Threshold Top-Antitop Production at the LHC

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Topics discussed

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- Phenomenology of top-antitop threshold production:
  - *electron-positron* vs *hadronic colliders*
  - *Tevatron* vs *LHC*
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- Phenomenology of top-antitop threshold production:
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  - *Tevatron* vs *LHC*
  - *top mass determination*
- Modern theory of threshold production:
  - *effective/perturbation theory*
  - *nonrelativistic renormalization group*
  - *unstable top*

Top-antitop threshold production at the ILC

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  - *Allows for the first principle QCD predictions*
  - *High order results are available (NNNLO is coming!)*

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Higgs, EWSB, GUT, SUSE, ...

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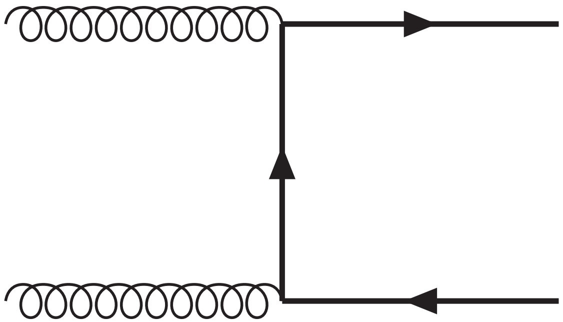
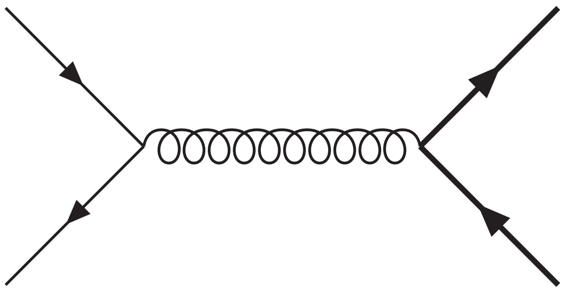
- Top mass determination
  - *single top reconstraction*
  - *top-antitop threshold production (new!)*

