Notes #2: Supersymmetry

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Supersymmetry (SUSY) is the most popular proposal for new physics beyond the SM. For every known particle, SUSY predicts that there exists a *superpartner* particle with the same charge but with spin differing by half a unit. The most attractive feature of SUSY is that it can resolve the electroweak hierarchy problem by inducing cancellations in the quantum corrections to the Higgs mass parameter. SUSY can also provide a dark matter candidate and account for the baryon asymmetry, it appears to lead to a unification of the SM gauge couplings close to the Planck scale, and it is an essential component in many attempts to construct a quantum theory of gravity. For these many reasons, SUSY is promising possibility for BSM physics and it is being searched for enthusiastically at the LHC.

To illustrate how SUSY addresses the electroweak hierarchy problem, suppose there exists a new fermion Ψ together with its superpartner boson $\widetilde{\Psi}$, both with a coupling y_Ψ to the Higgs field. The equality of this coupling is enforced by supersymmetry. Together, the net leading quantum correction to the Higgs quadratic parameter for this particle-superpartner pair is

$$\Delta\mu^2 \;\simeq\; \frac{y_\Psi^2}{(4\pi)^2}\left(M_{\widetilde{\Psi}}^2 - M_{\widetilde{\Psi}}^2\right) \;. \tag{1}$$

This correction is acceptably small, $\Delta\mu^2 \lesssim \mu^2$, provided the masses of the particle and its superpartner are not too different. Numerically, we have $\mu \sim 100$ GeV, so if SUSY is to address the hierarchy problem the superpartners of the SM particles should not be too much heavier than $M_{\widetilde{\Psi}} \lesssim (4\pi/y_\Psi)\,(100\ \text{GeV})$. This motivates LHC searches for superpartners with masses in the TeV range.

In these notes, we will give a general overview of SUSY and its phenomenological extension to the SM. We will begin with a practical overview of SUSY and discuss how to construct supersymmetric Lagrangians. Next, we will apply this to build a supersymmetric extension of the SM. Finally, we will come back to study the underlying structure of supersymmetric theories in slightly more detail. For more detailed discussions of supersymmetry and its applications to particle physics and beyond, I highly recommend the reviews of Refs. [1, 2] and the textbooks of Refs. [3, 4].

1 Supersymmetry Basics

We begin with a practical overview of SUSY and the structure of supersymmetric field theories. Before getting to SUSY itself, let us review a few key results for the Poincaré group.

1.1 Poincaré and Particles

Our Universe is symmetric locally under Lorentz transformations (boosts and rotations) and translations to within experimental accuracy. Together, this set of transfomations is called the Poincaré group. Because of the apparent invariance under Poincaré, it is convenient to construct theories of elementary particles using variables that transform linearly under the group. We will see that these nice variables can often be identified with particles of definite spin and momentum.[1]

A description of a group in terms of linear transformations is called a *representation* [5]. In particular, for every group element g there is a linear operator (matrix) $M(g)$ such that $M(1) = \mathbb{I}$ and $M(f \cdot g) = M(f)M(g)$. The Poincaré group is a *Lie group*, meaning that it is a group with infinitely many elements that we can parametrize with a finite set of coordinates $\{\alpha^a\}$. Any group element in a representation that is smoothly connected to the identity can be written in the form $M(\alpha^a) = \exp(-i\alpha^a t^a)$ for some set of linear operators t^a called *generators*. This form is convenient because it implies that to build a representation of the group (with infinitely many elements), we only need to find a represenation of a finite number of generators. For any Lie group, these generators can be shown to satisfy a *Lie algebra* such that

$$[t^a, t^b] = i f^{abc} t^c \qquad \text{for some constants } f^{abc} \tag{2}$$

$$[t^a, [t^b, t^c]] + [t^b, [t^c, t^a]] + [t^c, [t^a, t^b]] = 0 \qquad \text{(Jacobi Identity)} \tag{3}$$

The appearance of commutators and exponentiation should be familiar from quantum mechanics. Indeed, you have already been constructing representations of Lie groups for a while now, even though it may not have been presented to you in this language.

The Poincaré group has ten generators that we can choose to be $\{P^\mu,\, J^{\mu\nu}\}$, where $\mu = 0, 1, 2, 3$ are Lorentz indices and $J^{\mu\nu} = -J^{\nu\mu}$ is antisymmetric. The four P^μ generate spacetime translations and the six independent $J^{\mu\nu}$ generate Lorentz boosts ($J^{0\nu}$) and rotations (J^{ij}). Their commutation relations (Lie algebra) are

$$[P^\mu, P^\nu] = 0 \tag{4}$$

$$[P^\mu, J^{\rho\sigma}] = i\,(\eta^{\mu\rho} P^\sigma - \eta^{\mu\sigma} P^\rho) \tag{5}$$

$$[J^{\mu\nu}, J^{\rho\sigma}] = -i\,(\eta^{\mu\rho} J^{\nu\sigma} - \eta^{\nu\rho} J^{\mu\sigma} - \eta^{\mu\sigma} J^{\nu\rho} + \eta^{\nu\sigma} J^{\mu\rho})\ , \tag{6}$$

where $\eta^{\mu\nu}$ is just the Minkowski metric.

