B}_0$$

Current follows lines of **B,** oscillates at axion mass

How to detect an oscillating current?

Axion-sourced current

$v_{DM} \ll 1$

$$\nabla \times \mathbf{B}_r = \cancel{\frac{\partial \mathbf{E}_r}{\partial t}} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \cancel{\nabla} a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

(quasistatic approximation)

$$\implies \mathbf{J}_{\mathrm{eff}} = g_{a\gamma\gamma}\sqrt{2\rho_{\mathrm{DM}}}\cos(m_a t)\mathbf{B}_0$$

Current follows lines of **B,** oscillates at axion mass

How to detect an oscillating current?

- Radiated power (at infinity)
- Time-varying flux (locally)

Axion-sourced current

$v_{DM} \ll 1$

$$\nabla \times \mathbf{B}_r = \cancel{\frac{\partial \mathbf{E}_r}{\partial t}} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \cancel{\nabla} a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

(quasistatic approximation)

$$\implies \mathbf{J}_{\text{eff}} = g_{a\gamma\gamma}\sqrt{2\rho_{\text{DM}}}\cos(m_a t)\mathbf{B}_0$$

Current follows lines of **B,** oscillates at axion mass

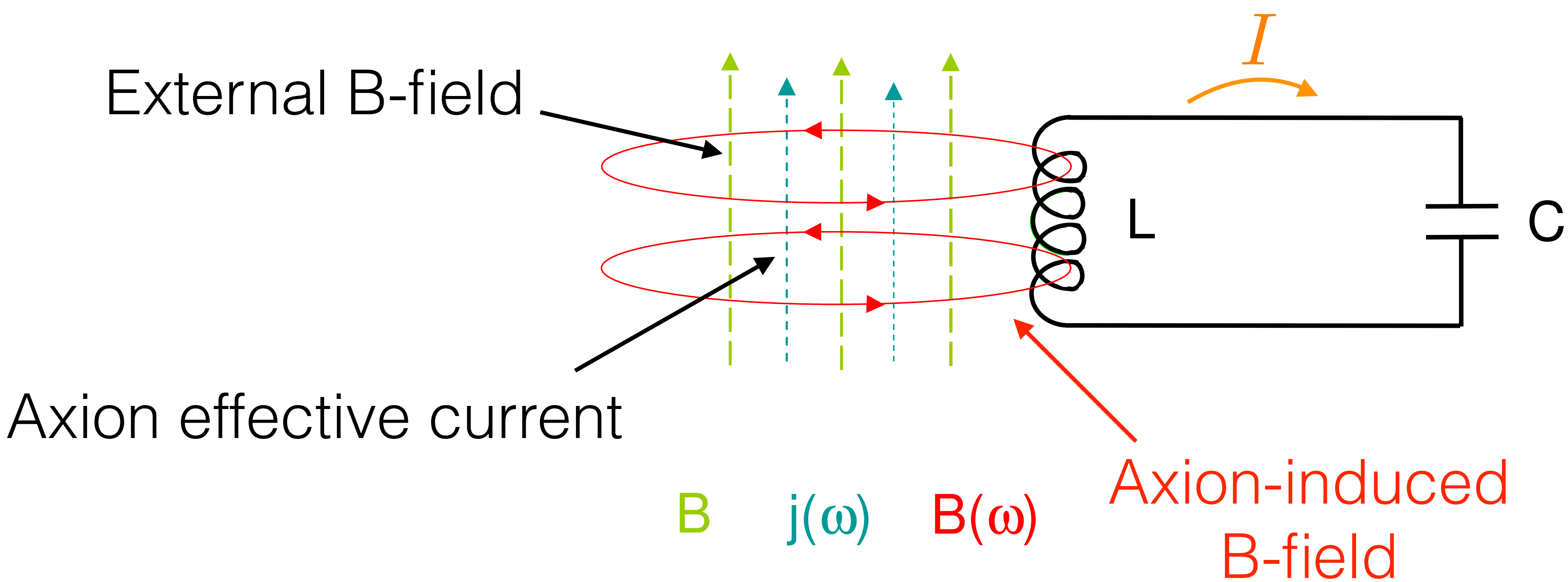
How to detect an oscillating current?

- Radiated power (at infinity)

- $\boxed{\text{Time-varying flux (locally)}}$

LC pickup

[S. Thomas and B. Cabrera; P. Sikivie et al, 1310.8545]



$$I = \frac{Q}{L} V_B g_{a\gamma\gamma} \sqrt{2\rho_{\mathrm{DM}}} B_0$$

- Measures coupling to $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ (like ADMX)
- Measurement taken in external B-field
- Same volume enhancement, sensitivity to small m_a

Summary: existing proposals

- Resonant conversion: mass pinned to size of cavity
- Require some kind of tuning to attain resonance
- Signal detection in external fields

Goal: cover low masses, keep volume enhancement, but detection in zero external field region

Summary: existing proposals

- Resonant conversion: mass pinned to size of cavity
- Require some kind of tuning to attain resonance
- Signal detection in external fields

Goal: cover low masses, keep volume enhancement, but detection in zero external field region

ABRACADABRA!

Summary: existing proposals

- Resonant conversion: mass pinned to size of cavity
- Require some kind of tuning to attain resonance
- Signal detection in external fields

Goal: cover low masses, keep volume enhancement, but detection in zero external field region

“**A** **B**roadband/**R**esonant **A**pproach to **C**osmic **A**xion **D**etection with an **A**mplifying **B**-field **R**ing **A**pparatus”

ABRACADABRA!

Summary: existing proposals

- Resonant conversion: mass pinned to size of cavity
- Require some kind of tuning to attain resonance
- Signal detection in external fields

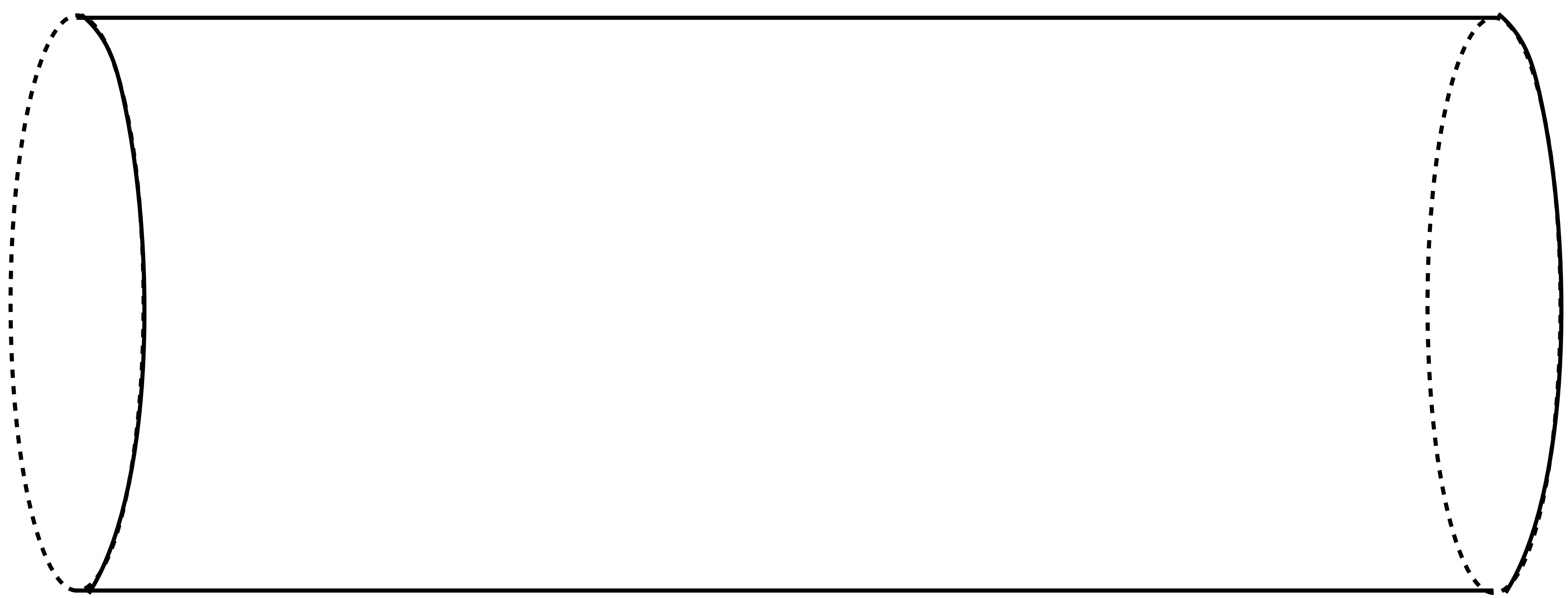
Goal: cover low masses, keep volume enhancement, but detection in zero external field region

"**A** **B**roadband/**R**esonant **A**pproach to **C**osmic **A**xion **D**etection with an **A**mplifying **B**-field **R**ing **A**pparatus"

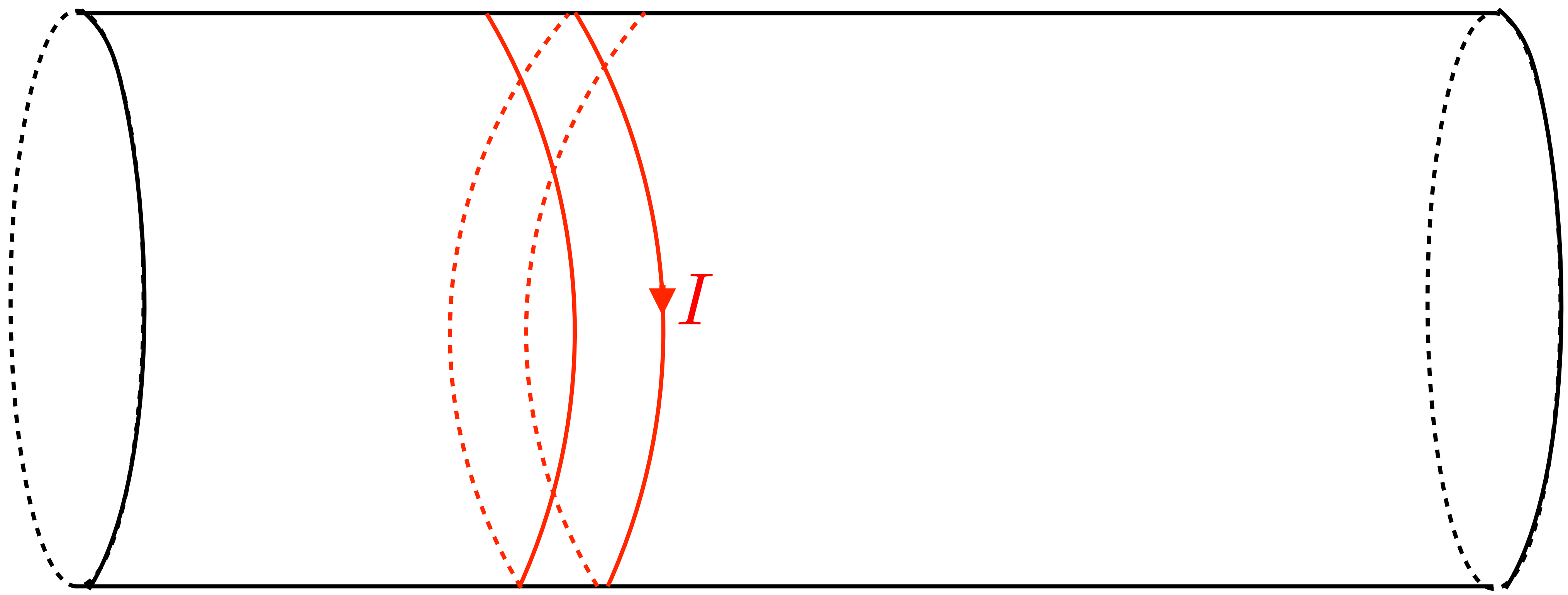
ABRACADABRA!

(J. Thaler)

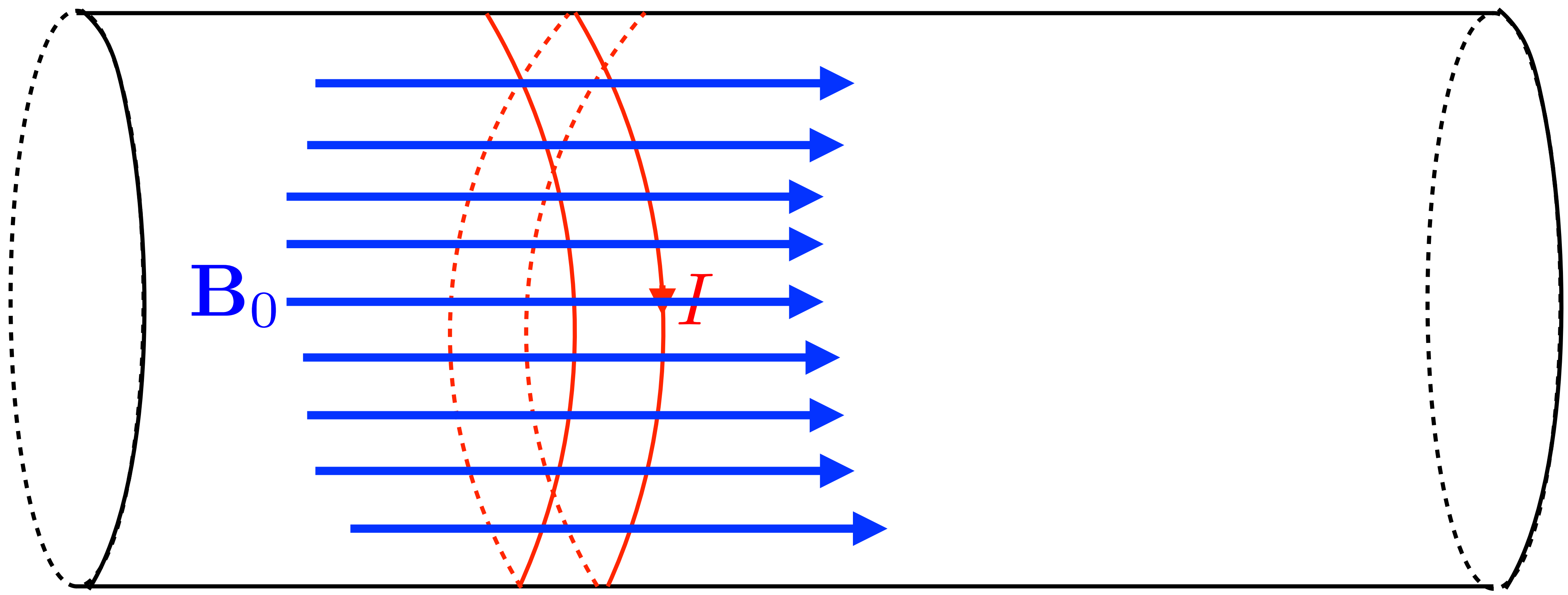
Toy example: solenoid



Toy example: solenoid

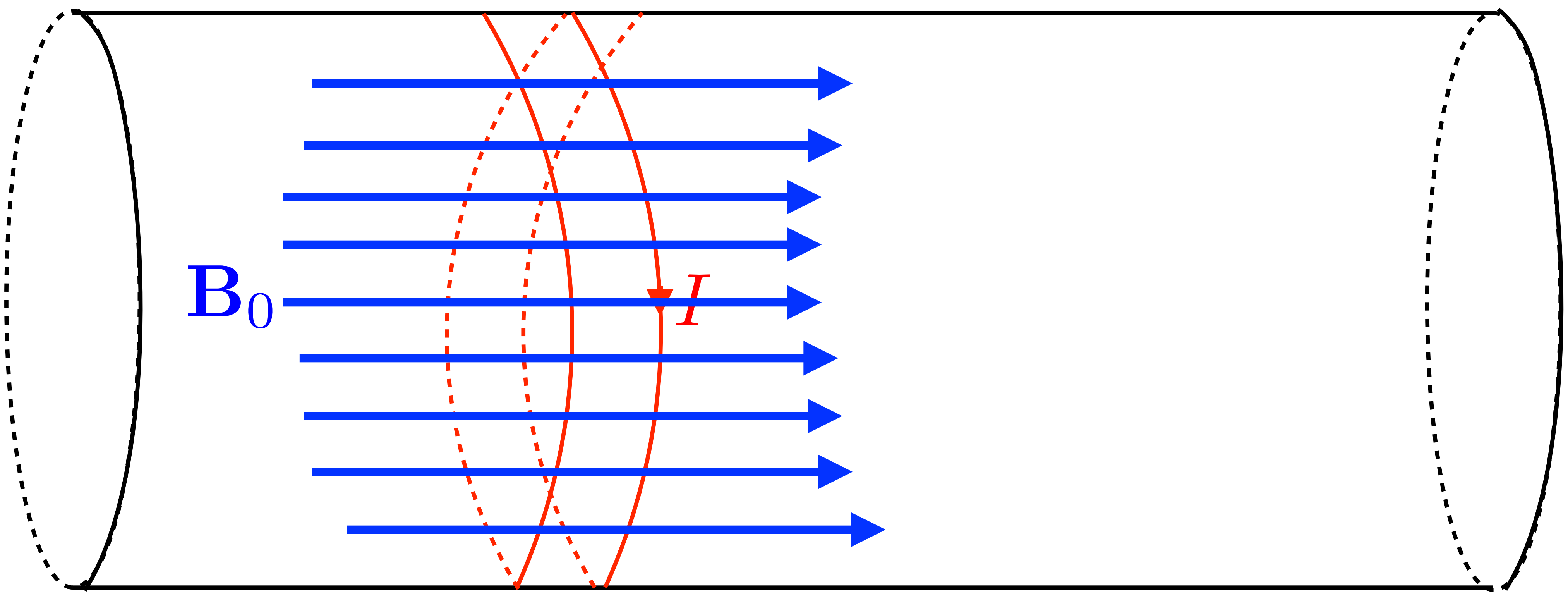


Toy example: solenoid



Toy example: solenoid

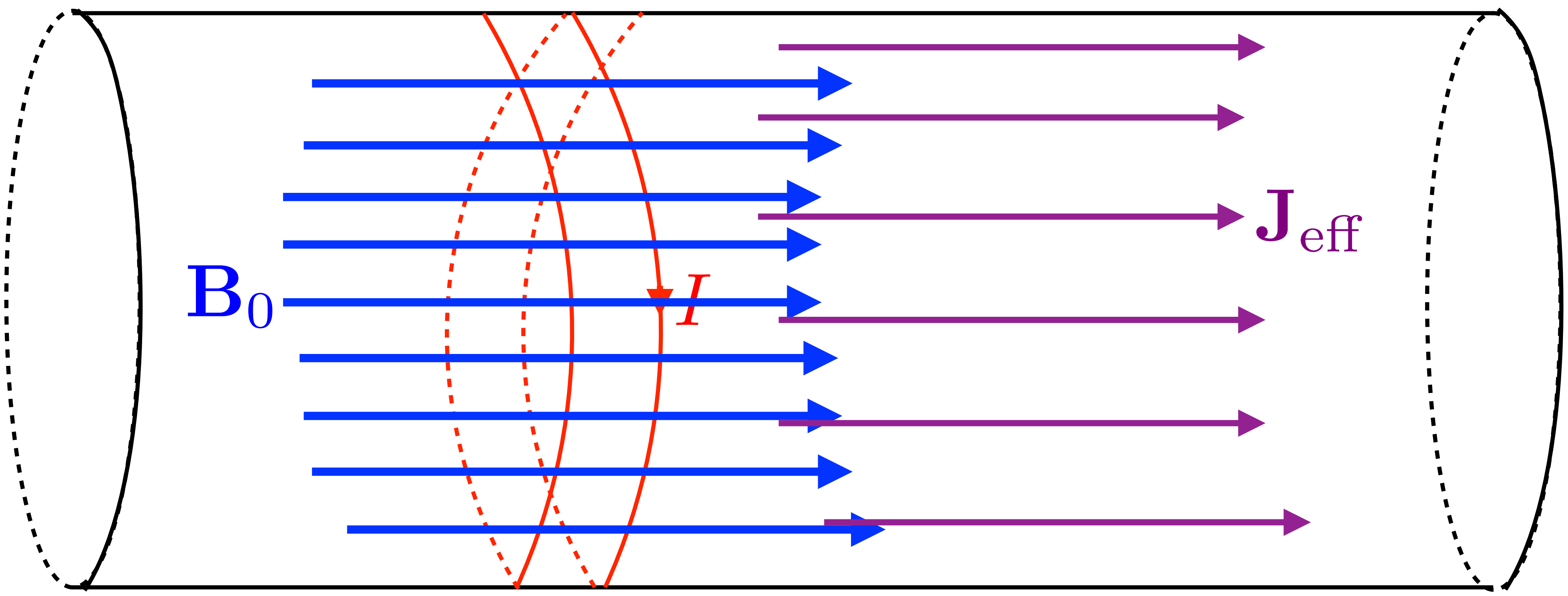
In the presence of axion DM:



$$\mathbf{J}_{\rm eff} = g_{a\gamma\gamma}\sqrt{2\rho_{\rm DM}}\cos(m_a t)\mathbf{B}_0$$

Toy example: solenoid

In the presence of axion DM:



$$\mathbf{J}_{\text{eff}} = g_{a\gamma\gamma}\sqrt{2\rho_{\text{DM}}}\cos(m_a t)\mathbf{B}_0$$

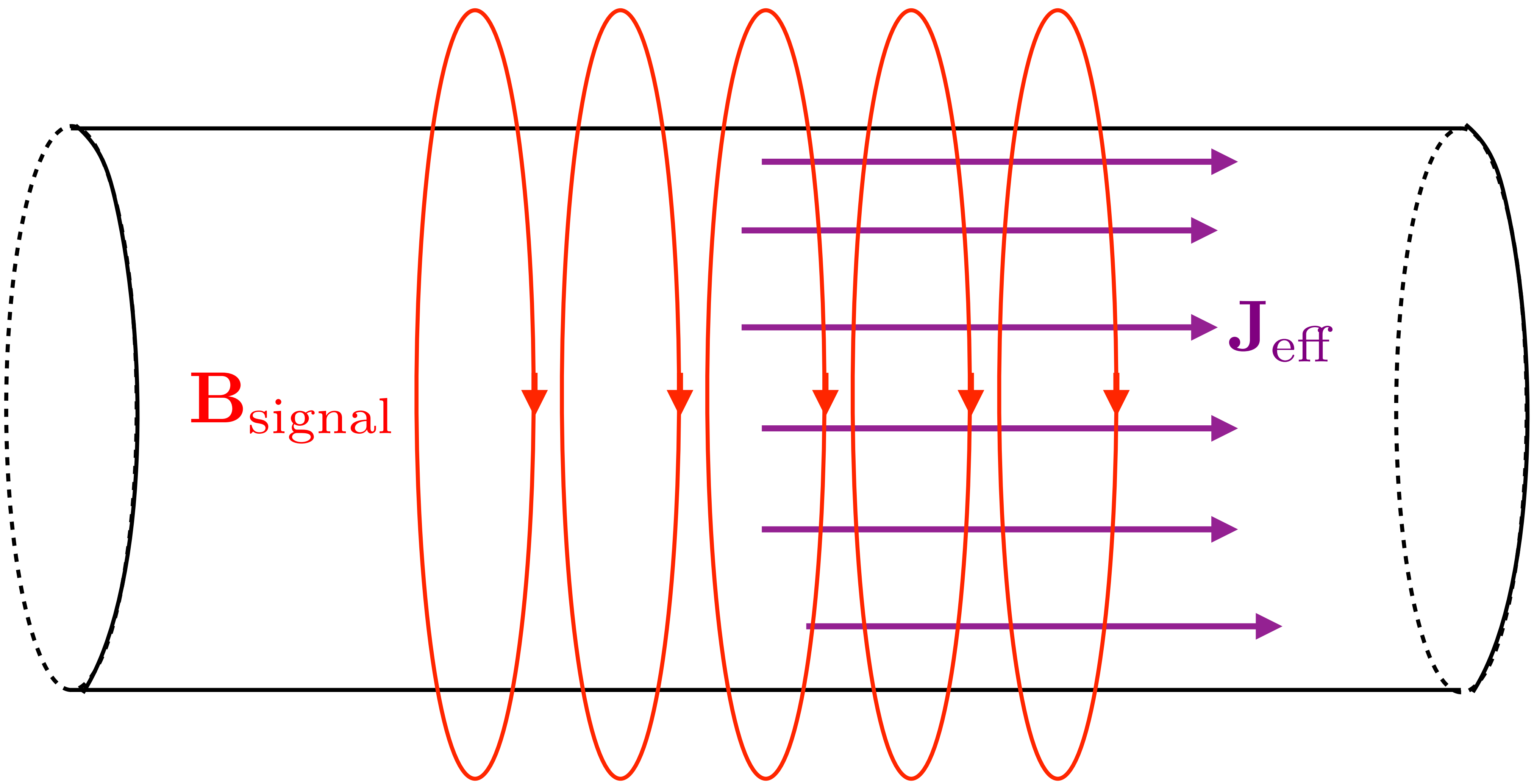
Toy example: solenoid

In the presence of axion DM:

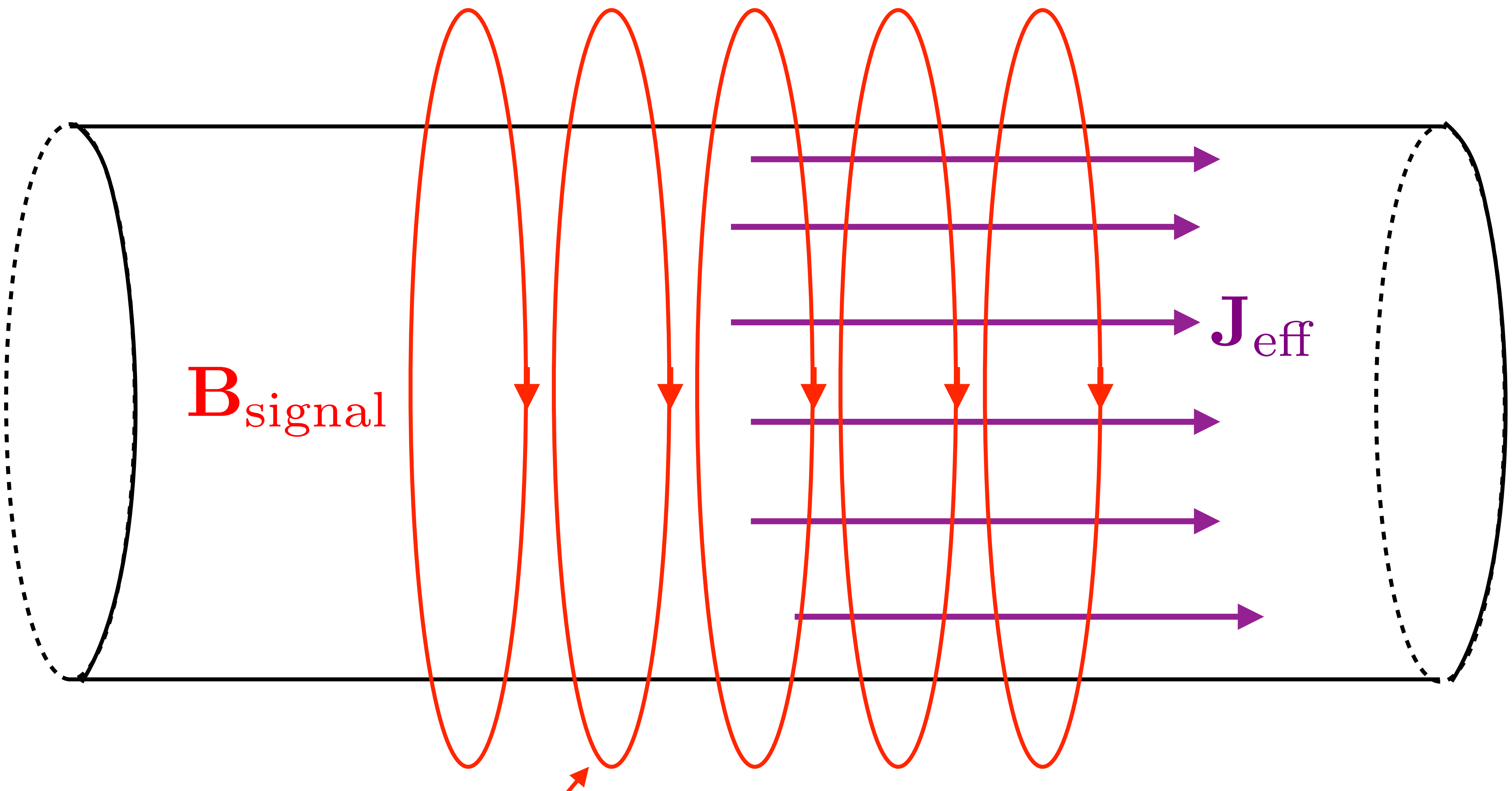
$$\mathbf{J}_{\rm eff} = g_{a\gamma\gamma}\sqrt{2\rho_{\rm DM}}\cos(m_a t)\mathbf{B}_0$$

Current-carrying wire

Current-carrying wire



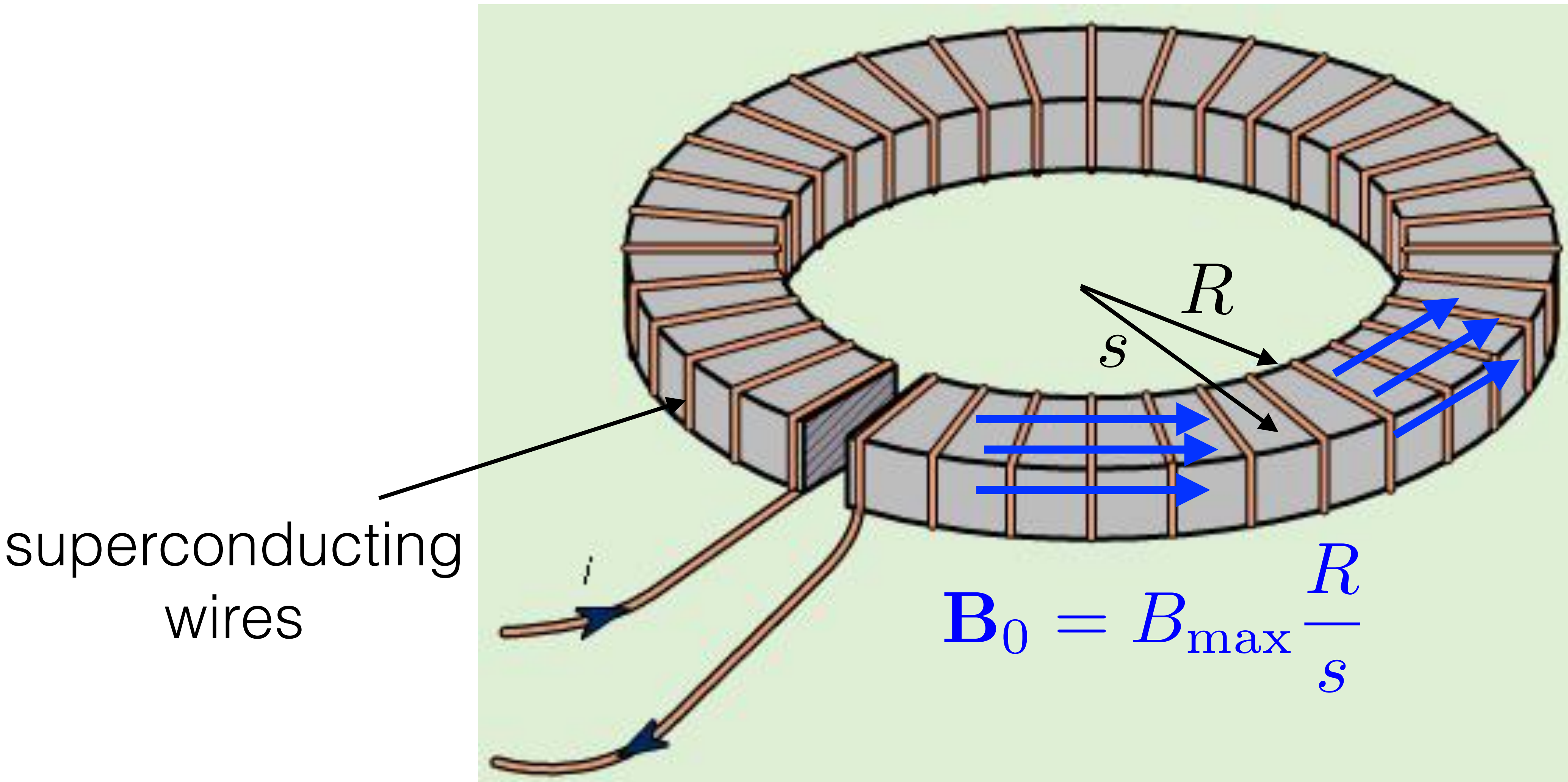
Current-carrying wire



Can detect axion-induced flux outside field region!