Partonic processes

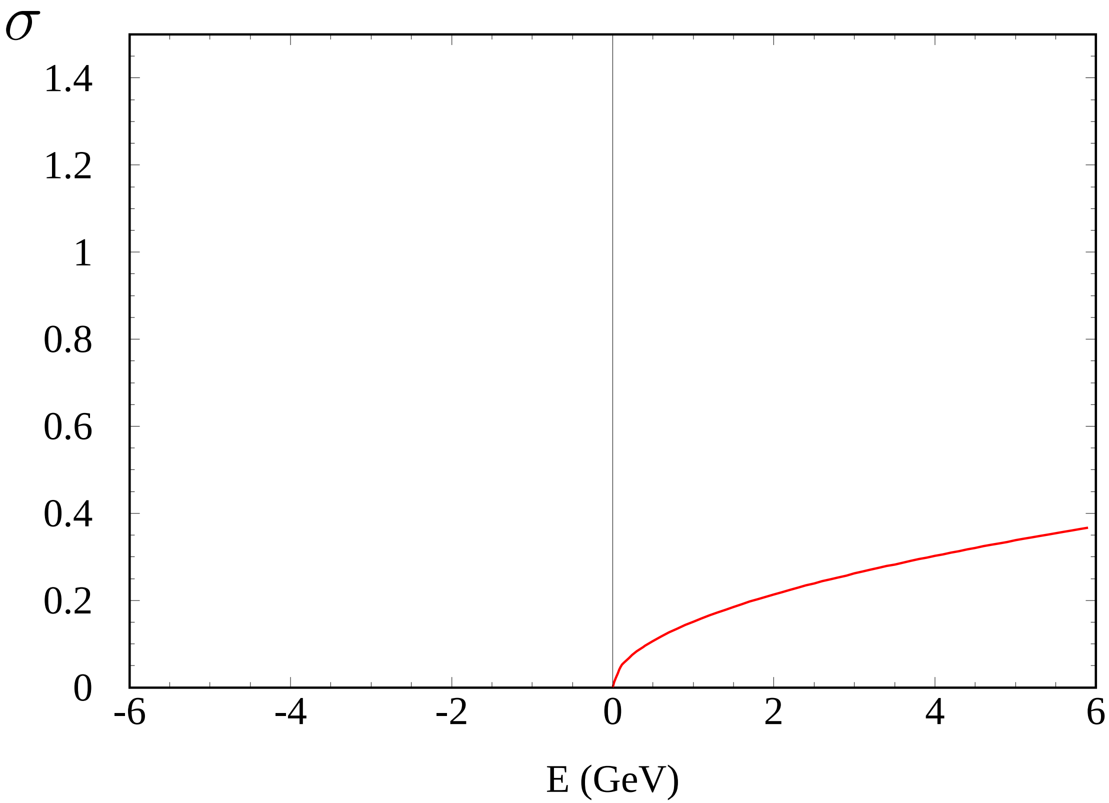


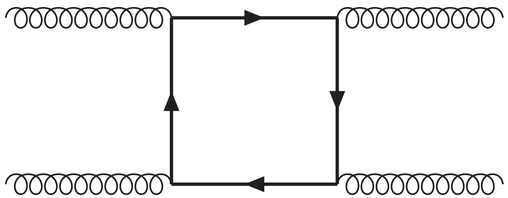
$$gg \to t\bar{t}(^1S_0^{[0]})$$
$$gg \to t\bar{t}(^1S_0^{[8]})$$



$$q\bar{q} \to t\bar{t}(^3S_1^{[8]})$$

Binding effects





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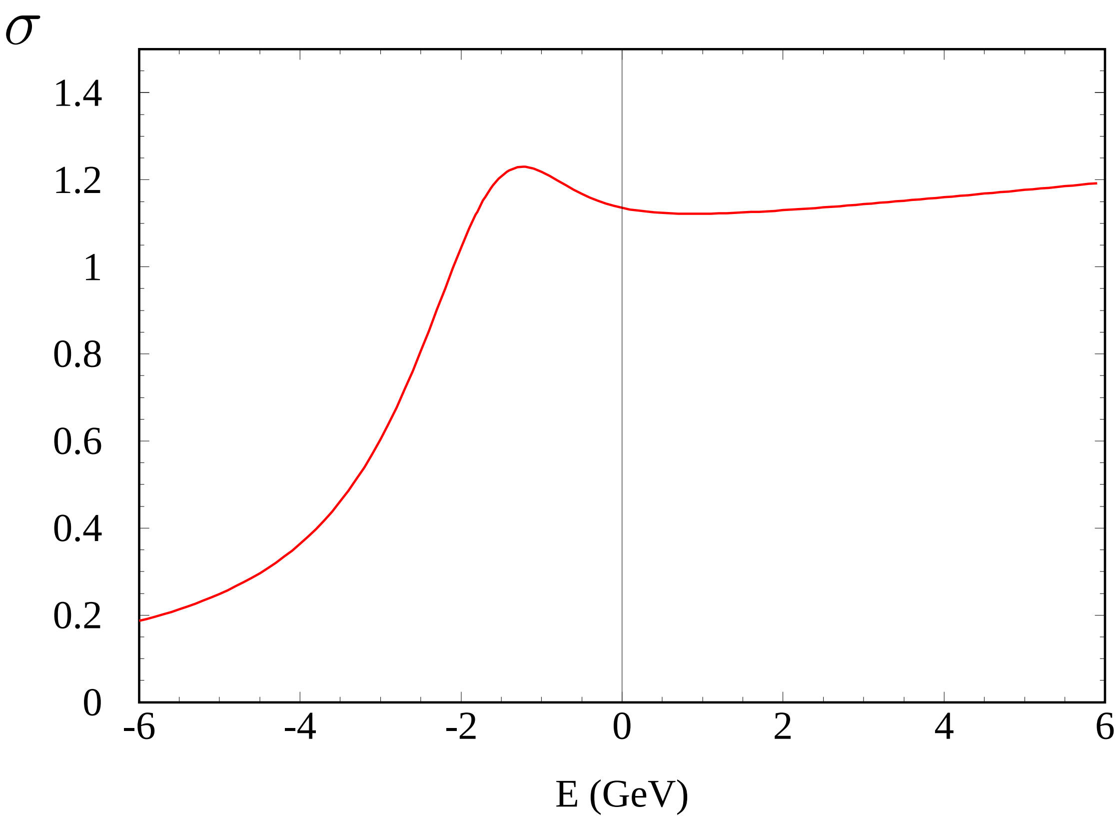
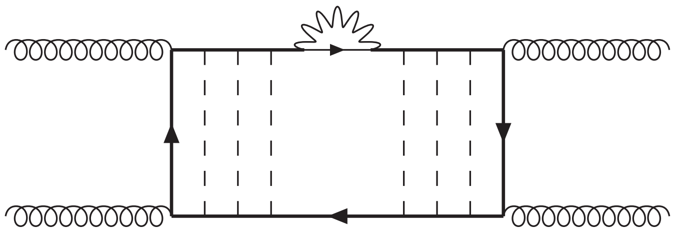




