Tutorial # 2

1. **Integrating Out with Equations of Motion**
   Please be so kind as to consider the following theory of a real scalar ϕ, and the heavy and light Dirac fermions X and ψ:

$$-\mathscr{L} \supset y\phi\overline{X}\psi + y\phi\overline{\psi}X + M_X\overline{X}X \ .$$

   Let us assume that the masses of ϕ and ψ negligible, but also that M_X is much larger than the energies at which we probe the theory.

   a) Compute the tree-level contributions to $\phi\psi \to \phi\psi$ scattering in this theory.

   b) Expand your result in powers of momentum over M_X. Use this to figure out the leading operator induced by X in the EFT obtained by integrating X out (at tree-level).

   c) A second way to obtain this (tree-level) result is by replacing X and $\overline{X}$ by their classical equations of motion in the effective Lagrangian. Find the equations of motion for these fields and plug them back in to the original Lagrangian to find the leading operator induced by X in the low-energy EFT. You should reproduce your previous result.
   Hint: derivative terms can be neglected because they are smaller than other terms by powers of p/M_X.

2. **Cancellations**
   In some theories, quadratic divergences cancel between fermion and boson loops. An example of such a cancellation can be seen in a theory of a complex scalar H and a pair of chiral fermions T_L and T_R (written as four-component projections) together with the scalars $\widetilde{T}$ and $\widetilde{T}^c$ with the interactions

$$-\mathscr{L} \supset \tilde{y}^2|H|^2(|\widetilde{T}|^2 + |\widetilde{T}^c|^2) + \tilde{y}^2|\widetilde{T}|^2|\widetilde{T}^c|^2 + \left[yH\overline{T}_R T_L + (h.c.)\right] \ .$$

   a) Work out Feynman rules for all the interactions given above. Make sure you keep track of all the arrows!

   b) Compute the two-point function for $H^\dagger H$ at zero external momentum. Don't worry about evaluating the integrals, but show that the quadratically divergent parts cancel for $y = \tilde{y}$.