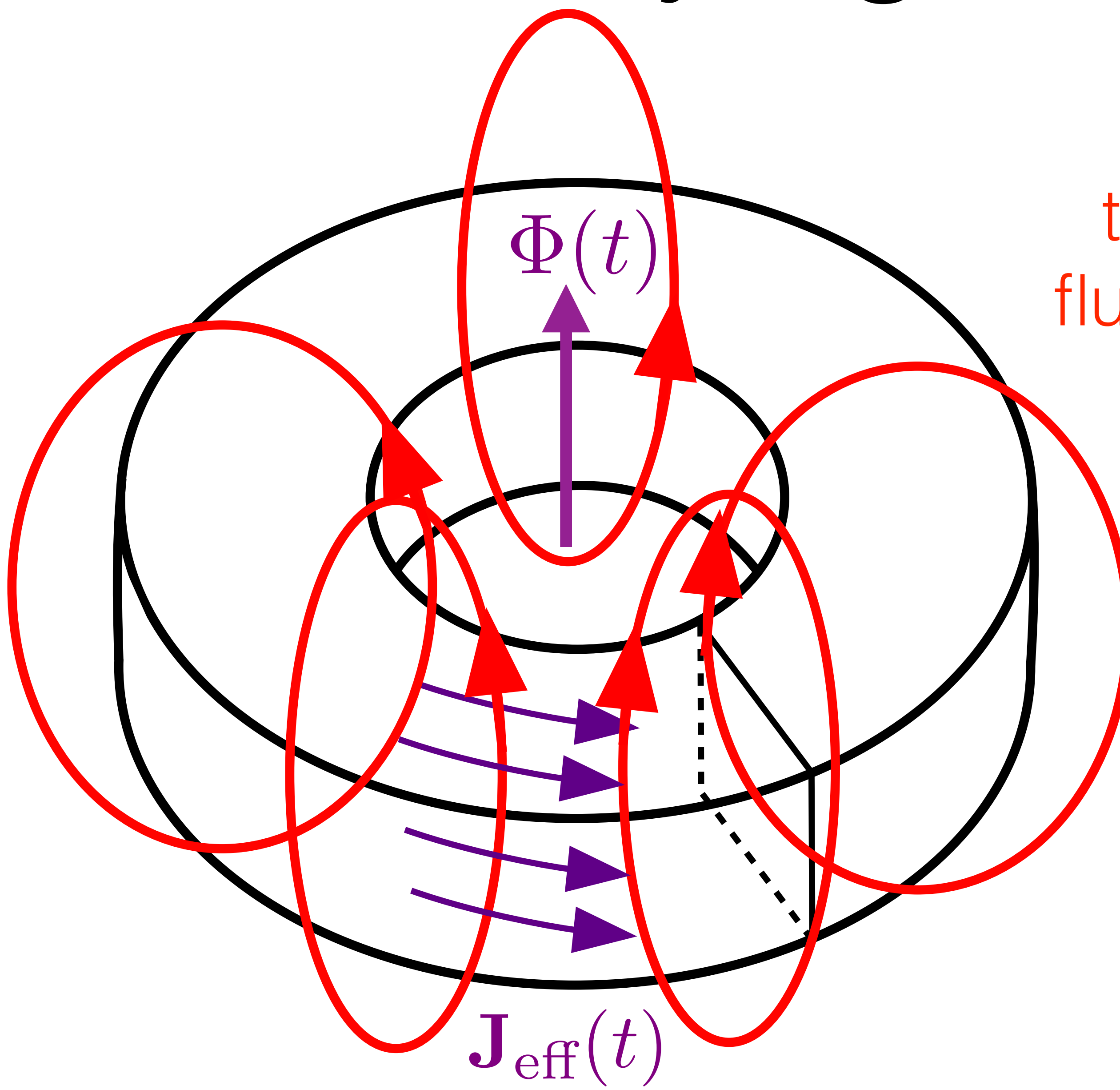
Toroid: no axion

Bend solenoid around into a toroid to reduce fringe fields



Magnetic field confined to toroid:
no flux through center

Toroid with axion: current-carrying loop



toroidal geometry:
flux adds **coherently**
in center

Signal: time-varying flux through center

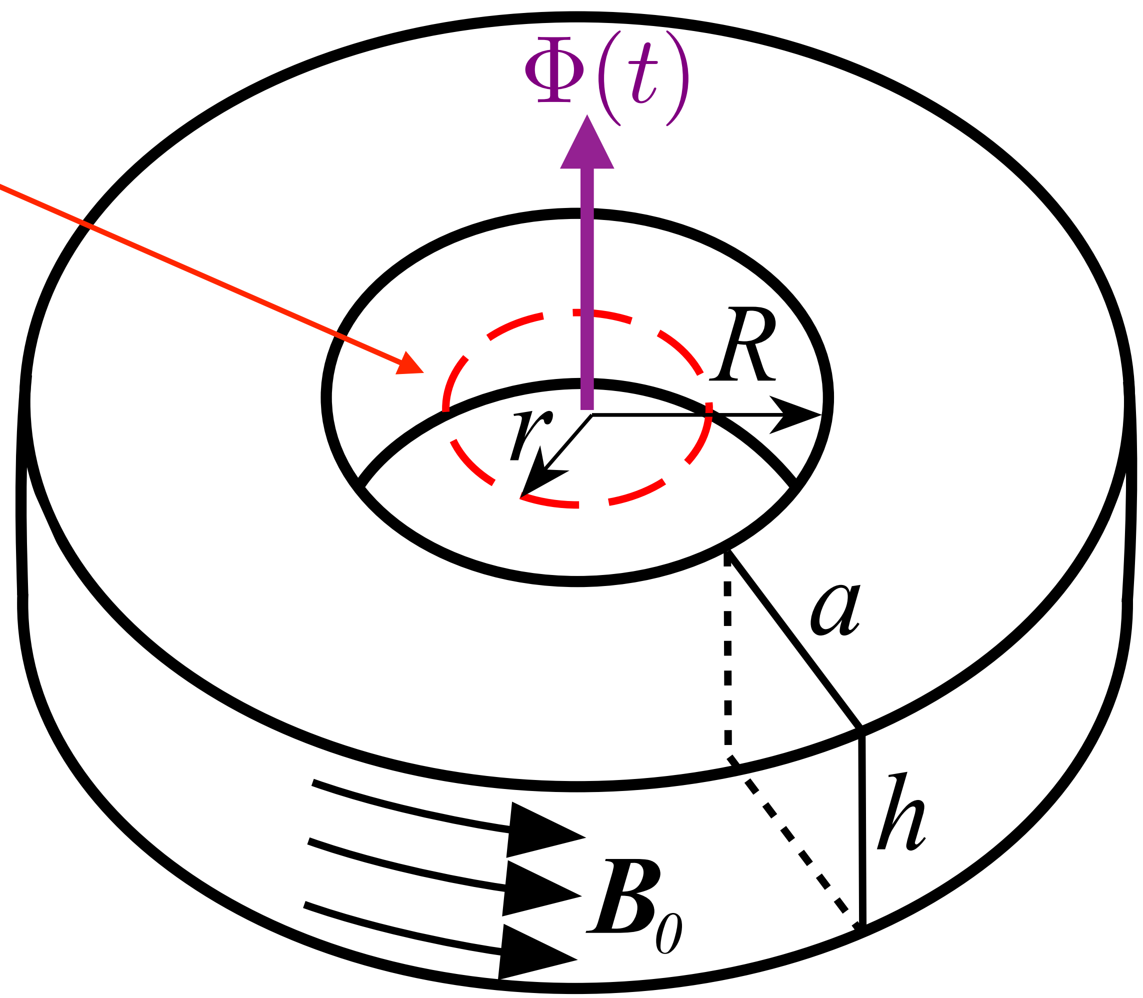
Key point: measure signal in zero-static-field region!

Signal flux

pickup loop

$R, h, a \ll m_a^{-1}$

(quasistatic)



$\mathbf{B}_0 = B_{\max}\dfrac{R}{s}$

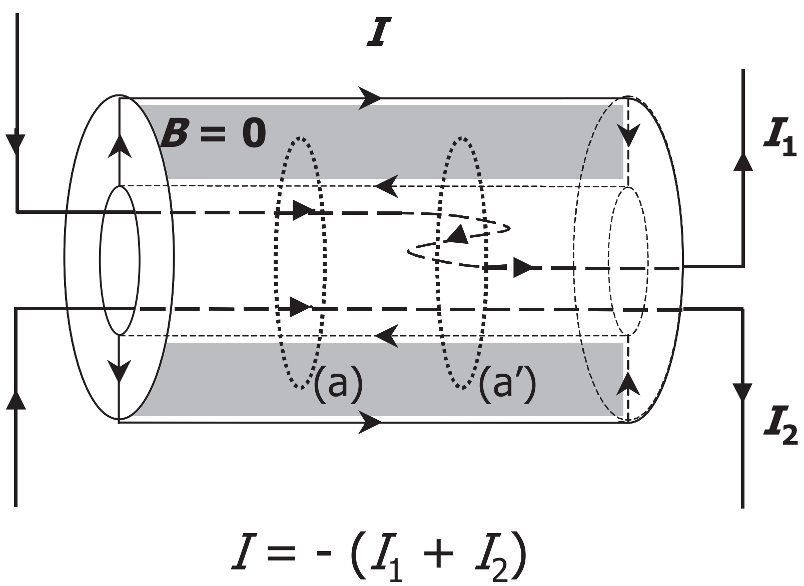
Effective toroid volume

$$\Phi_{\text{pickup}}(t) = g_{a\gamma\gamma}\, B_{\max}\, \sqrt{2\rho_{\text{DM}}}\, \cos(m_a t)\, V_B$$

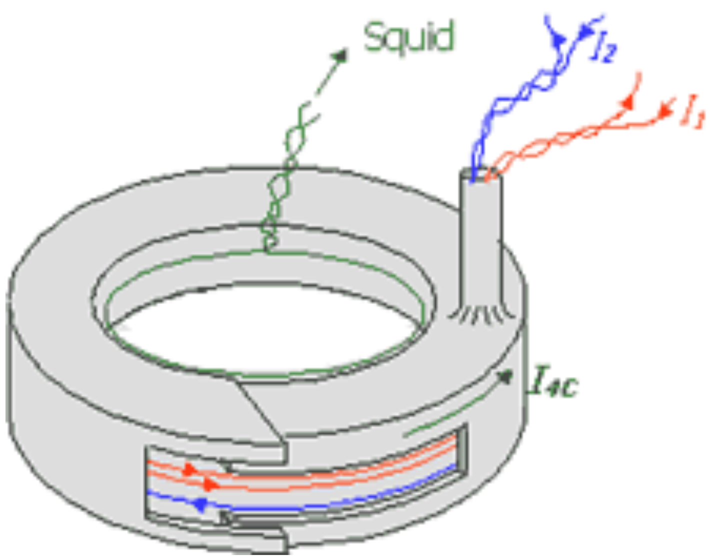
Couple this flux into SQUID magnetometer through either broadband or resonant readout circuit

Self-screening?

Borrow analysis of cryogenic current comparators



solenoid



toroid

Meissner return current
actually generates signal!

Some rough numbers

GUT-scale KSVZ axion: $|g_{a\gamma\gamma}| = 2.2 \times 10^{-19}\,\mathrm{GeV}^{-1}$

R = r = a = h/3= 4 m: $V_B = 100\ \mathrm{m}^3$

Average axion-induced B-field for B_{max} = 5 T:

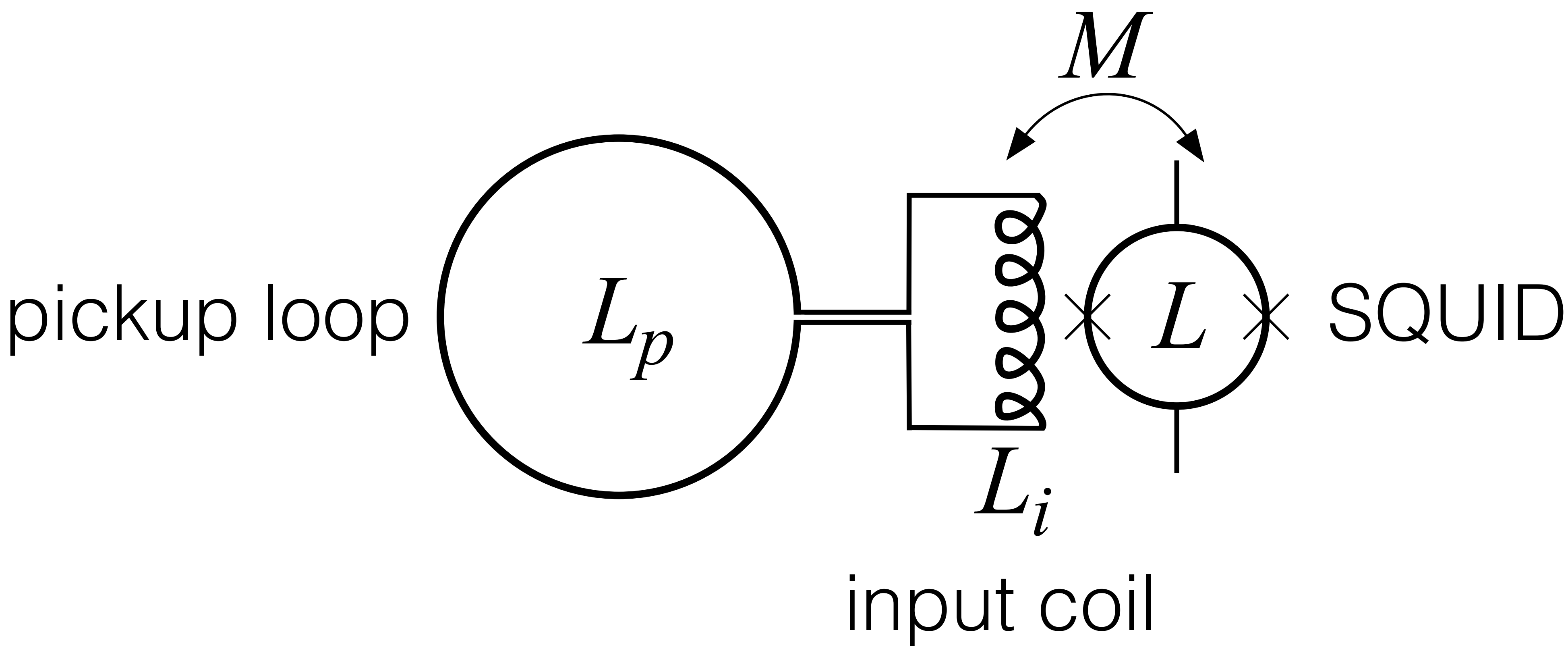
$$\boxed{B_{\mathrm{avg}} = 2.5 \times 10^{-23}\ \mathrm{T}}$$

For 1 year of measurement, can achieve signal-to-noise of 1 with $S_\Phi^{1/2} = 1.2 \times 10^{-19}\ \mathrm{Wb}/\sqrt{\mathrm{Hz}}$

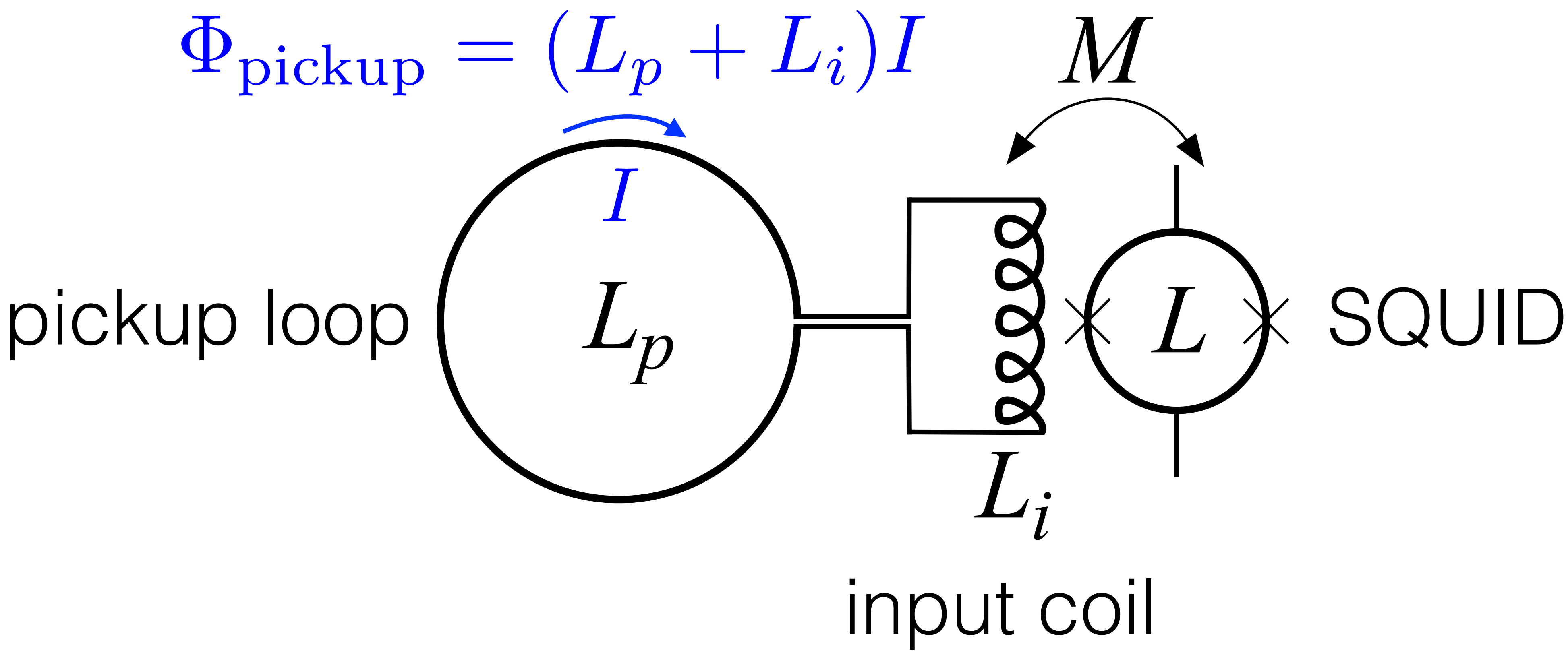
achievable by coupling to commercial SQUIDS!

Assuming axion is all of DM, only free parameter is $g_{a\gamma\gamma}$ as a function of m_a