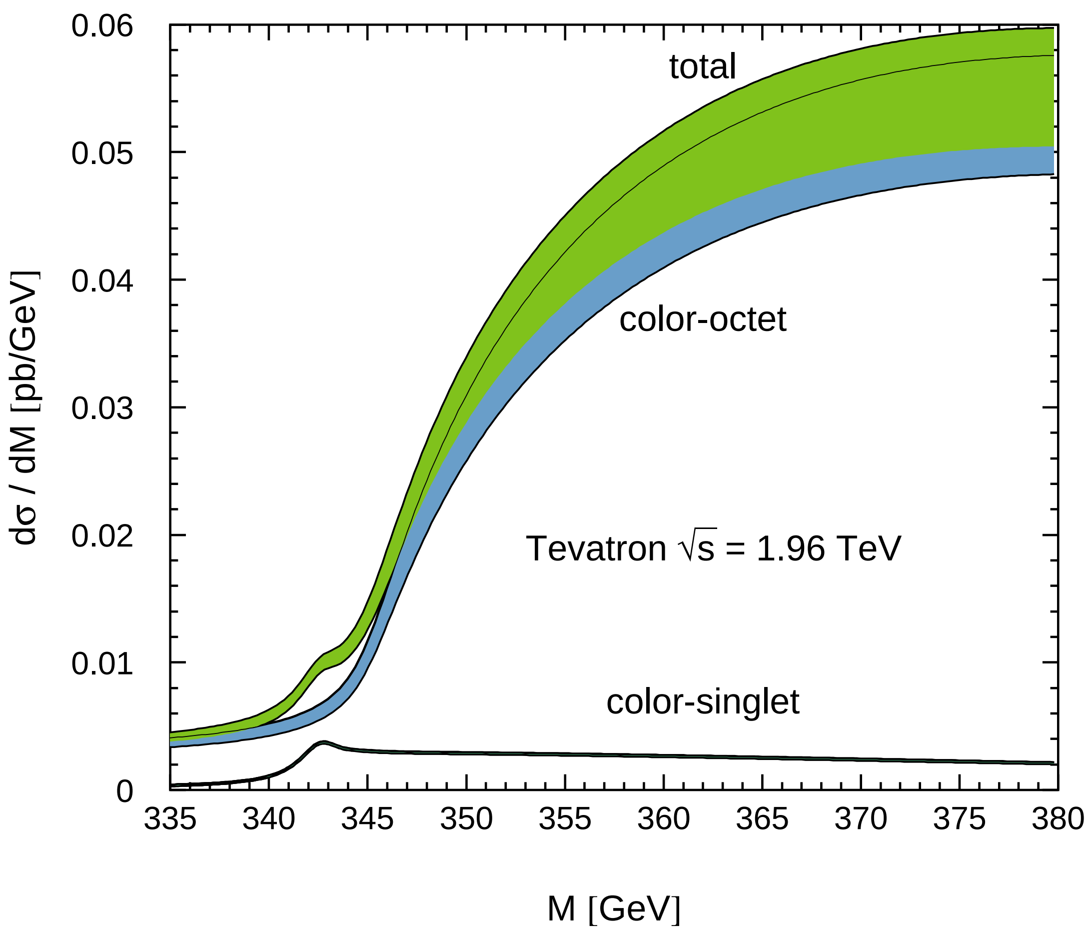
$$\sigma_{\rm res} \sim \frac{\alpha_s^5}{m_t\Gamma_t}$$

Invariant mass distribution

$$\ldots^2 \frac{d}{dM^2}\sigma_{pp\to t\bar t}(s, M^2) \propto \sum_{partons} \sigma_{parton}(M^2)\int_{M^2/s}^{1} \frac{dz}{z} F(z)\mathcal{L}'(M^2/zs)$$

$t\bar{t}$ threshold production at the Tevatron

(Kiyo, Kuhn, Moch, Steinhauser, Uwer)



