PSI Dark Matter Homework #2

Due: Apr. 5, 2013

1. Dark matter freeze out, from start to finish.

   a) Starting from the interactions

   $$-\mathscr{L} \supset y_\chi \bar{\chi}\chi\,\phi + y_f \bar{f} f\,\phi \tag{1}$$

   compute the leading contribution to $\langle \sigma v \rangle$ for $\chi\bar{\chi} \to f\bar{f}$ using the non-relativistic prescription described in `notes-2`. You may treat f as massless but keep the masses of ϕ and χ. You should also account for the fact that ϕ is unstable by including its decay width Γ_ϕ in the propagator: $i/(q^2 - m_\phi^2 + i m_\phi \Gamma_\phi)$.

   b) Compute x_f and $\Omega_\chi h^2$ using the approximate expressions in `notes-2`.

   c) Plot your results as a function of the χ mass over the range 10 GeV $< m_\chi <$ 1000 GeV for $y_\chi = y_f = 0.2$ and $m_\phi = 300$ GeV. Take the width of ϕ to be $\Gamma_\phi = m_\phi/50$.

   You should now be equipped to compute the DM relic density of your favourite theory!

2. Direct detection scattering.

   a) Consider a theory of a dark matter particle χ interacting with a visible fermion f through the interaction of Eq. (1) give above. Compute the amplitude and the summed and squared matrix element for the elastic-scattering process $\chi(p_1) + f(p_2) \to \chi(p_3) + f(p_4)$ mediated by this interaction.

   b) Work out the kinematics for this process in the non-relativistic limit where χ comes in along the z axis with velocity $v \ll 1$, and strikes the fermion f at rest. Treat the fermion and χ as both being very massive ($m_\chi v \ll m_\chi$, m_f, m_ϕ) and compute the momentum and the energy of the outgoing fermion in terms of the masses, v, and its scattering angle θ relative to the z axis.
   Hint: non-relativistic $\Rightarrow E = m + p^2/2m$, $p = mv$.

   c) Evaluate the leading non-zero term in the summed and squared matrix element found in a) in an expansion in $v \ll 1$.
   Hint: it should be independent of v.

   d) Compute the differential scattering cross section, $d\sigma/dq^2$, where $q = |\vec{p}_4|$ is the momentum transferred to the fermion f in the reaction. For this, you may assume that $m_\phi^2 \gg (p_1 - p_3)^2$ and use this fact to simplify the intermediate ϕ propagator factor.
   Hint: recall that $\delta(f(x)) = \delta(x - x_0)/|f'(x_0)|$, *where* $f(x_0) = 0$.

3. AA operators.
Consider the interaction of DM with a fundamental $s = 1/2$ fermion N via the interaction $a_{\chi N}\,(\chi\gamma^{\mu}\gamma^{5}\chi)\,(\bar{N}\gamma_{\mu}\gamma^{5}N)$.

a) Compute the amplitude for $\chi(p_1) + N(p_2) \to \chi(p_3) + N(p_4)$ elastic scattering.

b) Find the summed and squared matrix element for this process. You should be able to write it in the form $(\#)\chi_{\mu\nu}N^{\mu\nu}$, where $\chi_{\mu\nu}$ depends only on the DM momenta p_1 and p_3 and $N^{\mu\nu}$ depends only on the N momenta p_2 and p_4

c) Evaluate $\chi^{\mu\nu}$ in the lab frame with N at rest and χ impinging upon it with three-momentum $p\hat{z} \ll m_{\chi}, m_N$. Next, evaluate $\chi^{\mu\nu}$ in the extreme non-relativistic limit of $p \to 0$.

d) Compute the summed and squared matrix element in this extreme non-relativistic limit and compare to what you found in part 2.c). For this comparison, take $a_{\chi N} = y_{\chi}y_f/m_{\phi}^2$. What is the relative factor between the two squared matrix elements?