PSI Dark Matter Homework #1

Due: Apr. 12, 2012

1. Dark Matter Profiles:

   a) Make plots (using a plotting program of your choice) of the NFW, Iso-Core, and Einasto ($\alpha = 1.7$) galactic DM profiles using the functional forms listed in the notes. Put them on the same set of axes.

   b) We will see that the cosmic ray signal strength from given point in the sky from DM annihilation in our galaxy is proportional to $\langle \sigma v \rangle n_\chi^2$, where n_χ is the local DM number density. Assuming a constant cross section, make plots of the contributions to the signal strengths (as a function of the distance from the galactic center) for the NFW and Einasto profiles relative to the Iso profile.

2. Boltzmannology:

   a) The collision term for the Boltzmann equation for n_χ we derived in class for the $\chi\chi \to f\bar{f}$ process was

   $$-\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} g_\chi^2 \sigma v \left(f_1 f_2 - f_{1_{eq}} f_{2_{eq}}\right) , \tag{1}$$

   where "1" and "2" refer to the two χ particles. Assuming $f(E,t) = \xi(t) f_{eq}(E)$, re-express this quantity in terms of n_χ, $n_{\chi_{eq}}$, and $\langle \sigma v \rangle$.

   b) Do the integral over p^0 in $\int d^4p\, \delta(p^2 - m^2)\theta(p^0)$. What does this imply for how the change in $(d\Pi) = d^3p/(2\pi)^3 2E$ under Lorentz transformations?

   c) Derive for yourself the rewriting of the Liouville term $(dn_\chi/dt + 3Hn_\chi)$ in terms of $Y_\chi = n_\chi/s$ as a function of $x = m/T$.

3. Cross Sections

   a) Consider the interaction

   $$-\mathscr{L} \supset \frac{1}{\Lambda^2}\, \bar{\chi}\gamma^\mu\gamma^5\chi\, \bar{f}\gamma_\mu f \ . \tag{2}$$

   This could arise from integrating out a heavy vector boson that couples to both DM and the fermion f (which would give $\Lambda^2 = m_V^2/g_V^2$). Compute the squared matrix element for this process, summed over final spins and averaged over initial spins. You may assume the fermion f is massless, but keep the mass of χ around. Write your answer in terms of dot products involving the two incoming momenta p_1 and p_2, and the outgoing momenta p_3 and p_4.

   b) Evaluate this summed and squared matrix element in the center-of-mass frame.

   c) Turn this matrix element into σv by integrating it over the final-state phase space ($\int d\Pi_3 d\Pi_4 (2\pi)^4 \delta^{(4)}(...)$) and multiplying by the appropriate factors. You should be able to write everything in terms of the Mandelstam variable $s = (p_1 + p_2)^2$ and the mass of χ.

d) Expand s to quadratic order in $v = p/E$ in the non-relativistic limit $v \ll 1$ (working in the CM frame). Plug this into your expression for $\sigma\, v$. You should find that $\sigma v \propto v^2$. Compute the thermal average $\langle \sigma v \rangle$ following the non-relativistic prescription described in class.

e) What would you get if you replaced $\bar{\chi}\gamma^\mu\gamma^5\chi$ by $\bar{\chi}\gamma^\mu\chi$ in the interaction?
Hint: aside from tracking signs in a few Dirac traces, you basically shouldn't have to do any more work.