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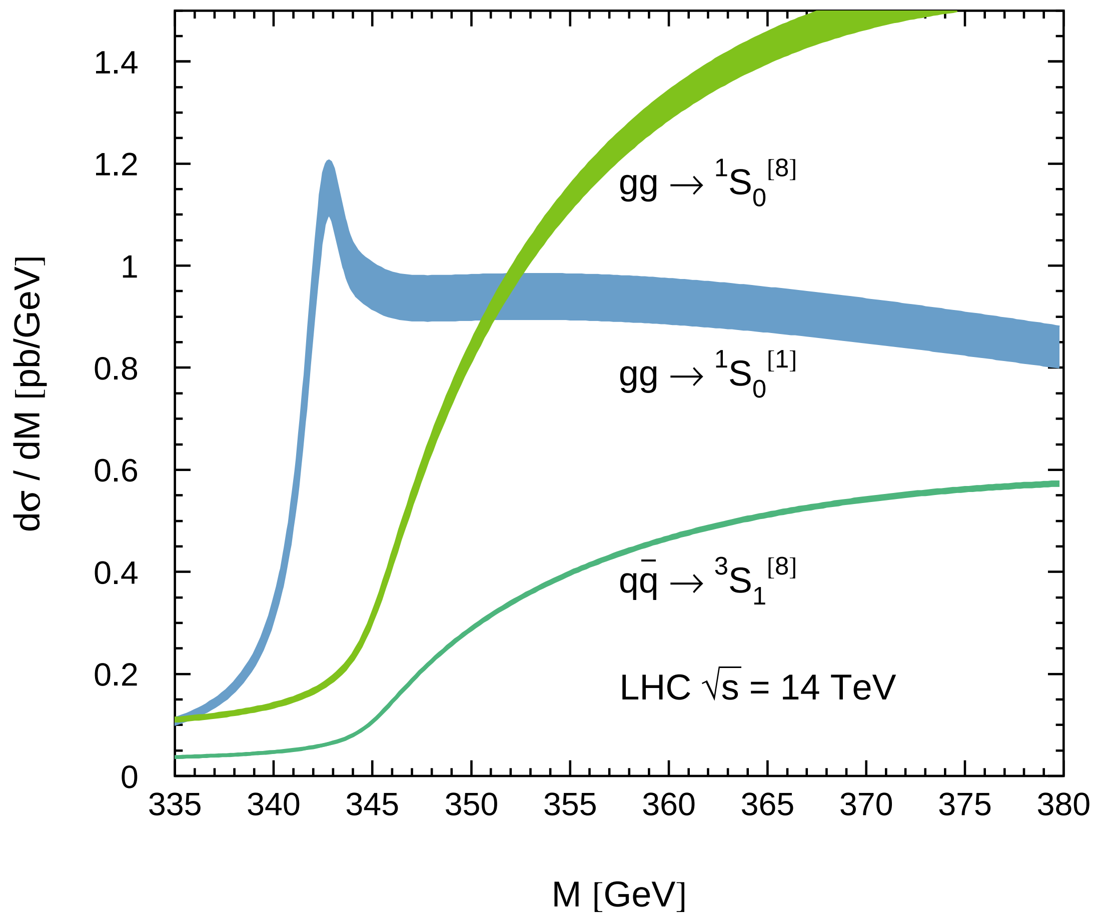

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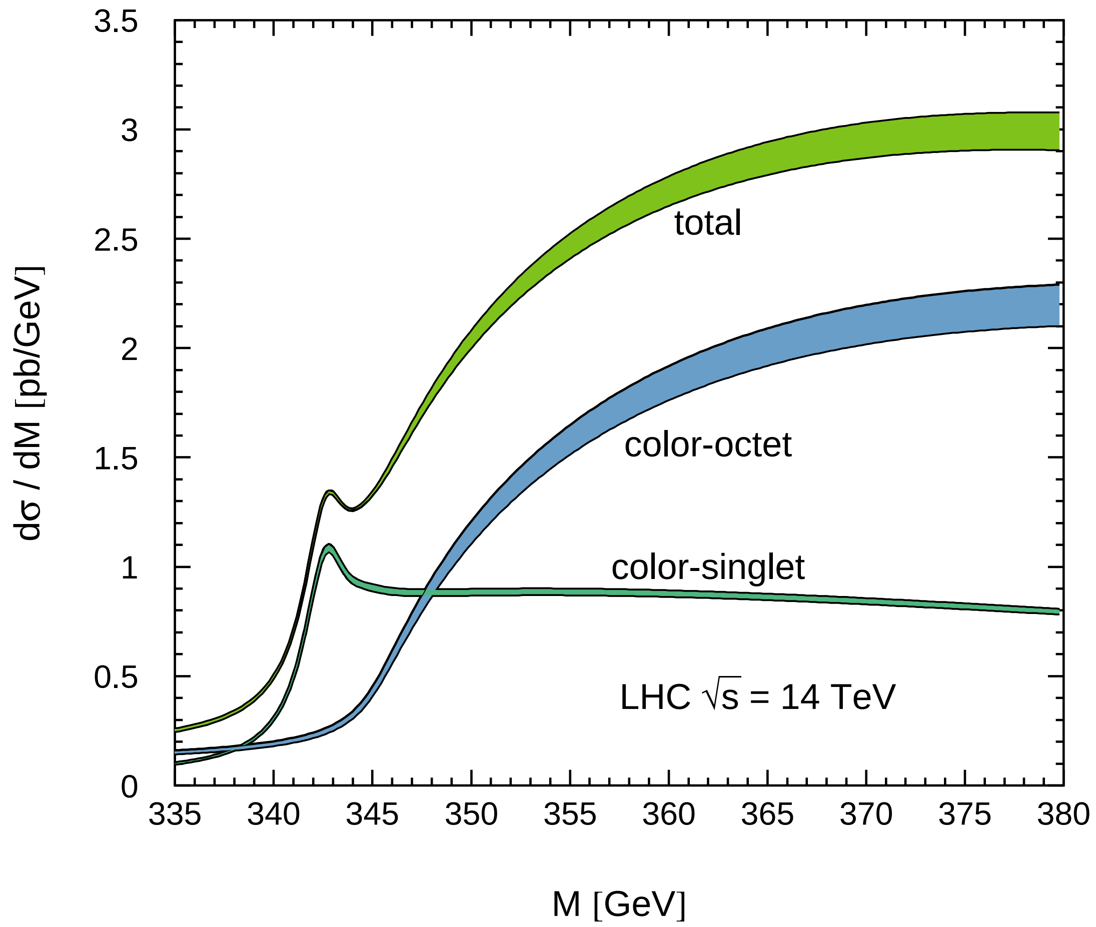