Quantum fields are the standard variables used to describe elementary particles. To build a general quantum field theory that is invariant under Poincaré transformations, we begin with field variables that transform under a definite representation of Poincaré, and then put them together to form an invariant action. The general form of a Poincaré transformation on a quantum field $\phi_A(x)$ such that $x \to \Lambda\, x + a$ is

$$\phi_C(x) \to \left[\exp(-\frac{i}{2}\omega_{\alpha\beta} J^{\alpha\beta}_{j_1 j_2})\right]_C^{\ D} \phi_D(\Lambda^{-1}x - a)\ . \tag{7}$$

[1] See my QFT lecture notes for a more detailed exposition of these topics [6].

Here, $\omega_{\alpha\beta}$ are a set of six real coordinates that specify a Lorentz transformation Λ such that $\Lambda^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + \omega^{\alpha}_{\beta} + \ldots$, C and D are the indices of the representation of the field, and $J^{\alpha\beta}_{j_1 j_2}$ is a set of finite-dimensional matrices.

It is not too hard to show that the $J^{\alpha\beta}_{j_1 j_2}$ matrices corresponding to the Lorentz generators can be built by combining pairs of $SU(2)$ representations with spins (j_1, j_2). The dimension of such a representation is just $(2j_1 + 1) \times (2j_2 + 1)$. The smallest representation is the trivial one with $(j_1 = 0, j_2 = 0)$ corresponding to $J^{\mu\nu}_{00} = 0$. This is just a scalar field whose excitations correspond to particles with spin $s = 0$. The standard Lorentz vector representation is $(1/2, 1/2)$, which has dimension four. Between the scalar and the vector, the simplest non-trivial representations are $(1/2, 0)$ and $(0, 1/2)$. These corresond to two-component $s = 1/2$ fermion fields, and they will play an important role in supersymmetry. Higher representations describe fields with higher spins.

So far, we have concentrated on representations of the Poincaré group on fields. Since we are interested in quantum theories, we should also consider the representations of the group on quantum states. This is slightly more complicated than for fields because the representation matrices on states must also be unitary, $M^{\dagger}(g) = M^{-1}(g) = M(g^{-1})$. It is convenient to classify the representations of states according to the eigenvalues of

$$P^2 = P^{\mu}P_{\mu}, \qquad W^2 = W^{\mu}W_{\mu} \tag{8}$$

where $W_{\mu} = -\epsilon_{\mu\nu\rho\sigma}J^{\nu\rho}P^{\sigma}/2$. These two operators form a maximal set that commutes with all the Poincaré generators; their eigenvalues are therefore Poincaré-invariant. The eigenvalue of P^2 corresponds to the total mass M of the state. For $M \neq 0$, we can transform to the rest frame of the state where $P^{\mu} = (M, \vec{0})$. This gives $W^2 = -M^2\vec{J}\cdot\vec{J}$ so its eigenvalues coincide with the total spin of the state. The situation is a bit different for massless states with $P^2 = 0$. There is no rest frame now, and the only two finite dimensional representations (after fixing the momentum) are characterized by the *helicity*, corresponding to the direction of the spin relative to the momentum.[2]

1.2 Fun with Fermions

To build supersymmetric theories, we will make extensive use of fields transforming in the $(1/2, 0)$ and $(0, 1/2)$ representations of the Lorentz subgroup of Poincaré. These are often called two-component or *Weyl* fermions, and chances are they are less familiar to you than four-component Dirac fermions. For this reason, let us review a few important points about them. More details can be found in Refs. [6].

The general form of the $(1/2, 0)$ representation matrix is

$$M(\alpha^a) = \;=\; e^{-i\alpha^a\sigma^a/2}\ , \tag{9}$$

where $\alpha^a = (\theta^a - i\beta^a)$. The 2×2 matrix $M(\alpha)$ looks just like a regular $SU(2)$ transformation, but now with a set of three *complex* parameters. The real part of α^a corresponds to a rotation

[2] There are also infinite-dimensional representations that have not been observed [7].

about the a-th spatial axis, and the imaginary part corresponds to a boost in the a-th spatial direction.