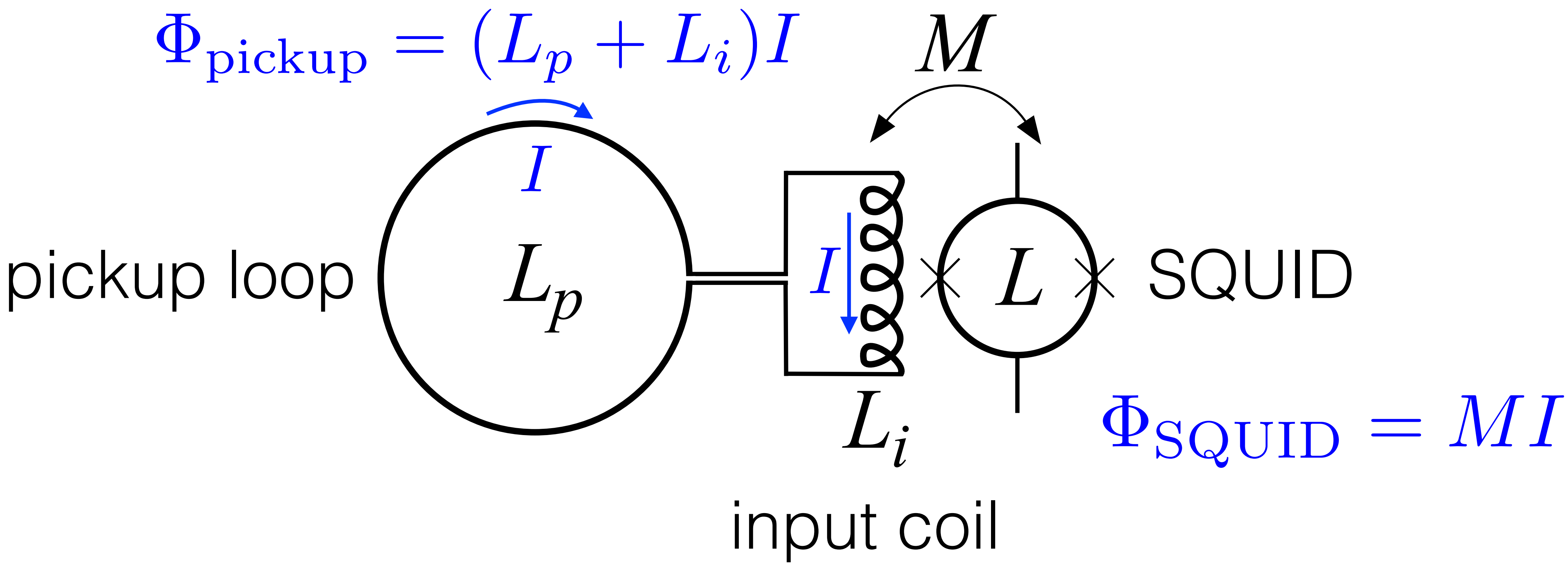
Broadband: readout circuit



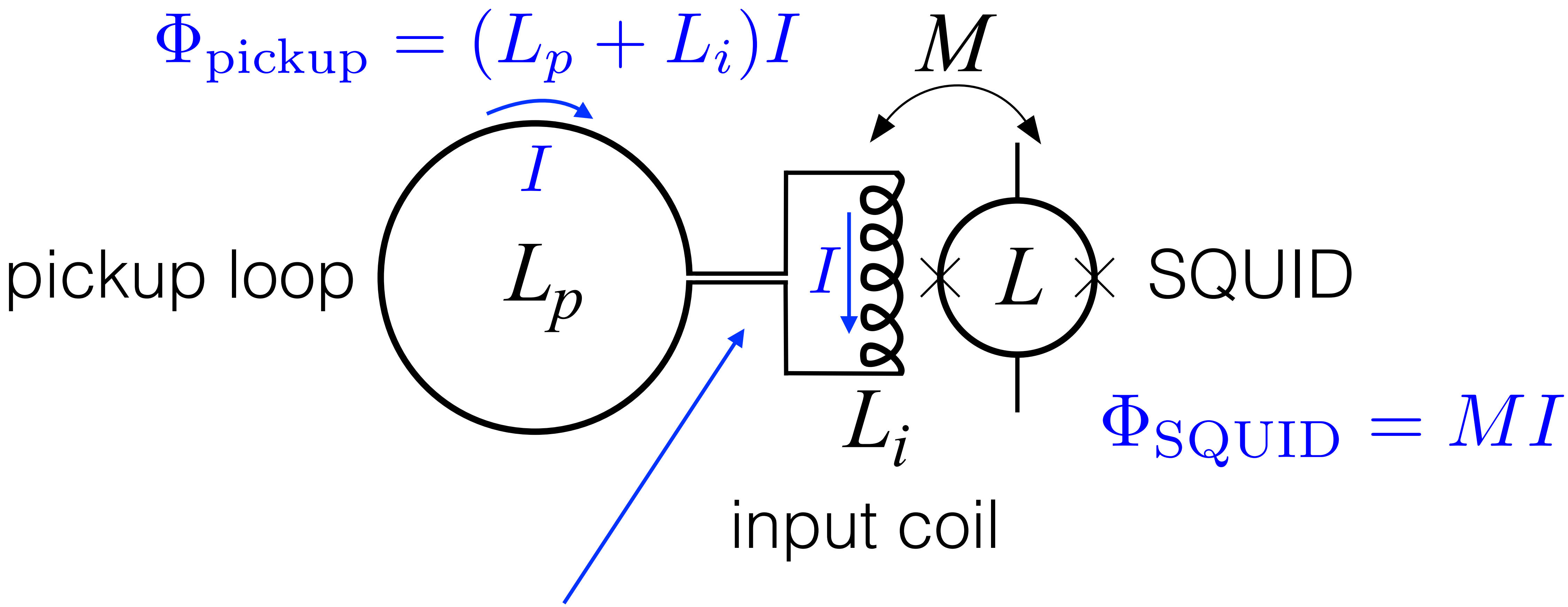
Broadband: readout circuit



Broadband: readout circuit



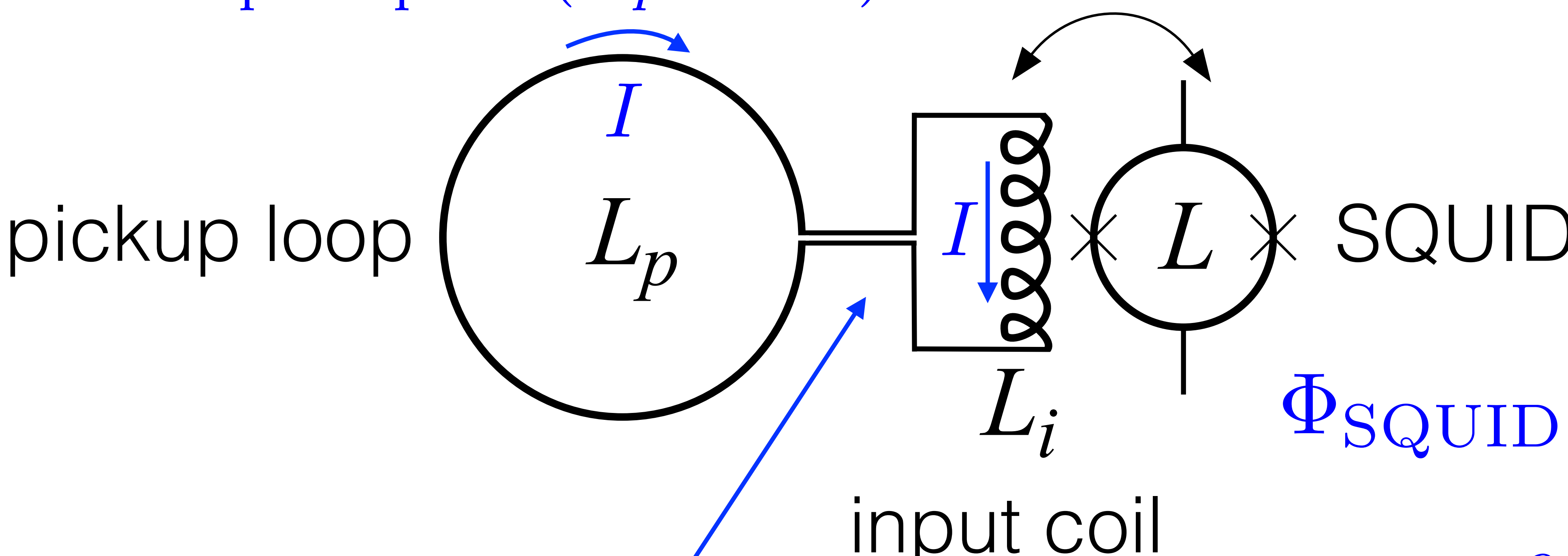
Broadband: readout circuit



pure superconducting = **zero** thermal noise (at low freq.)

Broadband: readout circuit

$$\Phi_{\rm pickup} = (L_p + L_i)I$$

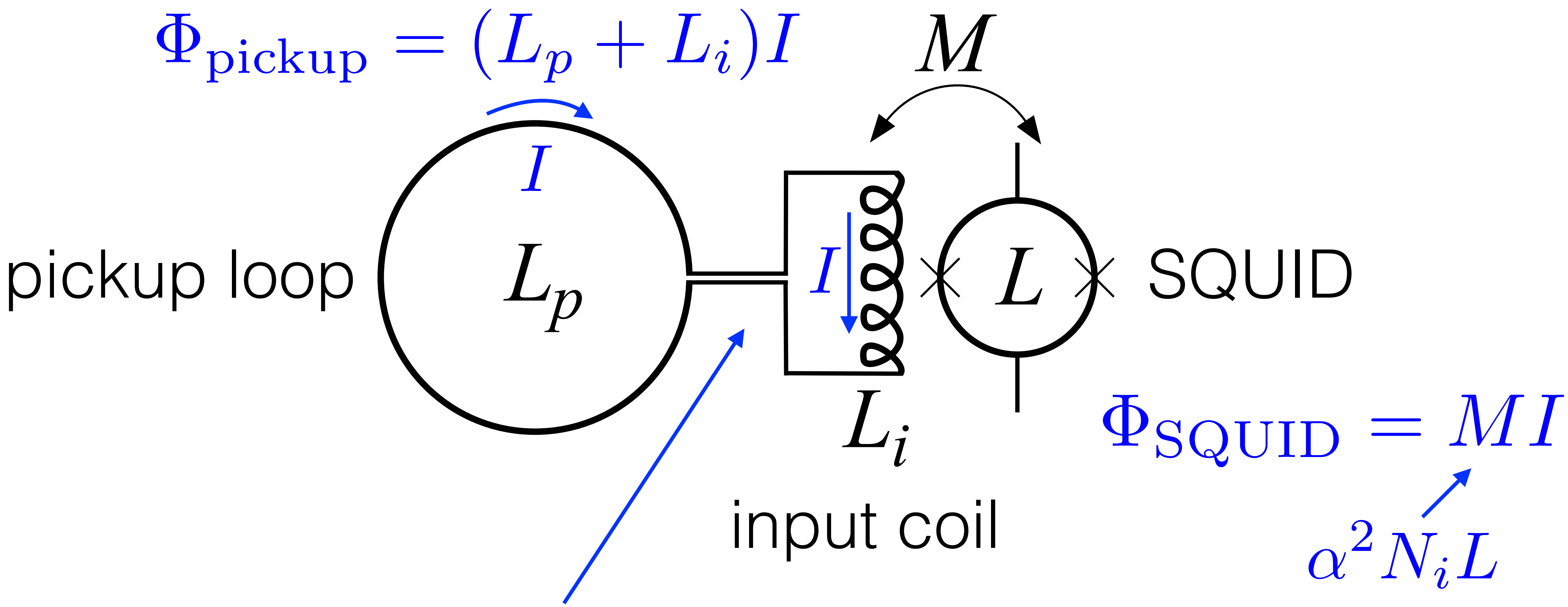


$$\Phi_{\rm SQUID} = \underbrace{M}_{\alpha^2 N_i L} I$$

pure superconducting = **zero** thermal noise (at low freq.)

Inductance matching: $L_i \approx L_p \implies \Phi_{\rm SQUID} \approx \frac{\alpha}{2}\sqrt{\frac{L}{L_p}}\Phi_{\rm pickup}$

Broadband: readout circuit

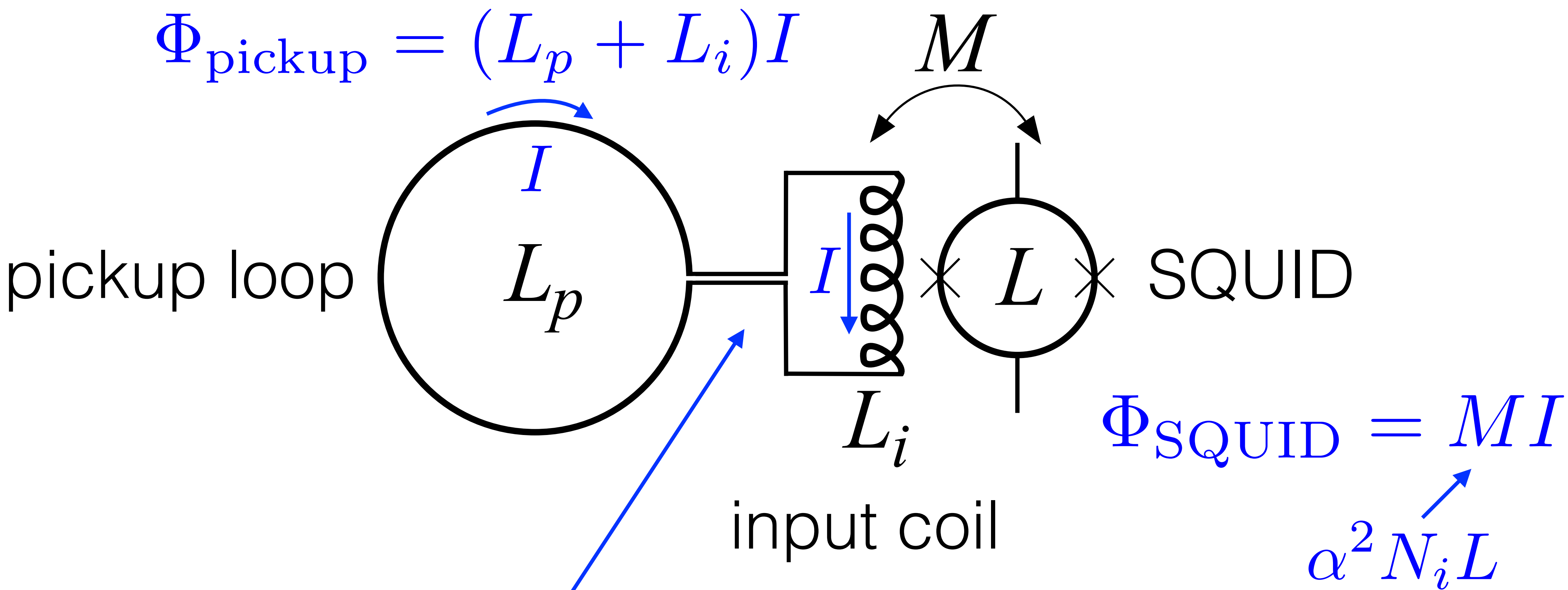


pure superconducting = **zero** thermal noise (at low freq.)

Inductance matching: $L_i \approx L_p \implies \Phi_{\text{SQUID}} \approx \underbrace{\frac{\alpha}{2}\sqrt{\frac{L}{L_p}}}_{\approx\, 0.01} \Phi_{\text{pickup}}$

Optimal coupling: $\frac{1}{2}\int \mathbf{B}^2\, dV = \frac{\Phi^2}{2L_p} \longrightarrow \approx 0.01$

Broadband: readout circuit



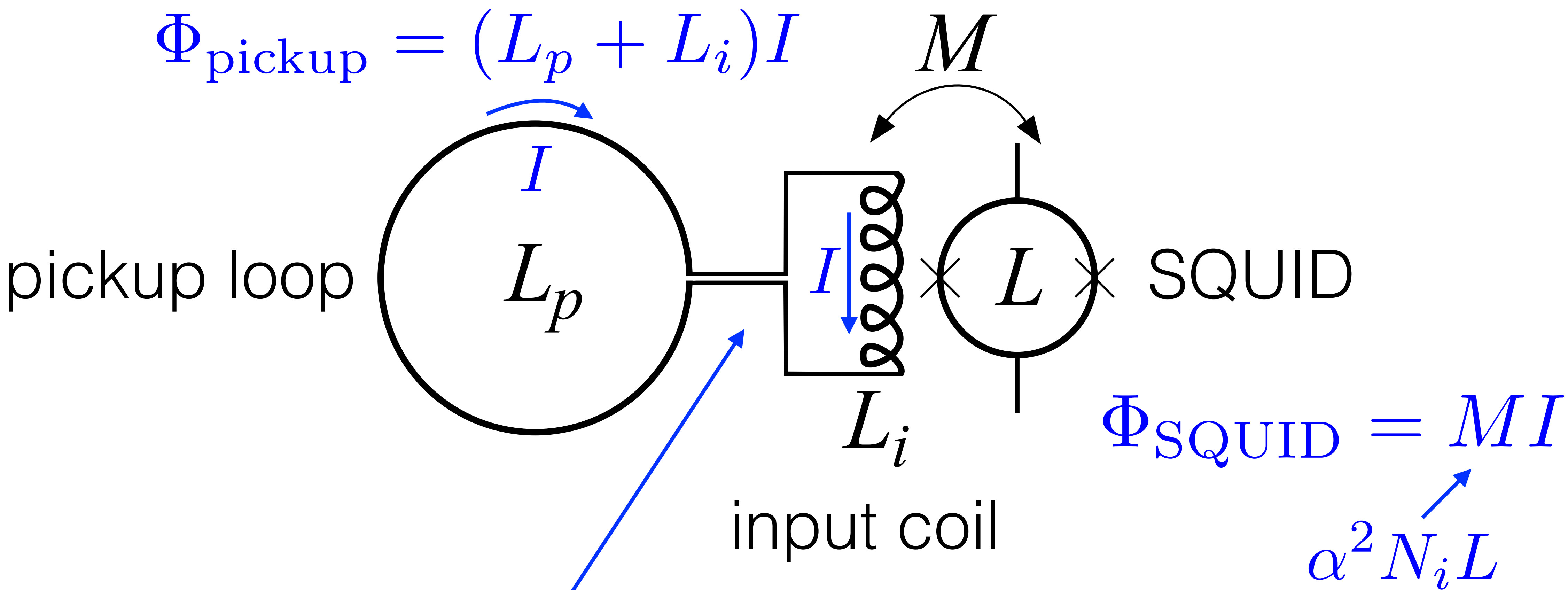
pure superconducting = **zero** thermal noise (at low freq.)

Inductance matching: $L_i \approx L_p \implies \Phi_{\mathrm{SQUID}} \approx \underbrace{\frac{\alpha}{2}\sqrt{\frac{L}{L_p}}}_{\approx 0.01}\Phi_{\mathrm{pickup}}$

Optimal coupling: $\frac{1}{2}\int \mathbf{B}^2\, dV = \frac{\Phi^2}{2L_p}$

huge area = amplification

Broadband: readout circuit



pure superconducting = **zero** thermal noise (at low freq.)

Inductance matching: $L_i \approx L_p \implies \Phi_{\text{SQUID}} \approx \underbrace{\frac{\alpha}{2}\sqrt{\frac{L}{L_p}}}_{\approx\, 0.01} \Phi_{\text{pickup}}$

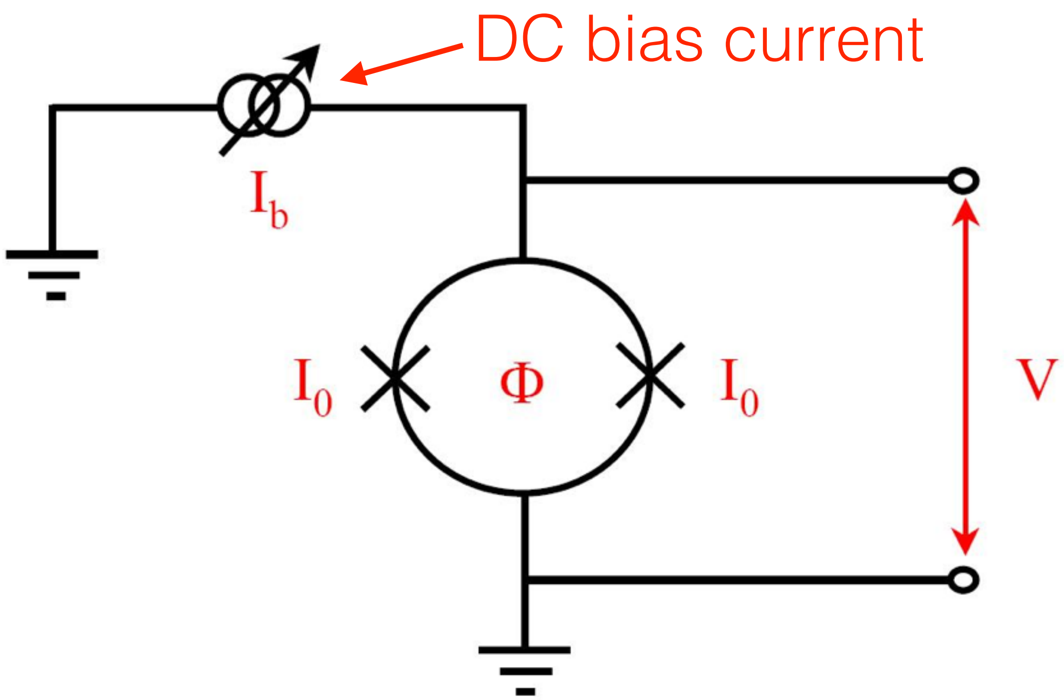
Optimal coupling: $\frac{1}{2}\int \mathbf{B}^2\, dV = \frac{\Phi^2}{2L_p}$

huge area = amplification

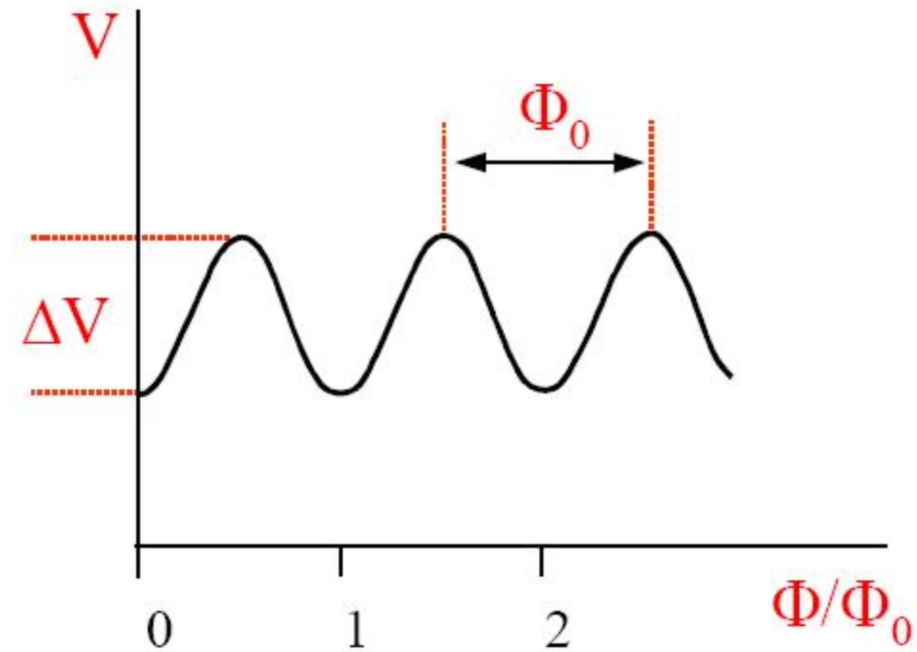
Broadband: response is frequency-independent!