The top mass from invariant mass distribution

$$M_{\text{res}} = 2m_t + E_1 + \delta^{\Gamma_t} E_{\text{res}}$$

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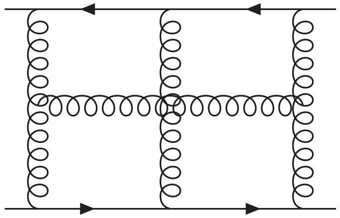
➥ *Full N^{3}LO analysis is mandatory*

N^3LO ground state energy

(Penin, Steinhauser)

$$\frac{\delta E_1^{N^3LO}}{E_1^{LO}} = \alpha_s^3 \left(58.205 + 15.297 \ln(\alpha_s) + 26.654\right)$$

Renormalon contribution



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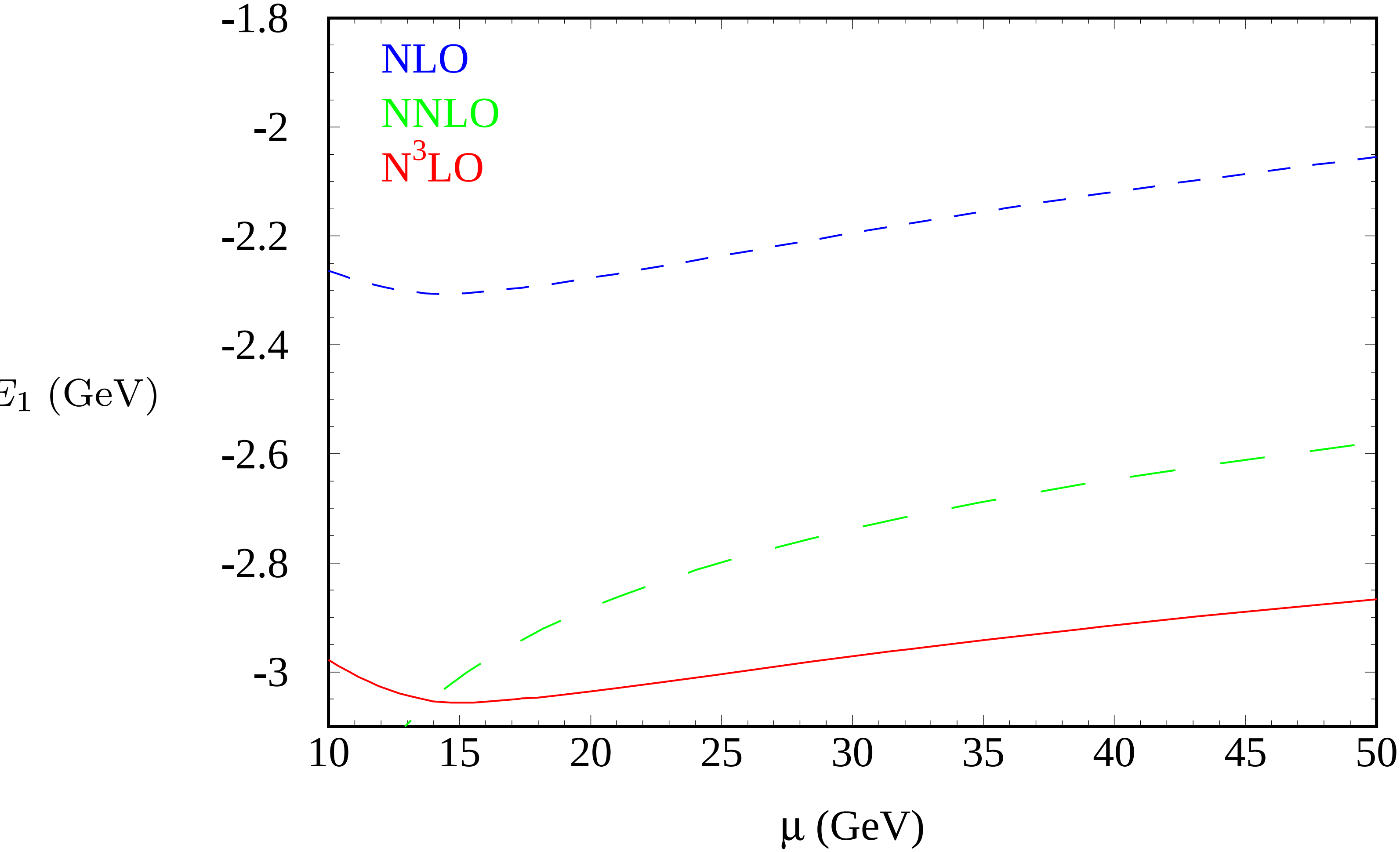