The Lorentz transformation of a $(1/2, 0)$ spinor field ψ_α is therefore

$$\psi_\alpha(x) \;\to\; [M(\alpha^a)]_\alpha^{\;\beta}\psi_\beta(\Lambda^{-1}x) \;, \tag{10}$$

where α, $\beta = 1, 2$. The use of Greek indices to label the components of the spinor ψ_α is traditional but unfortunate – make sure you don't confuse them with 4-vector indices.

Given the form of $M(\alpha^a)$, we can build a Lorentz-invariant bilinear operator using the ϵ trick:

$$M^t(\alpha^a)\epsilon M(\alpha^a) = \epsilon \;. \tag{11}$$

This implies that given any two $(1/2, 0)$ spinors ψ and χ, the combination $[\chi^t\epsilon\psi]$ is Lorentz-invariant. Putting in indices,

$$[\chi^t\epsilon\psi] = \chi_\beta\epsilon^{\beta\alpha}\psi_\alpha = -(\epsilon^{\alpha\beta}\chi_\beta)\psi_\alpha \;. \tag{12}$$

For this reason, it is standard to define a spinor with a raised index,

$$\chi^\alpha := \epsilon^{\alpha\beta}\chi_\beta \;. \tag{13}$$

In terms of this, the Lorentz-invariant bilinear is written as

$$\chi\psi := \chi^\alpha\psi_\alpha \;. \tag{14}$$

We would also like to be able to lower the spinor index. Using $\epsilon\bar{\epsilon} = 1$, it follows that we can do this with $\bar{\epsilon}_{\alpha\beta}$:

$$\psi_\alpha = \bar{\epsilon}_{\alpha\beta}\psi^\beta \;. \tag{15}$$

The bar on $\bar{\epsilon}$ is usually not written explicitly. Instead, the standard notation has $\epsilon^{\alpha\beta}$ antisymmetric with $\epsilon^{12} = +1$, and $\epsilon_{\alpha\beta}$ also antisymmetric with $\epsilon_{12} = -1$. Thus, $\epsilon^{\alpha\lambda}\epsilon_{\lambda\beta} = \delta^\alpha_{\;\beta}$.

The $(0, 1/2)$ representation is very similar in form to the $(1/2, 0)$, with a general transformation A general finite element is therefore

$$\begin{aligned} \overline{M}(\alpha^a) &= e^{-i(\theta^a+i\beta^a)\sigma^a/2} && (16)\\ &= e^{-i(\alpha^a)^*\sigma^a/2} \;, && (17) \end{aligned}$$

where θ^a and β^a are exactly the same way as before.

Given the similarity of this form to the $(1/2, 0)$ rep, we will use a peculiar but ultimately useful notation for the indices of a $(0, 1/2)$ spinor $\bar{\psi}(x)$ (where the bar on the field is part of its name, not some sort of conjugation operation):

$$\bar{\psi}(x) \;\to\; [\overline{M}(\alpha^a)]^{\dot{\alpha}}_{\;\dot{\beta}}\bar{\psi}^{\dot{\beta}}(\Lambda^{-1}x) \;, \tag{18}$$

where the indices $\dot{\alpha}, \dot{\beta} = 1, 2$. With this transformation law, we can immediately form a Lorentz-invariant bilinear operator from a pair of $(0, 1/2)$ spinors $\bar{\chi}$ and $\bar{\psi}$ using our ϵ *trick*:

$$\bar{\chi}\bar{\psi} := \bar{\chi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = (\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\chi}^{\dot{\beta}})\bar{\psi}^{\dot{\alpha}} \ , \tag{19}$$

where $\epsilon_{\dot{\alpha}\dot{\beta}}$ corresponds to the matrix $\bar{\epsilon}$. Similarly, we can raise indices using $\epsilon^{\dot{\alpha}\dot{\beta}}$,

$$\bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\chi}_{\dot{\beta}} \ . \tag{20}$$

The components of $\epsilon^{\dot{\alpha}\dot{\beta}}$ are equal to those of $\epsilon^{\alpha\beta}$, and the same for $\epsilon_{\dot{\alpha}\dot{\beta}}$ and $\epsilon_{\alpha\beta}$.

The notation we are using looks funny, but there is a good reason for it. Consider the transformation property of the $(1/2, 0)$ spinor ψ with a raised index,

$$\begin{aligned}
\psi^{\alpha} &\to \epsilon^{\alpha\lambda}[M(\alpha^a)]_{\lambda}^{\ \beta}\psi_{\beta} &(21)\\
&= \epsilon^{\alpha\lambda}[M(\alpha^a)]_{\lambda}^{\ \beta}\epsilon_{\beta\kappa}\psi^{\kappa} &(22)\\
&= \left[e^{+i\alpha^a(\sigma^a)^t/2}\right]^{\alpha}_{\ \kappa}\psi^{\kappa} \ , &(23)
\end{aligned}$$

where we have arranged the index structure of the last matrix to make it consistent. Comparing Eq. (23) to the transformation of Eq. (17), we see that $(\psi^{\alpha})^*$ transforms in exactly the same way under Lorentz as a $(0, 1/2)$ spinor!