(DC) SQUID basics

Cartoon picture: extremely sensitive flux-to-voltage amplifier



change in flux induces current across junction (DC Josephson effect)



measure extremely small fractions of Φ_0 by fitting sine curve

$$\Phi_0 = \frac{h}{2e} = 2.1 \times 10^{-15}\ \mathrm{Wb} = 2.1 \times 10^{-15}\,\mathrm{T} \cdot \mathrm{m}^2$$

Broadband: SQUID noise

Typical SQUID noise (thermal voltage and current fluctuations):

$$S_{\Phi,0}^{1/2} \sim 10^{-6}\Phi_0/\sqrt{\mathrm{Hz}}$$

$A_{\mathrm{SQUID}} \sim (30\,\mu\mathrm{m})^2$

$\Longrightarrow$ field sensitivity of $2\,\mathrm{pT}/\sqrt{\mathrm{Hz}}$ at SQUID

Ultimate limit is shot noise:

$$S_{\Phi}^{1/2} = L\,S_{J,0}^{1/2} = \sqrt{\frac{11}{8}hL}/\sqrt{\mathrm{Hz}}$$

dominates below ~ 60 mK

For L ~ 1 nH, only ~ 0.5 x typical noise,
not much improvement possible

Broadband: S/N and sensitivity

Take data for time t:

If $t < \tau$, S/N improves like $\sqrt{t}$ (random walk)

Our regime is $t \gg \tau$: $S/N \sim |\Phi_{\mathrm{SQUID}}|\,(t\tau)^{1/4}/S_{\Phi,0}^{1/2}$

$$S/N = 1$$

$\Longrightarrow$ sensitivity to

$$g_{a\gamma\gamma} > 6.3 \times 10^{-18}\ \mathrm{GeV}^{-1} \left(\frac{m_a}{10^{-12}\ \mathrm{eV}} \frac{1\ \mathrm{year}}{t}\right)^{1/4} \frac{5\ \mathrm{T}}{B_{\max}} \times \left(\frac{0.85\ \mathrm{m}}{R}\right)^{5/2} \sqrt{\frac{0.3\ \mathrm{GeV/cm^3}}{\rho_{\mathrm{DM}}} \frac{S_{\Phi,0}^{1/2}}{10^{-6}\Phi_0/\sqrt{\mathrm{Hz}}}}$$

improves at low masses from coherence time

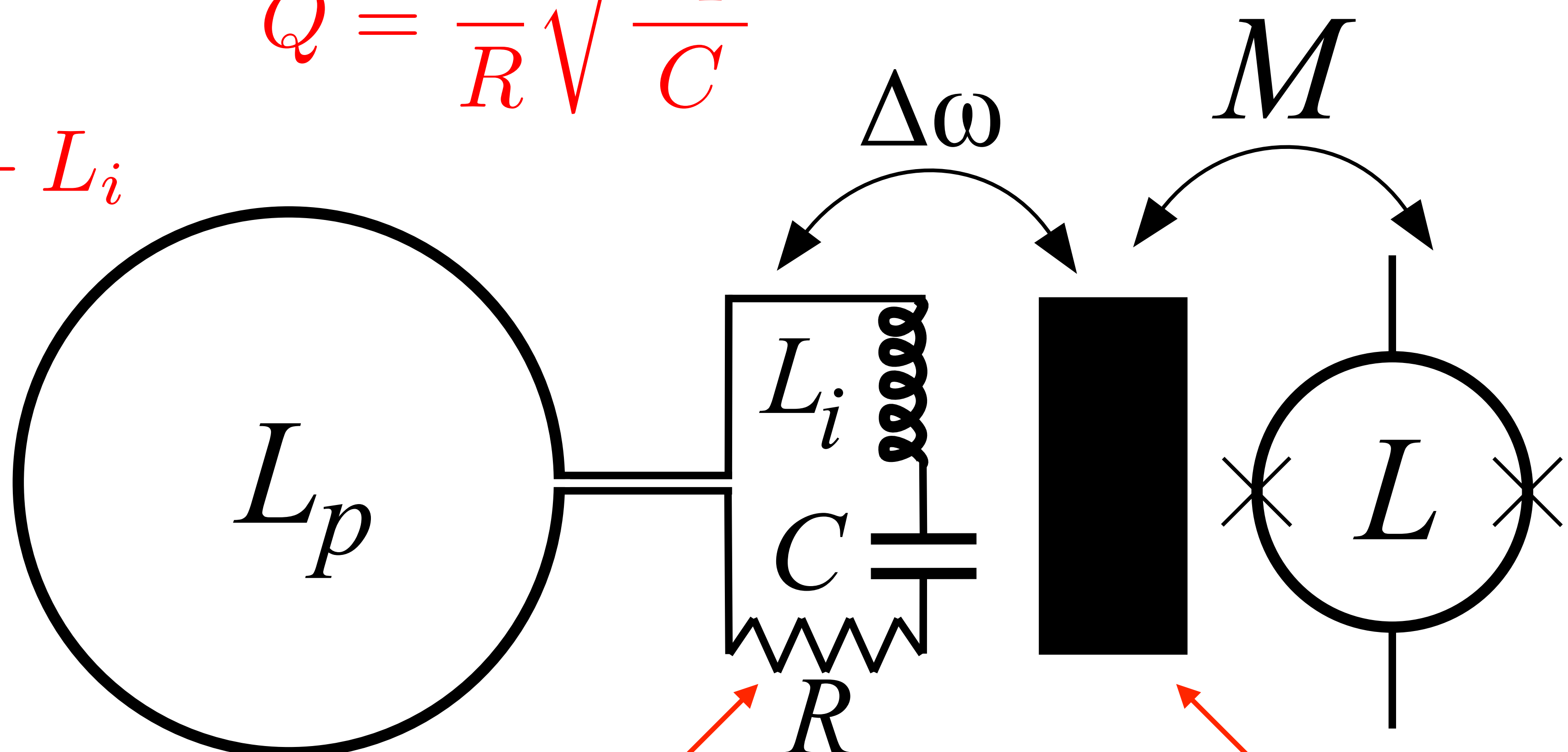
R = r = a = h/3: tall toroid increases B-field energy

Scale up dimensions to 4m, can probe GUT-scale axions!

Resonant: readout circuit

$$Q = \frac{1}{R}\sqrt{\frac{L_T}{C}}$$

$$L_T = L_p + L_i$$



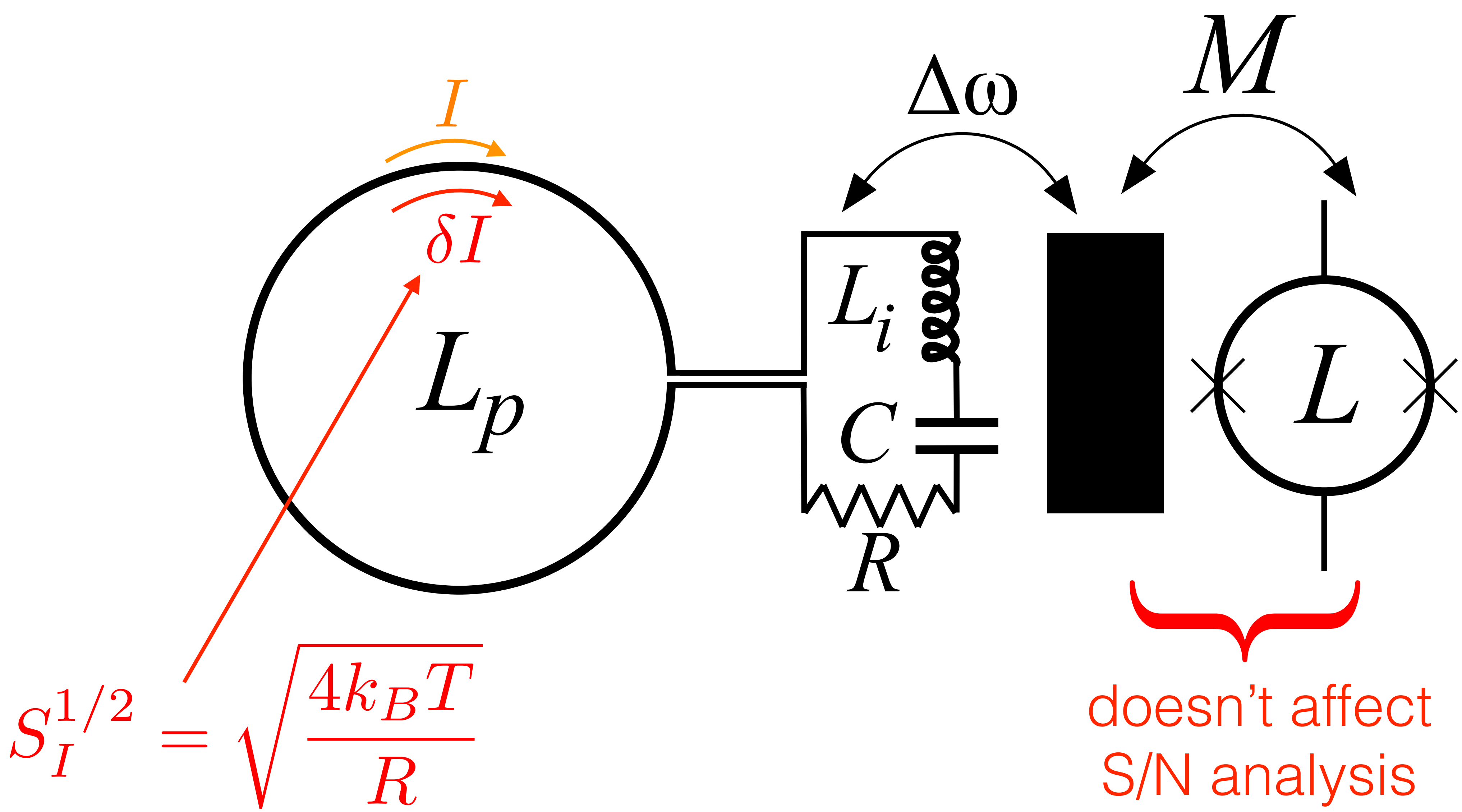
irreducible resistance
(resistance in series rather than parallel)

black box: match circuit bandwidth to signal

New source of noise: thermal noise in pickup RLC circuit

Resonant: noise

Can show thermal noise dominates at 0.1 K up to $Q = 10^8$



Resonant: S/N and sensitivity

$$P_S = Q_0 \frac{m_a \Phi^2_{\text{pickup}}}{2L_T}, \qquad P_N = k_B T \sqrt{\frac{m_a}{2\pi t_{\text{e-fold}}}}$$

energy stored
in tank circuit

each e-fold of frequency
scanned for equal time

$$P_S / P_N = 1$$

$\Longrightarrow$ sensitivity to

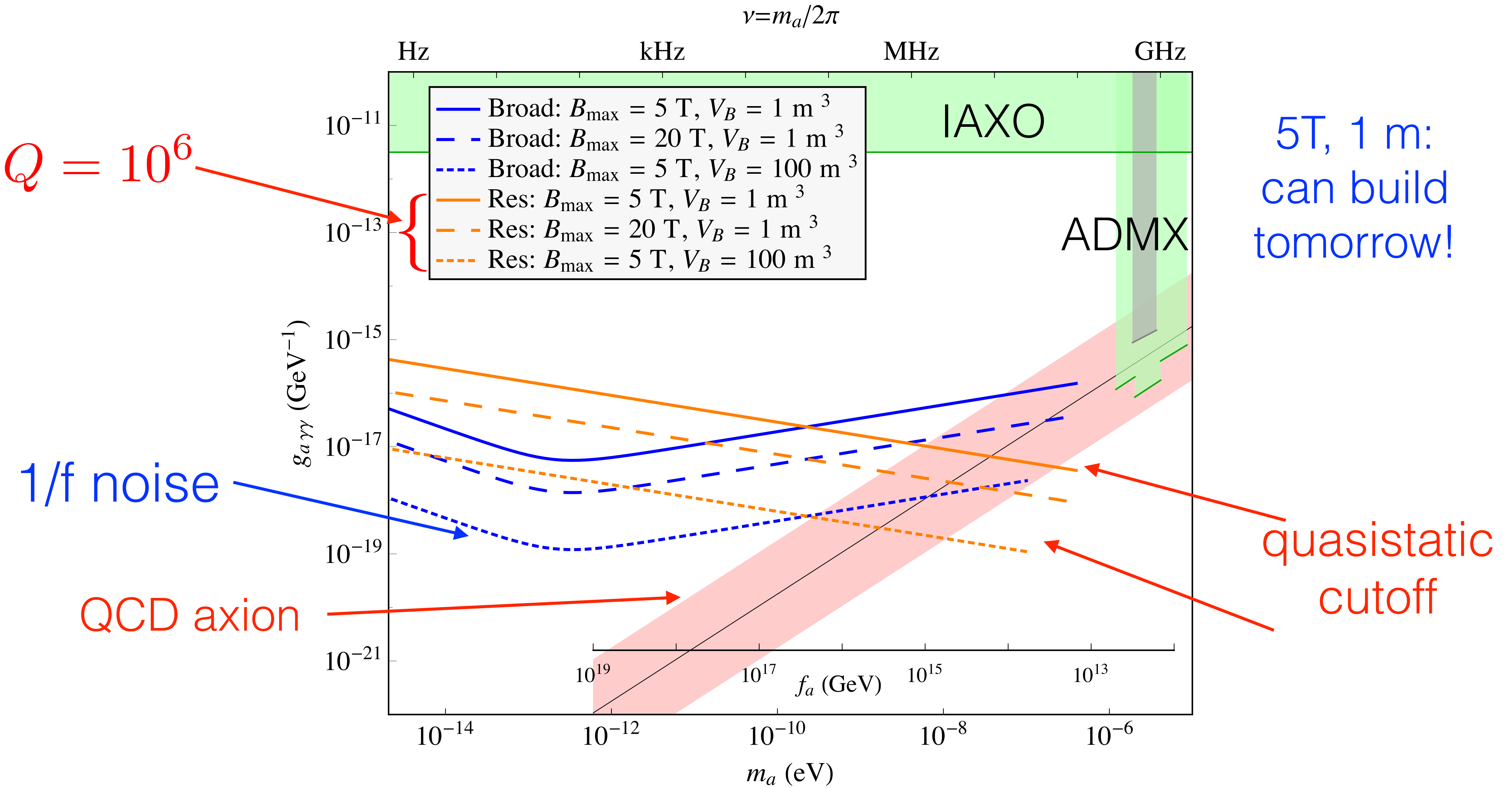
$$g_{a\gamma\gamma} > 9.0 \times 10^{-17}\ \text{GeV}^{-1} \left(\frac{10^{-12}\ \text{eV}}{m_a} \frac{20\ \text{days}}{t_{\text{e-fold}}} \right)^{1/4} \times \frac{5\ \text{T}}{B_{\max}} \left(\frac{0.85\ \text{m}}{R} \right)^{5/2} \sqrt{\frac{0.3\ \text{GeV/cm}^3}{\rho_{\text{DM}}} \frac{10^6}{Q_0} \frac{T}{0.1\ \text{K}}}$$

improves at high masses

improves at low temp

Broadband and resonant reach

1 year **total** measurement time



With same experimental parameters,

broadband for low frequencies, resonant for high frequencies

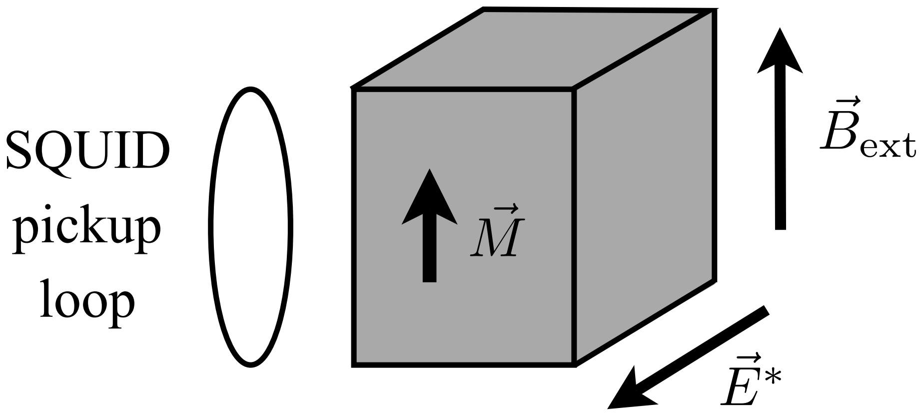
[Irwin et al. currently developing optimal readout combining both strategies]