Formula of Success

pNRQCD + Dim.Reg. = N^{3}LO

Nonrelativistic Effective Theory

(Caswell, Lepage; ...)

- Characteristic momentum regions:

Hard	$p_0 \sim m_q, \quad \boldsymbol{p} \sim m_q$
Soft (static quarks)	$p_0 \sim v m_q, \quad \boldsymbol{p} \sim v m_q$
Potential (static gluons)	$p_0 \sim v^2 m_q, \quad \boldsymbol{p} \sim v m_q$
Ultrasoft	$p_0 \sim v^2 m_q, \quad \boldsymbol{p} \sim v^2 m_q$

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- Schrödinger theory & multipole radiation
- Dynamical degrees of freedom

Potential NRQCD

(Pineda, Soto; ...)

Schrödinger equation

$$(\mathcal{H}-E)\,G(\boldsymbol{r}\,,\boldsymbol{r},E)=\delta(\boldsymbol{r}-\boldsymbol{r}')$$

Effective Hamiltonian

$$\mathcal{H} \;=\; -\frac{\boldsymbol{\partial}^2}{m_q}+V_{\rm Coulomb}(\boldsymbol{r})+\delta\mathcal{H}$$

Multipole interaction to ultrasoft gluons

$$g_s(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot\boldsymbol{E}+g_s(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\boldsymbol{B}+\ldots$$

Loops in the Effective Theory

Problem: Separation of regions in virtual momentum space

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" ...there is a moral here for us. The artificial separation of high and low frequencies, which are handled in different ways, must be avoided"

(Schwinger, Particles, sources and fields, Vol.2)

Loops in the Effective Theory

Regions are separated in dimensional regularization!

Effective theory in dimensional regularization

(Pineda, Soto; Beneke, Smirnov; Czarnecki, Melnikov, Yelkhovsky; Kniehl, Penin, Smirnov, Steinhauser)

gauge, Lorenz invariance + automatic matching

➡ *ideal for high-order calculations*

NLO corrections to HFS

Spin-flip potential:

$$V_S(\boldsymbol{q}^2) = \frac{4\pi C_F \alpha_s \boldsymbol{S}^2}{3m_q^2}$$

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Spin-flip potential:

$$V_S(\boldsymbol{q}^2) = \frac{4\pi C_F \alpha_s \boldsymbol{S}^2}{3m_q^2}\left\{1 + \frac{\alpha}{\pi}\left[\left(1+\frac{7}{8}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{m_q^2}\right)\right)C_A - \frac{1}{2}C_F + \frac{6-6\ln 2+i3\pi}{4}T_F\right] + \frac{\alpha}{\pi}\left[\left(-\frac{7}{18}-\frac{7}{8}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\boldsymbol{q}^2}\right)\right)C_A - \frac{5}{9}T_F n_l\right]\right\}$$

hard contribution

QCD on-shell on-threshold amplitude

soft contribution

NQCD, static heavy quark propagator

Coulomb potential:

$$V_C(\boldsymbol{q}^2) = -\frac{4\pi C_F \alpha_s(\boldsymbol{q}^2)}{\boldsymbol{q}^2}\left[1 + \frac{\alpha_s}{\pi}\left(\frac{31}{36}C_A - \frac{5}{9}T_F n_l\right)\right]$$