Comparison to existing proposals

ADMX

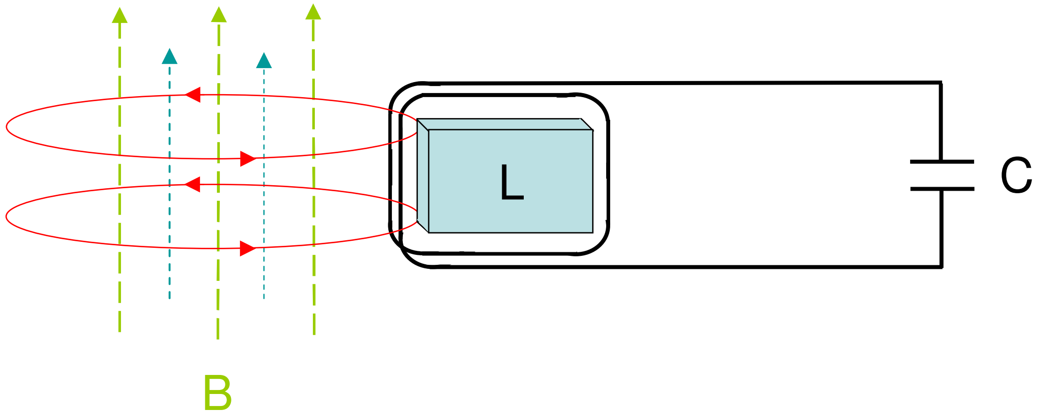
Complementary: we probe lower masses with volume enhancement

CASPEr



Complementary: we measure coupling to photons instead of nuclear EDMs, probe QCD axion for $f_a < M_{\text{GUT}}$

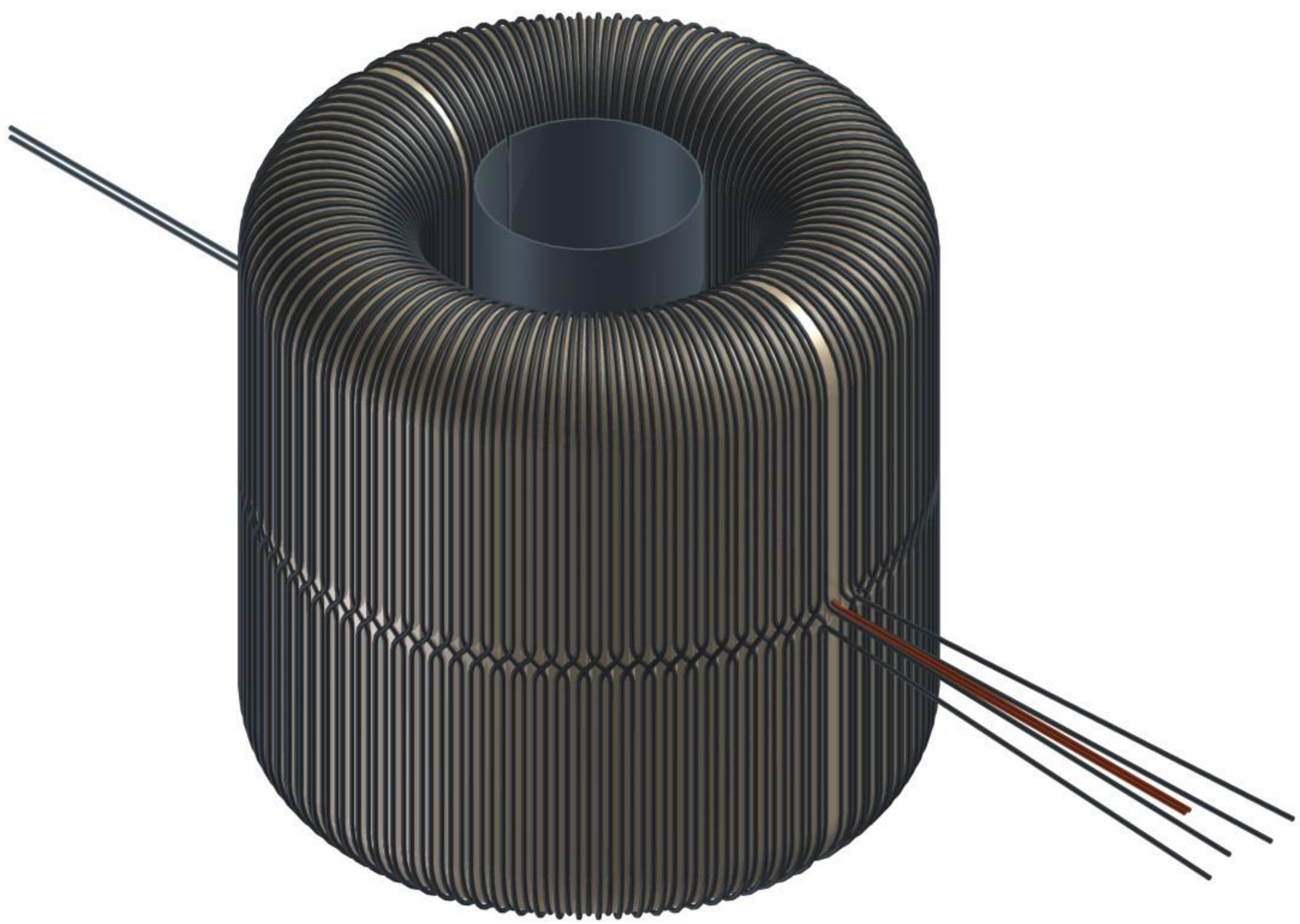
Thomas/Cabrera/Sikivie LC circuit

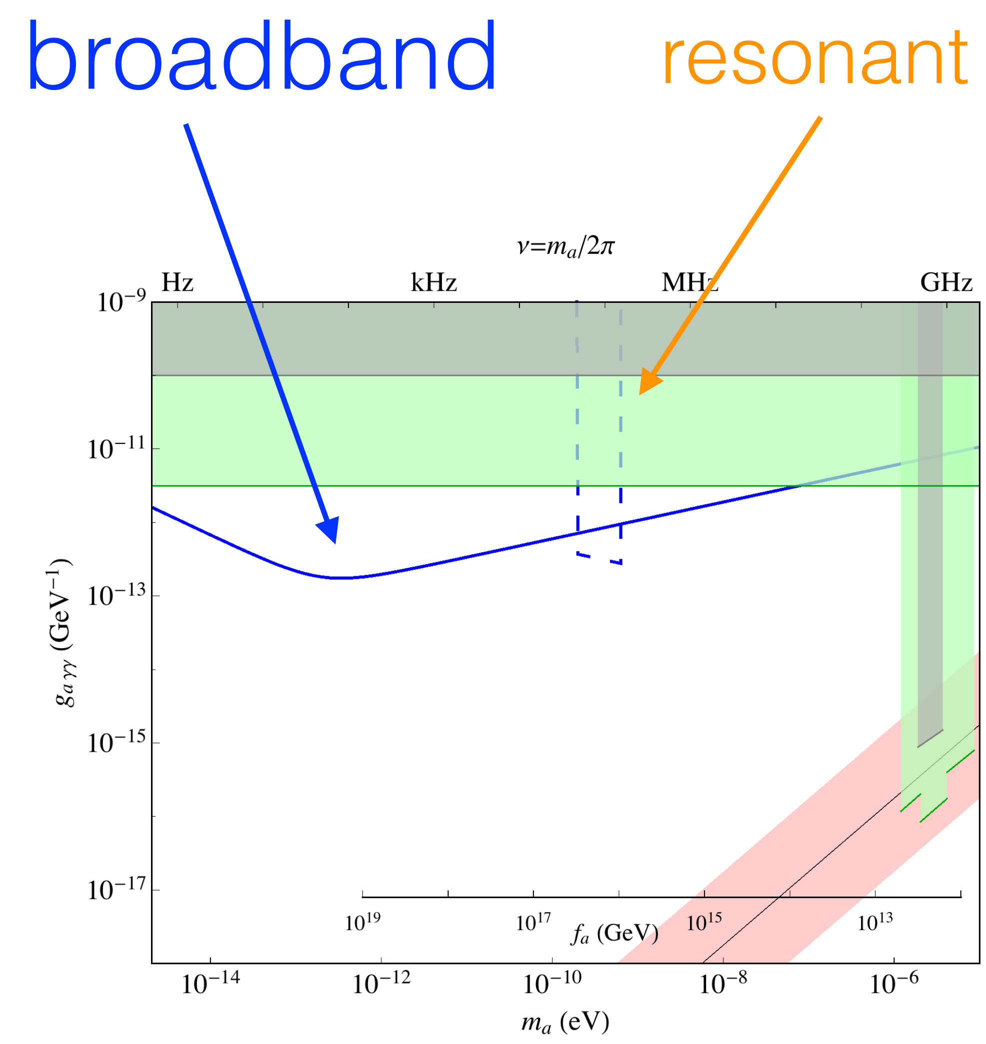


Our pickup is in zero-field region, advantages at low frequencies with broadband readout

MIT prototype

R ~ 10 cm



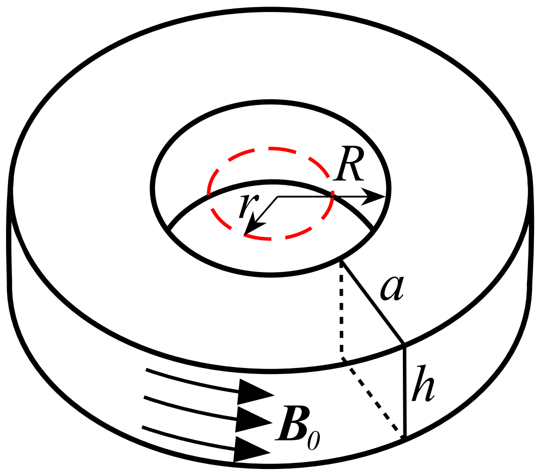


(1 month data-taking)

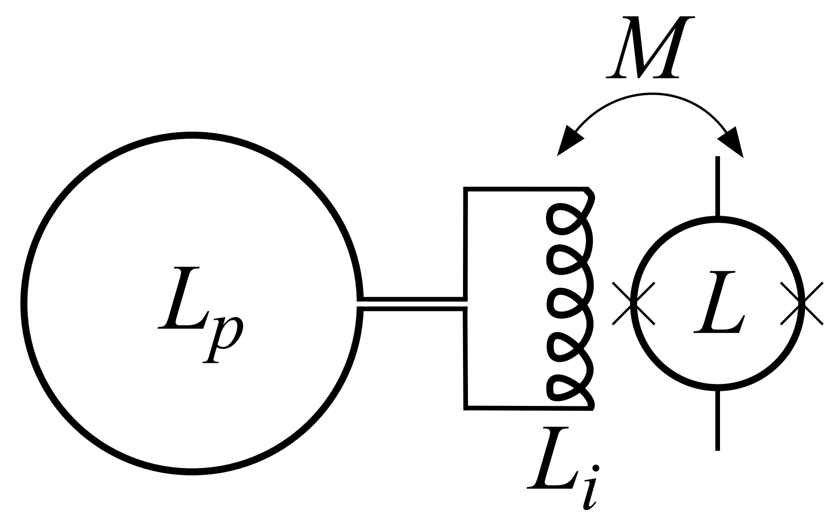
Interesting physics with first-stage experiment!

Summary

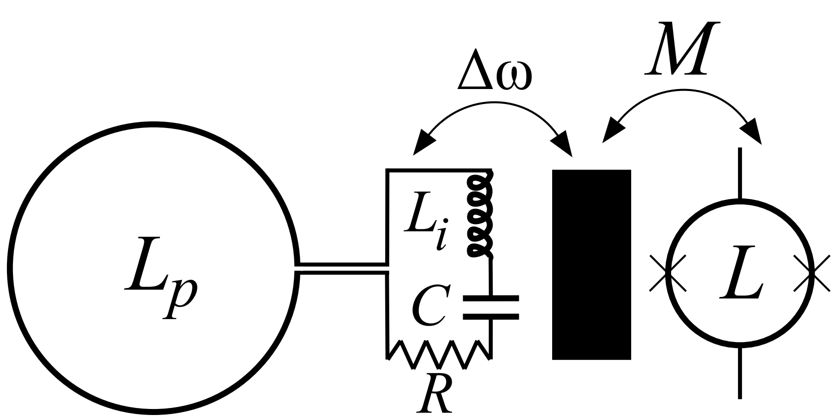
I. Zero-field pickup geometry



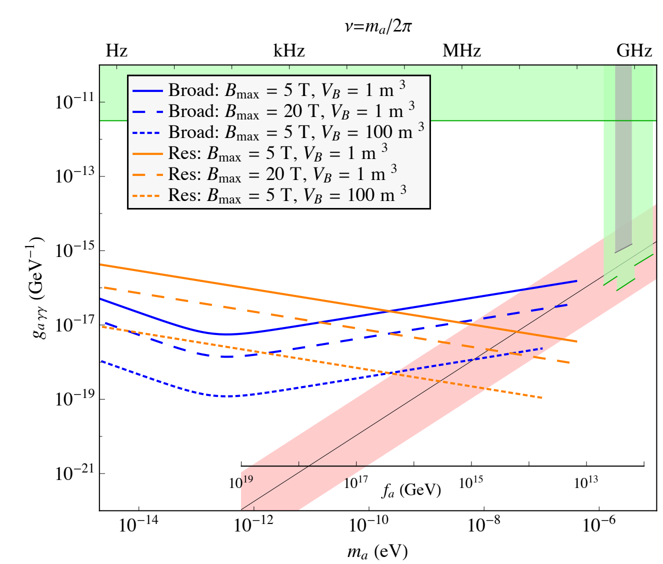
II. Broadband readout for low frequencies



III. Resonant readout for high frequencies



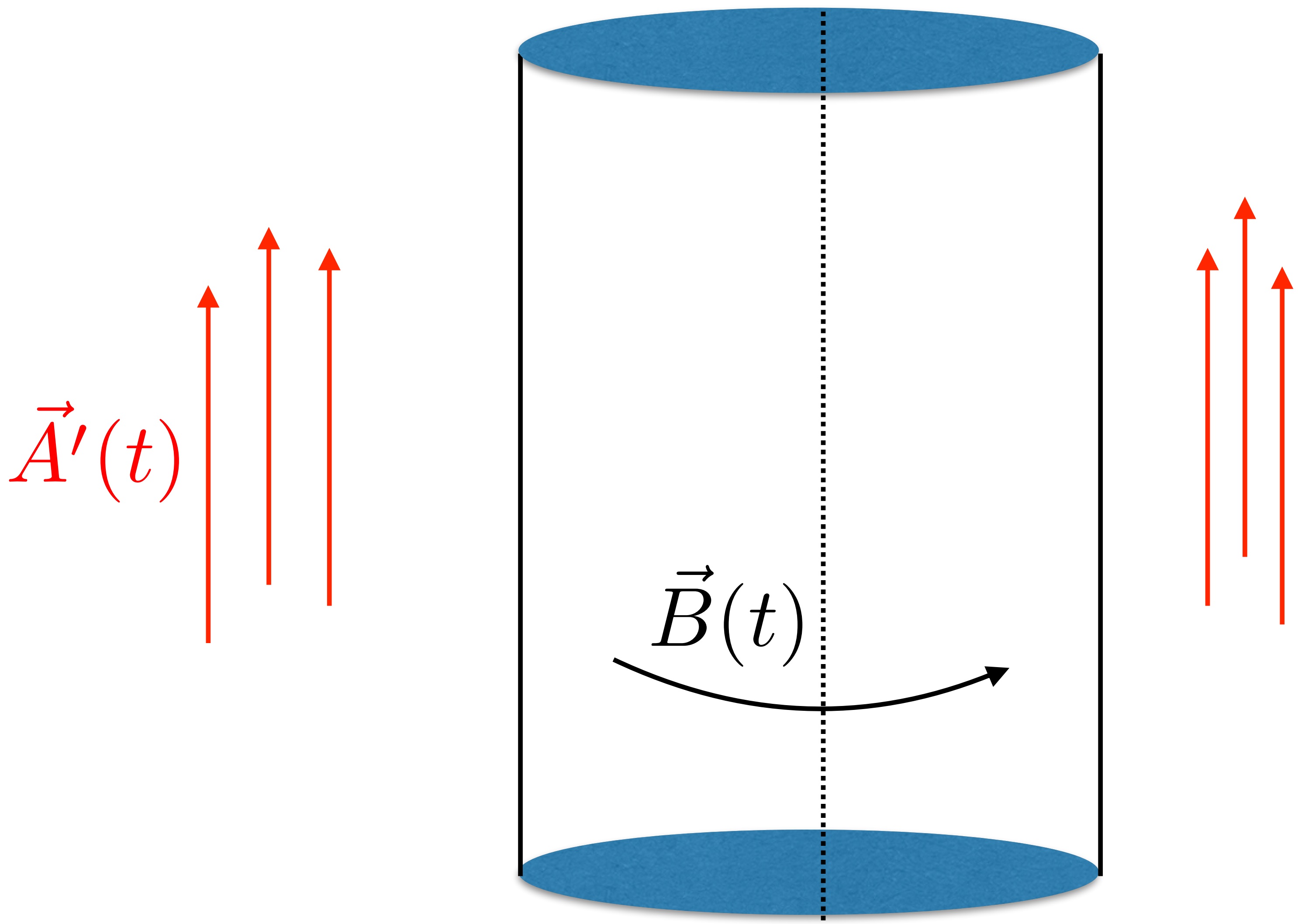
IV. Can probe the GUT-scale QCD axion!



Backup slides

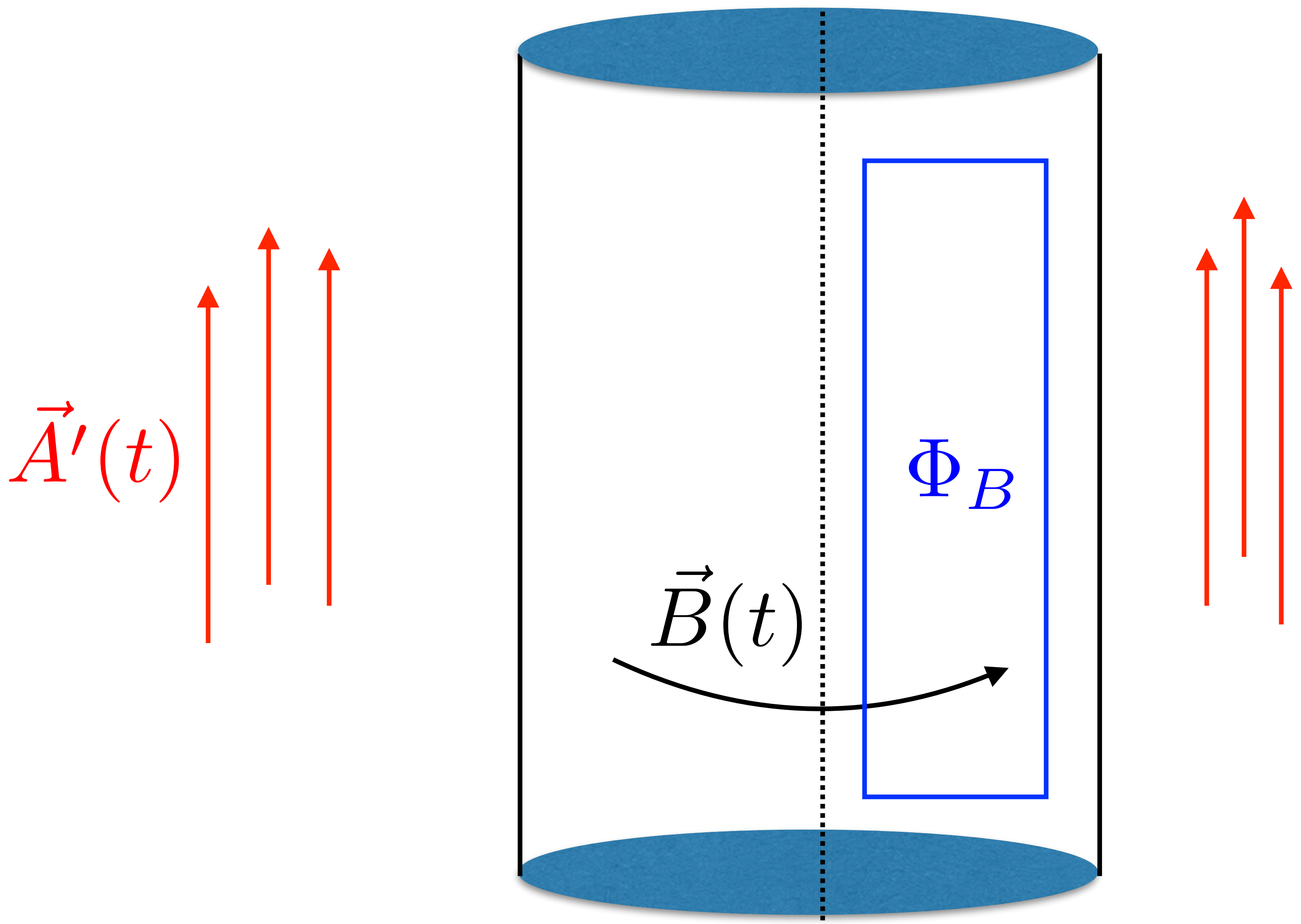
Dark photon applications

$$\mathcal{L} = \mathcal{L}_{EM} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m^2_{\gamma'}A'_\mu A'^\mu - \epsilon e J^\mu_{EM} A'_\mu$$



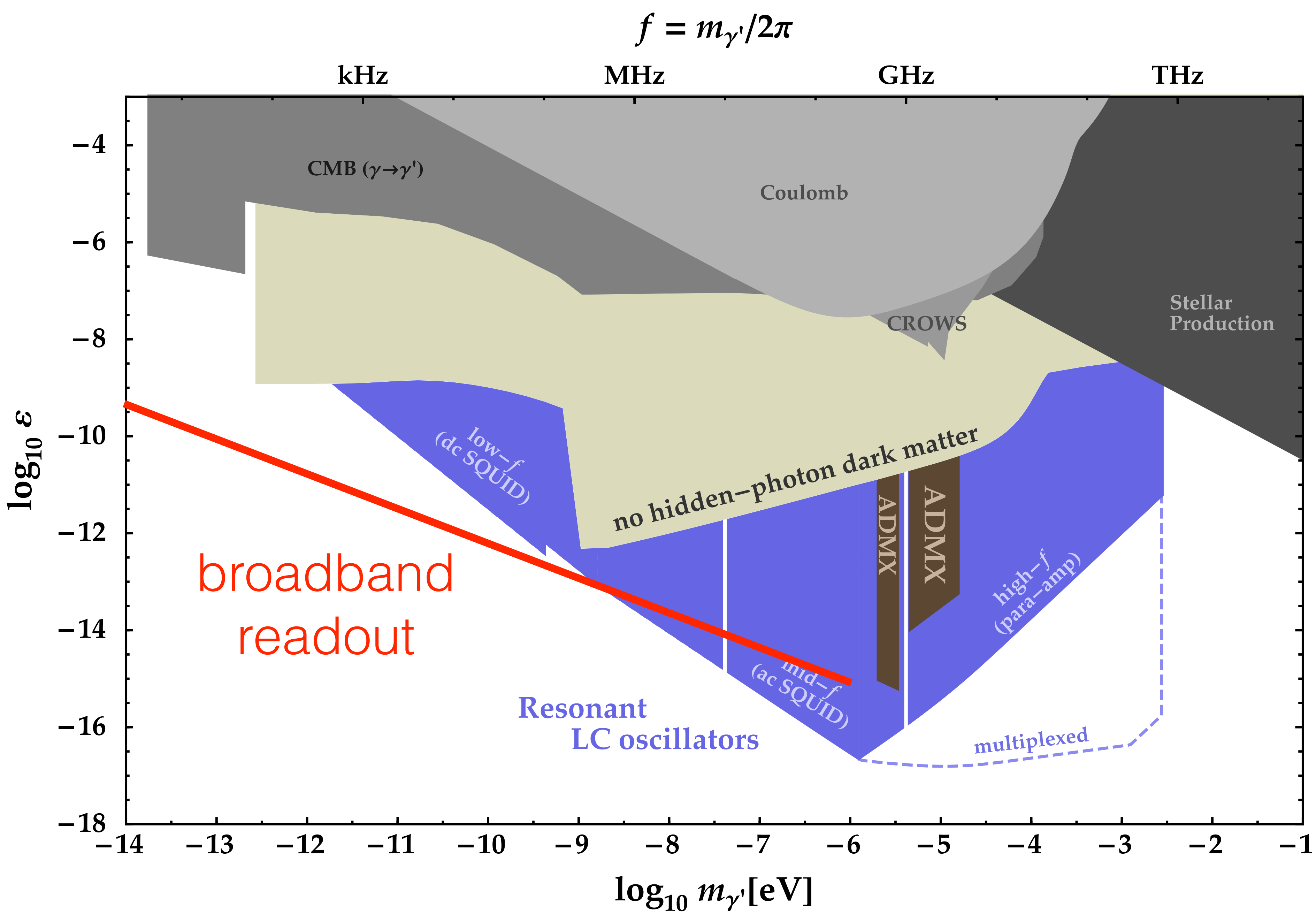
Dark photon applications

$$\mathcal{L} = \mathcal{L}_{EM} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m^2_{\gamma'}A'_\mu A'^\mu - \epsilon e J^\mu_{EM}A'_\mu$$



Try detecting B-field with broadband strategy?
No tuning necessary!

Broadband dark photon reach

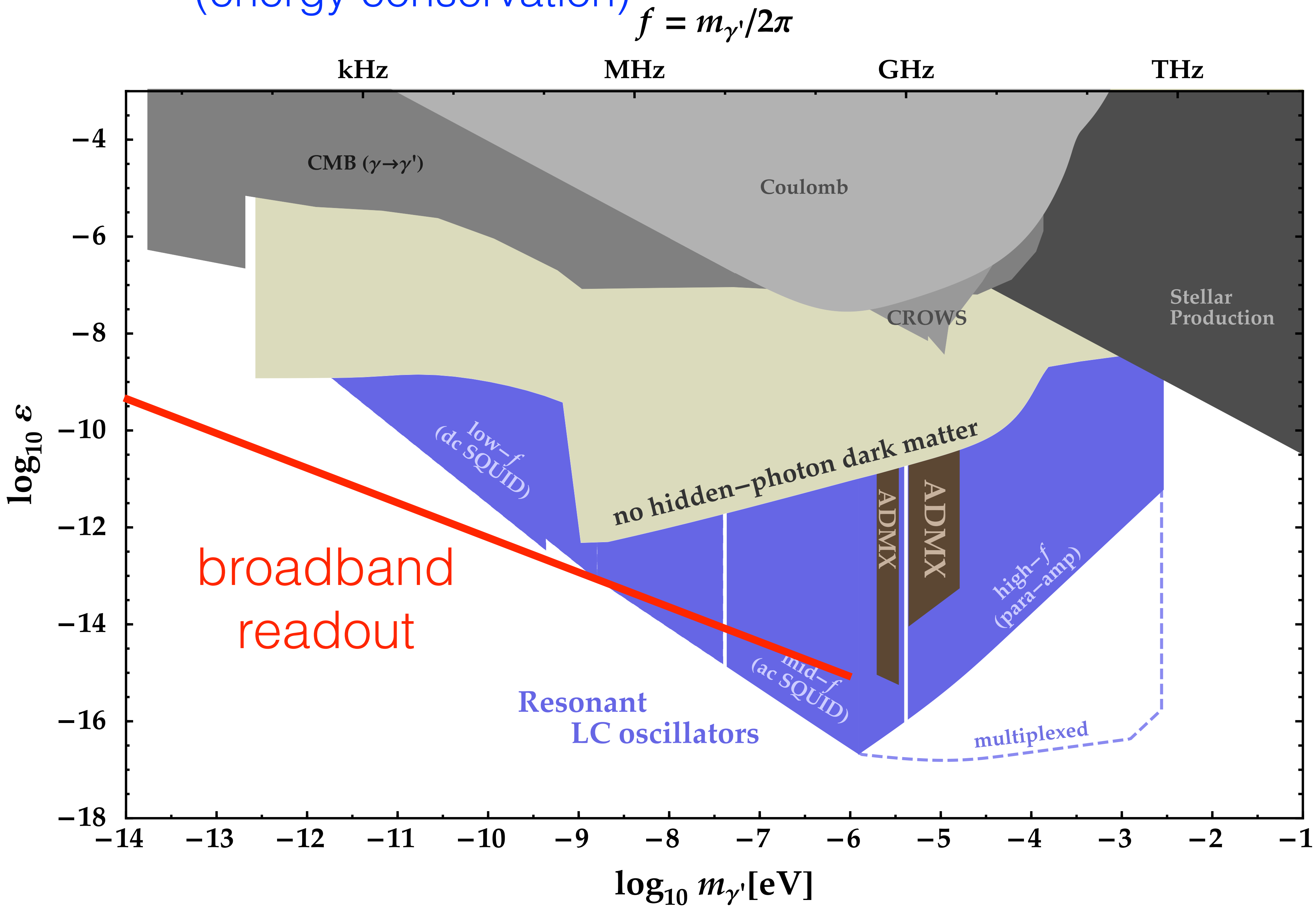


[Chaudhuri et al., 1411.7382]

Broadband dark photon reach

Assume $\Phi_{\text{SQUID}} = 0.025\Phi_B$, 1 year running as with axions

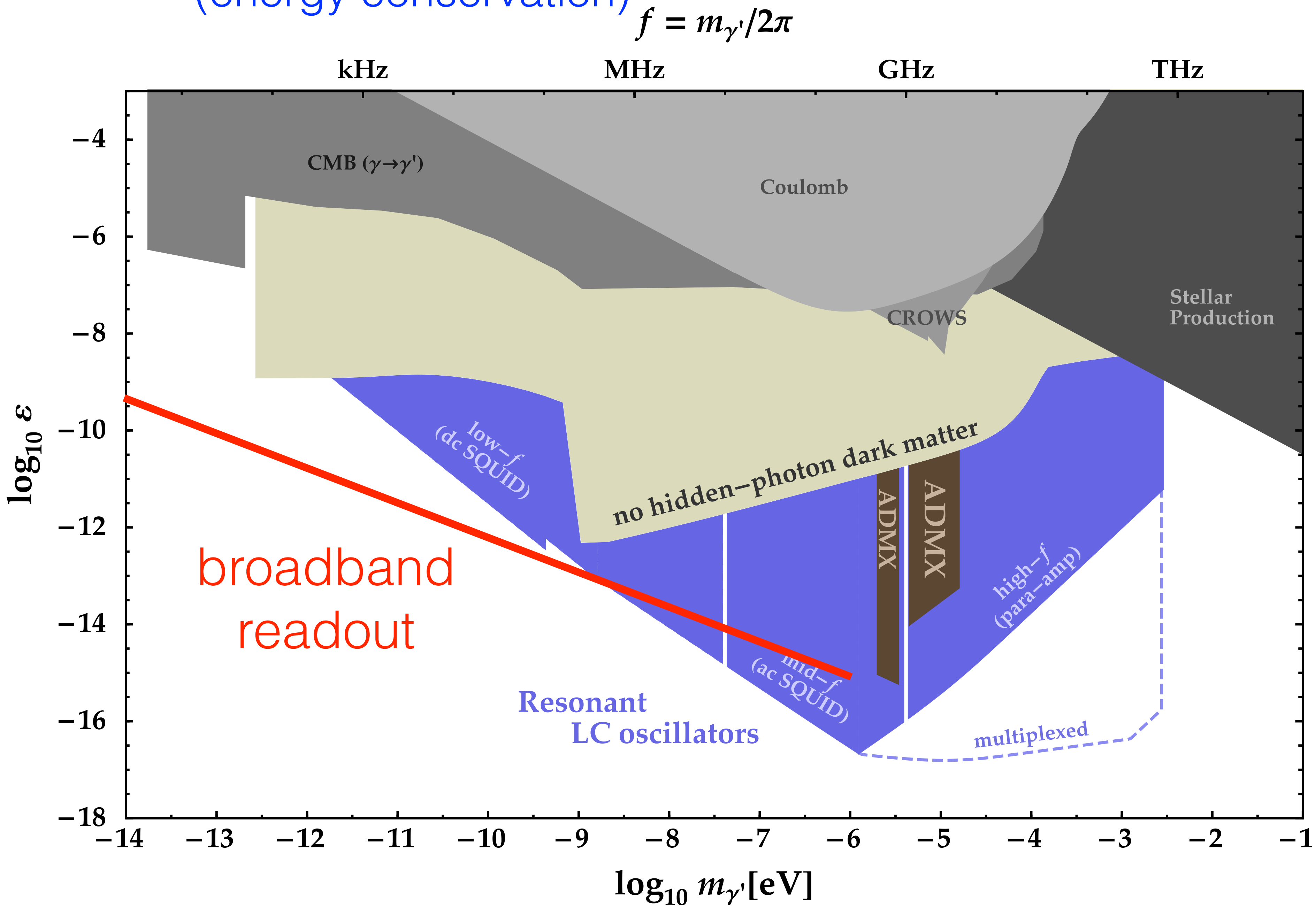
(energy conservation)



[Chaudhuri et al., 1411.7382]

Broadband dark photon reach

Assume $\Phi_{\mathrm{SQUID}} = 0.025\Phi_B$, 1 year running as with axions
(energy conservation)



[Chaudhuri et al., 1411.7382]

Complementary to high-frequency LC readout

Resonant: bandwidth matching

$v_{DM} = 10^{-3} \implies$ intrinsic signal bandwidth:

$$\frac{\Delta\omega}{\omega} = 10^{-6}$$

Intrinsic LC circuit bandwidth: $\frac{\Delta\omega_{\rm LC}}{\omega} = \frac{1}{Q}$ $\quad Q = \frac{1}{R}\sqrt{\frac{L_T}{C}}$

have to wait at least one cycle: $\Delta\omega_{\rm LC} > \frac{2\pi}{\Delta t}$

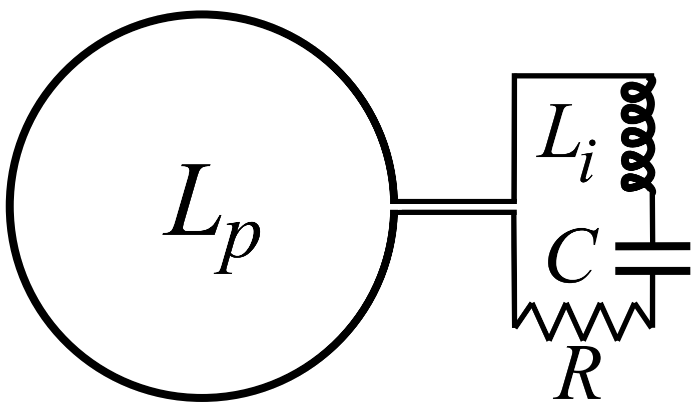
Can potentially use "black box" (e.g. feedback damping) to broaden bandwidth without decreasing Q: take Q = 10^{6}* but larger may be possible

*comparable to existing Nb superconducting LC circuits

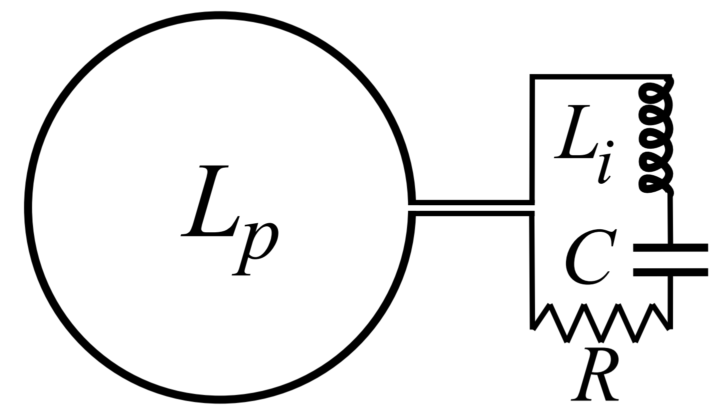
Broadband $\neq$ non-resonant!