NLO corrections to HFS

Quantum Mechanical PT *(potential contribution)*

$$\delta^{NLO}E_{\rm hfs} = \langle\psi_n^{\rm Coulomb}|V_S^{1-{\rm loop}}|\psi_n^{\rm Coulomb}\rangle + 2\langle\psi_n^{1-{\rm loop}}|V_S^{\rm tree}|\psi_n^{\rm Coulomb}\rangle$$

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Result for general n

$$E_{\rm hfs}^{NLO}(n) = \frac{1}{3} \frac{C_F^4 \alpha_s^4}{n^3} m_q \left\{ 1 + \frac{\alpha_s}{\pi} \left[\frac{7\, C_A}{4} \ln\left(\frac{C_F \alpha_s}{n}\right) - \frac{C_F}{2} + \frac{3}{2}(1 - \ln 2) T_F \right.\right.$$
$$+ \frac{-15 - 11\, n + 12\, n^2\, \Psi_2(n)}{9\, n} n_f\, T_F$$
$$\left.\left. + \frac{393 + 95\, n + 126\, \gamma_E\, n + 126\, n\, \Psi_1(n) - 264\, n^2\, \Psi_2(n)}{72\, n} C_A \right] \right\}$$

(Penin, Steinhauser)

Nonrelativistic Renormalization Group

- Several scales: m_q, $m_q v$, $m_q v^2$
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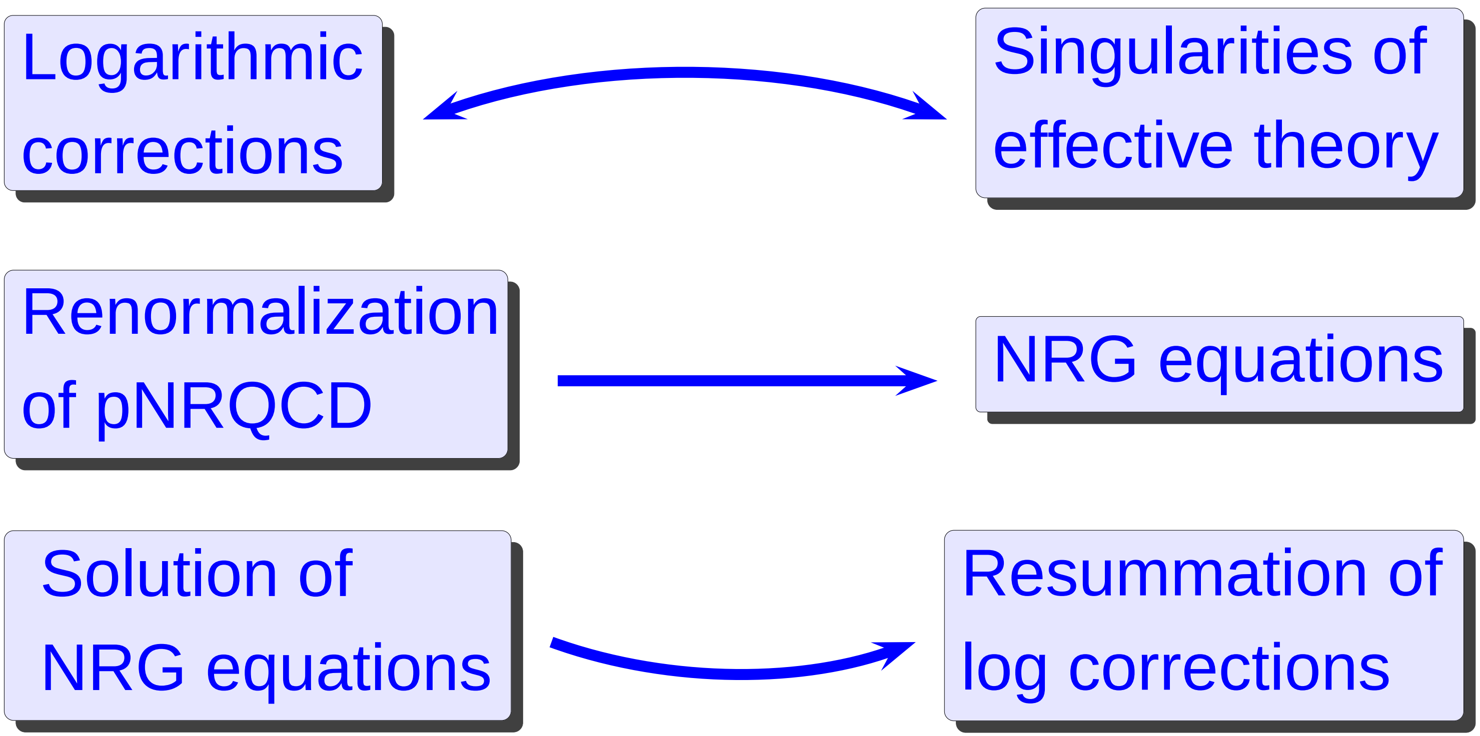