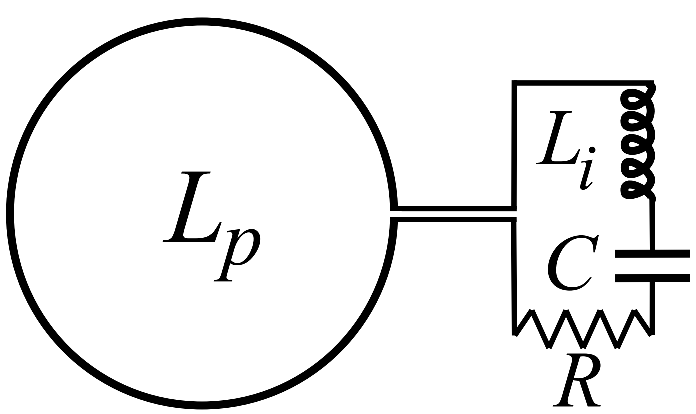
$$Q = \frac{1}{R}\sqrt{\frac{L_T}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

more noise,
wider bandwidth



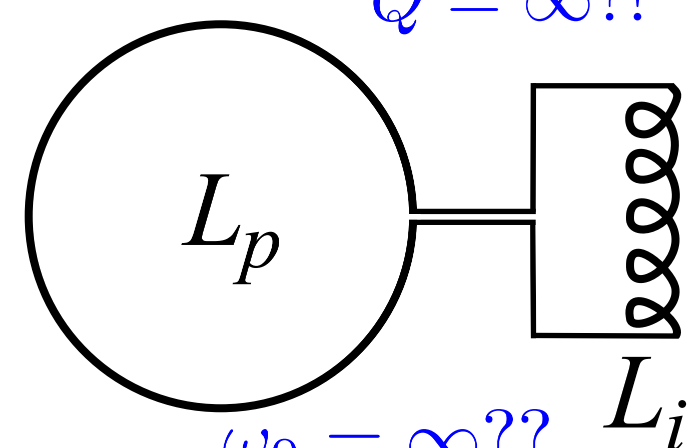
$Q \to 1,\ \ L_T, C$ fixed





$R, C \to 0,\ \ L_T$ fixed

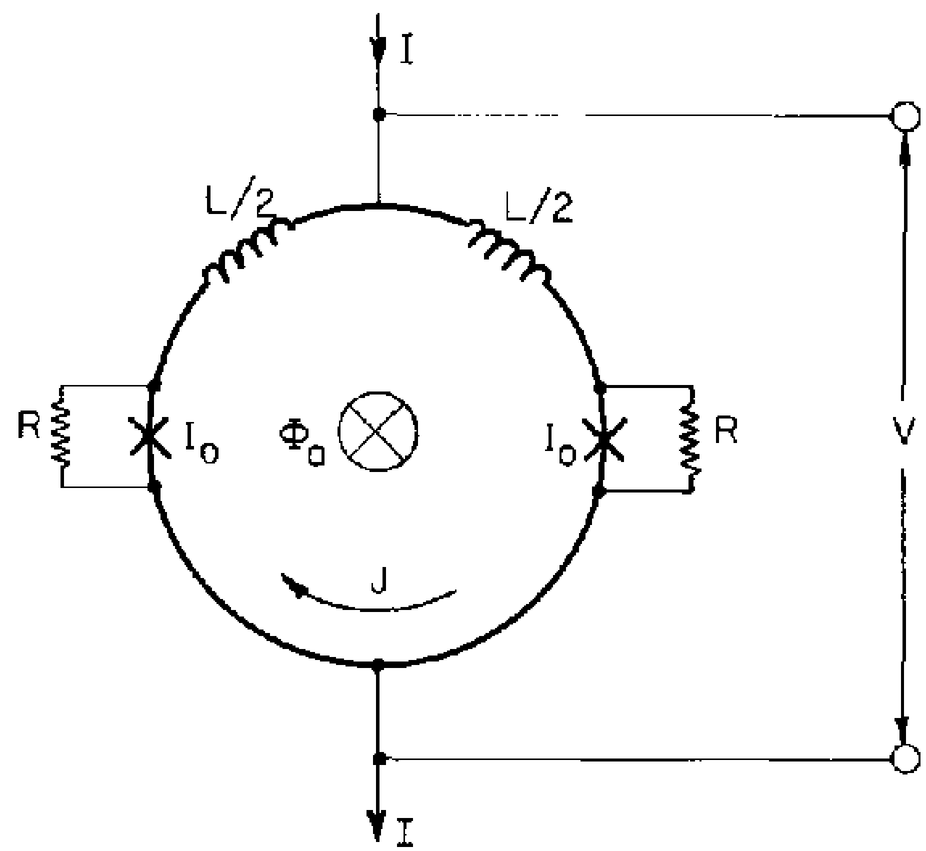
$Q = \infty$?!



$\omega_0 = \infty$??

Q is not an appropriate variable
to describe a purely inductive circuit

Origin of SQUID noise



Junction shunt resistance introduces thermal noise:

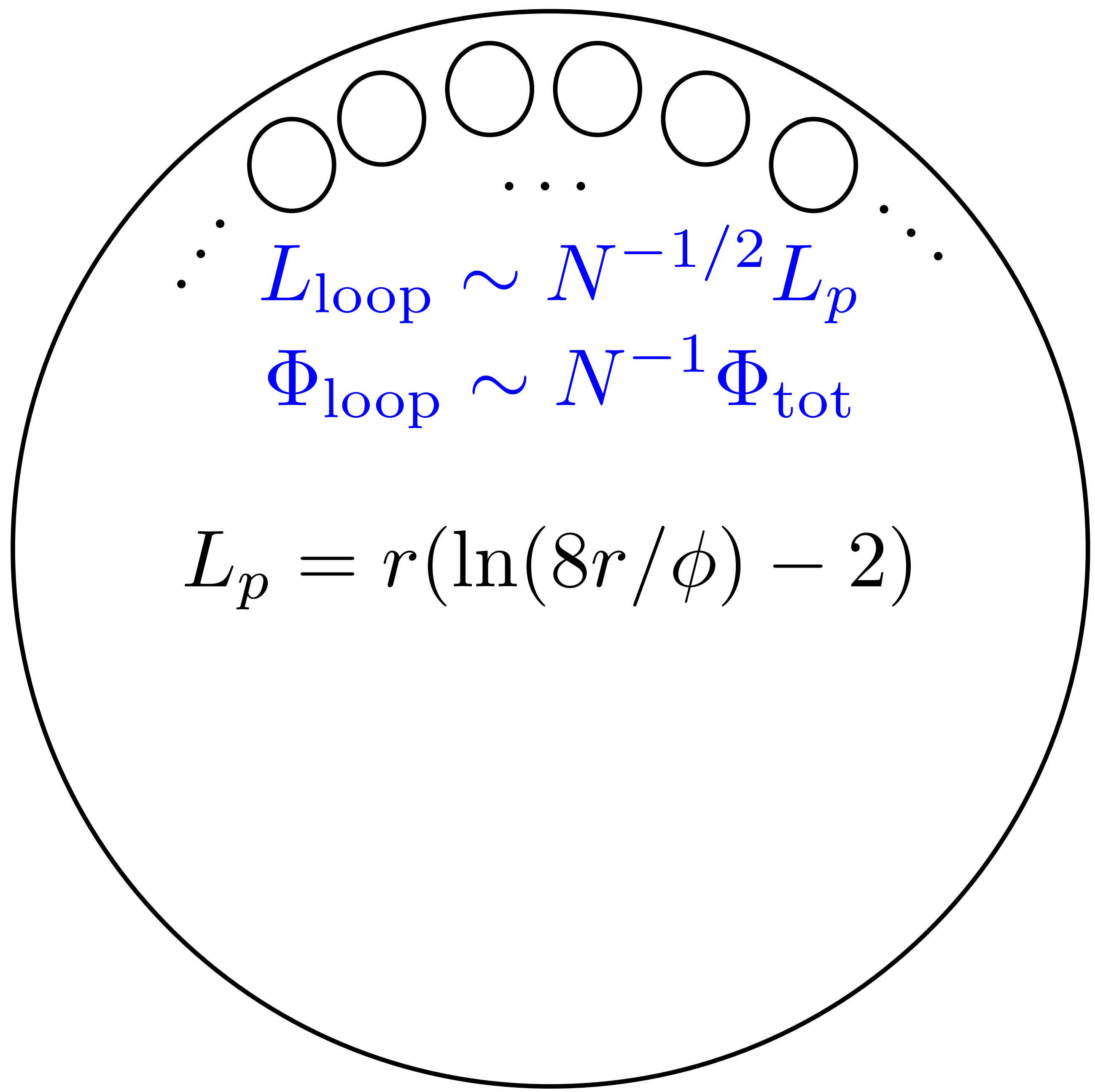
$$S_V \approx \cancel{16 k_B T R}$$

$$S_J \approx 11 k_B T / R$$

always subdominant
in resonant circuit
(suppressed by narrow bandwidth)

Inductance matching

N loops
in parallel:



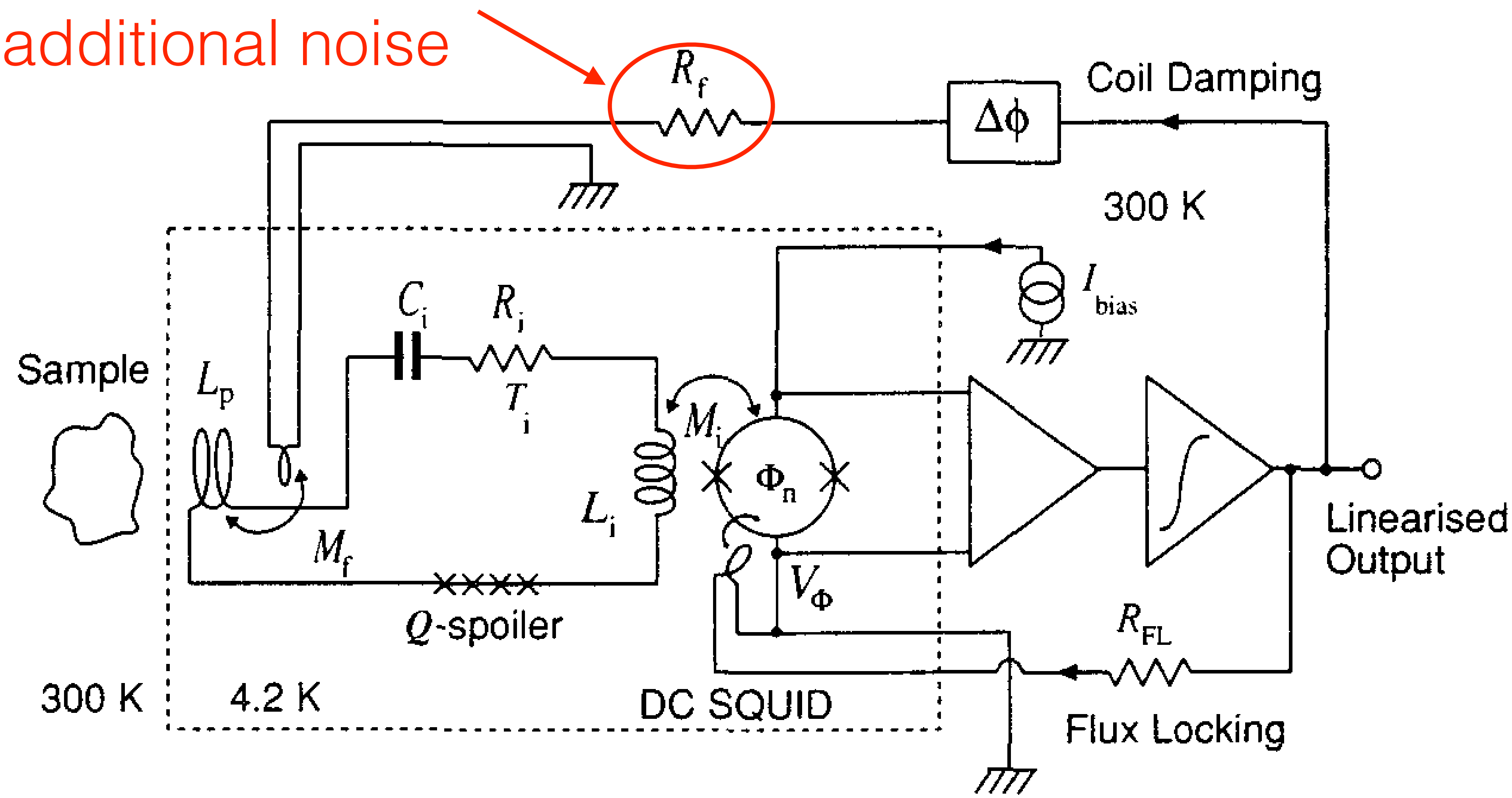
$$\implies L_{\text{eff}} \sim N^{-1/2} L_p$$

Could also use “pie-slice” loops (fractional-turn magnetometer),
or slitted sheath as in 1411.7382

Resonant: feedback damping

Trick from SQUID magnetometry:
can widen bandwidth without increasing thermal noise

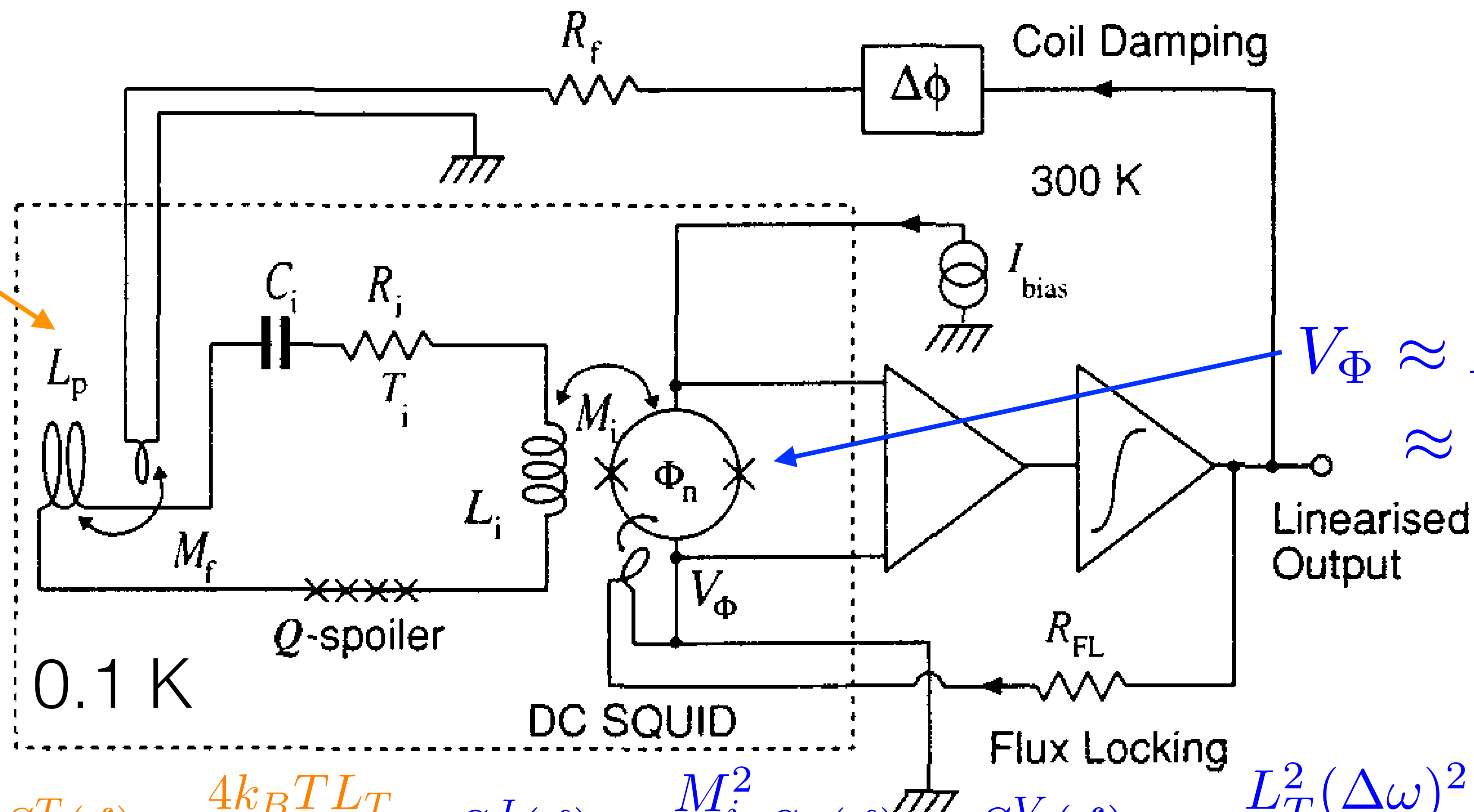
decreases loaded Q,
but not part of resonant circuit
so no additional noise



[Seton et al. *Mag. Res. Mat.* 1999]

Dominance of thermal noise in resonant circuit

N_s turns



$V_\Phi \approx R/L$
$\approx 10^9$ Hz

$$S_\Phi^T(f) = \frac{4k_BTL_T}{N_s^2\omega Q_0} \qquad S_\Phi^J(f) \approx \frac{M_i^2}{N_s^2}S_J(f) \qquad S_\Phi^V(f) \approx \frac{L_T^2(\Delta\omega)^2}{N_s^2\omega^2 M_i^2 V_\Phi^2}S_V(f)$$

Optimize w.r.t. N_s:

thermal

$< 10^{-2}$

shot noise

$$S_\Phi(f) \approx \frac{4k_BTL_p}{\omega Q_0}\left[1 + \frac{4\times10^{-6}}{\alpha^2}\frac{Q_0\Delta\omega}{\omega}\cancel{\frac{\Delta\omega}{V_\Phi}} + 10^{-6}Q_0\alpha^2\frac{\omega}{V_\Phi}\left(\frac{11}{4} + \frac{S_{J,0}}{4k_BT}\right)\right]$$

Other noise sources

- Shielding noise: can reduce with superconducting shield
- Current noise: probably minimal if current-carrying wires are superconducting, but may contribute small azimuthal current. Can reduce with clever toroid winding, or envelop toroid in overlapping superconducting shield
- 1/f SQUID noise: dominant below 50 Hz, worse at low temperatures, maybe mitigate with modulation?