HFS in NLL approximation

Kniehl, Penin, Pineda, Smirnov, Steinhauser, Phys.Rev.Lett. 92, 242001 (2004)

$$
{}^{LL}_{s} = \frac{C_F^4 \alpha_s^4(\nu) m_q}{3} \left\{ \frac{27}{14} y^{-1} - \frac{13}{14} y^{-\frac{18}{25}} + \frac{\alpha_s(m_q)}{\pi} \left[\left(\frac{1037}{224} + \frac{405086361761\,\pi^2}{25617160800} \right.\right.\right.
$$

$$
\left. \ln 2 \right) \times y^{-1} - \frac{1024\,\pi^2}{143} y^{-\frac{39}{50}} - \left(\frac{102973}{26250} + \frac{184336\,\pi^2}{25725} \right) y^{-\frac{18}{25}} + \frac{1024\,\pi^2}{675} y^{-\frac{1}{2}} + \frac{671\,\pi^2}{1029} y^{-\frac{11}{25}}
$$

$$
\frac{\pi^2}{23} y^{-\frac{2}{25}} + \left(-\frac{13427921}{1260000} + \frac{88057\,\pi^2}{151200} \right) y^{\frac{7}{25}} + \frac{4\,\pi^2}{41} y^{\frac{16}{25}} + \frac{1377}{56} - \frac{1253587\,\pi^2}{227500} - \frac{629\,\pi^2}{7500} y
$$

$$
\frac{873\,\pi^2}{7182} y^{\frac{32}{25}}\, {}_2F_1\left(\frac{57}{25}, 1; \frac{82}{25}; \frac{y}{2} \right) + \frac{2873\,\pi^2}{3591} y^{-1}\, {}_2F_1\left(1, 1; \frac{82}{25}; -1 \right) + \left(\frac{675}{28} - \frac{533}{42} y^{\frac{7}{25}} \right)
$$

$$
\left(\frac{\mu}{C_F \alpha_s(\mu) m_q} \right) + \frac{85248\,\pi^2}{30625} y^{-1} \ln y + \left(-\frac{45834}{4375} y^{-1} + \frac{21216}{4375} - \frac{2873}{1575} y^{\frac{7}{25}} + \frac{243}{1250} y \right)
$$

$$
{}^2 \ln(2-y) \Big] \Big\} ,
$$

$$
y = \frac{\alpha_s(\mu)}{\alpha_s(m_q)}
$$

Finite lifetime effect

- Top decays into W and b with $\Gamma_t \approx 1.5$ GeV
  - *pole of the top propagator moves into the complex plain* $E \to E + i\Gamma_t$
  - *makes process perturbative in the whole threshold region*
  - *smears out Coulomb resonances*

(Fadin, Khoze)

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  - *new effective theory - nonrelativistic QCD with* $v \to \rho^{1/2}$

(Penin, Piclum, JHEP 2011)

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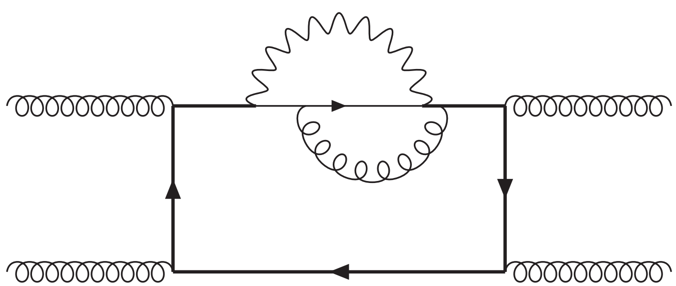

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- Problem:
  - *experimental resolution?*