Given a $(1/2, 0)$ spinor ψ_{α}, we can therefore construct a $(0, 1/2)$ spinor $\bar{\psi}^{\dot{\alpha}}$ by

$$\bar{\psi}^{\dot{\alpha}} := \epsilon^{\dot{\alpha}\dot{\beta}}(\psi^*)_{\dot{\beta}} \ , \tag{24}$$

where we have written $(\psi^*)_{\dot{\beta}} = (\psi_{\beta})^*$. Similarly, given a $(0, 1/2)$ spinor $\bar{\chi}^{\dot{\alpha}}$, we can form a $(1/2, 0)$ spinor through

$$\chi_{\alpha} := \epsilon_{\alpha\beta}(\bar{\chi}^*)^{\beta} \ . \tag{25}$$

Thanks to these handy relations, we only really ever need to deal with $(1/2, 0)$ spinors.

For our next trick, let us try to connect the $(1/2, 1/2)$ rep with the 4-vector rep. To do so, let us define the set of four 2×2 matrices σ^{μ} $(\mu = 0, 1, 2, 3)$ by

$$\sigma^{\mu}_{\alpha\dot{\alpha}} = (\mathbb{I}, \vec{\sigma})_{\alpha\dot{\alpha}} \ . \tag{26}$$

With this, we can form the object

$$\psi\sigma^{\mu}\bar{\chi} := \psi^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} \ . \tag{27}$$

Under Lorentz, we have

$$\begin{aligned}
\psi^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} &\to \epsilon^{\alpha\lambda}(M_{\lambda}^{\ \beta}\psi_{\beta})\sigma^{\mu}_{\alpha\dot{\alpha}}(\overline{M}^{\dot{\alpha}}_{\ \dot{\beta}}\,\bar{\chi}^{\dot{\beta}}) &(28)\\
&= -\psi_{\beta}[M^t\epsilon\sigma^{\mu}\overline{M}]^{\beta}_{\ \dot{\beta}}\bar{\chi}^{\dot{\beta}} &(29)
\end{aligned}$$

With some work, it can be shown that

$$[M^t \epsilon \sigma^\mu \overline{M}]^{\beta}{}_{\dot{\beta}} = \epsilon^{\beta\lambda} \Lambda^\mu{}_\nu \sigma^\nu_{\lambda\dot{\beta}} \ , \tag{30}$$

where Λ is the corresponding 4-vector transformation. It follows that under Lorentz

$$\psi\sigma^\mu\bar{\chi} \to \Lambda^\mu{}_\nu \, \psi\sigma^\nu\bar{\chi} \ , \tag{31}$$

which transforms like a 4-vector. This justifies our notation for the upper index on σ^μ, and it also shows how the 4-vector rep emerges from the $(1/2, 1/2)$ rep.

Let us also define

$$(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} = (\mathbb{I}, -\vec{\sigma})^{\dot{\alpha}\alpha} \ . \tag{32}$$

Following the same steps as before, one can show that for any $(1/2, 0)$ and $(0, 1/2)$ spinors ψ and $\bar{\chi}$,

$$\begin{aligned} \bar{\chi}\bar{\sigma}^\nu\psi \ &:= \ \bar{\chi}_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\psi_\alpha & (33) \\ &\to \ \Lambda^\mu{}_\nu \, \bar{\chi}\bar{\sigma}^\nu\psi \ . & (34) \end{aligned}$$

Numerically, one also has

$$(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot{\beta}} \ . \tag{35}$$

This relation means that the spinor indices on $\bar{\sigma}^\mu$ are consistent with raising and lowering with our good friend ϵ.

The σ^μ matrices also satisfy two very useful relations. The first follows from the tracelessness of the Pauli matrices, and reads

$$tr(\bar{\sigma}^\mu\sigma^\nu) = (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\sigma^\nu_{\alpha\dot{\alpha}} = 2\eta^{\mu\nu} \ . \tag{36}$$

The second relation is

$$\sigma^\mu_{\alpha\dot{\alpha}} \, (\sigma_\mu)_{\beta\dot{\beta}} = -2\epsilon_{\alpha\beta} \, \epsilon_{\dot{\alpha}\dot{\beta}} \ . \tag{37}$$

With all that spinor technology out of the way, we are now able to put together Lorentz-invariant Lagrangians for spinor fields. In doing so, however, there are two additional conditions. First, to describe a physical system, the action must be real. Since a spinor ψ is an inherently complex object, we must therefore have $\bar{\psi} = \epsilon\psi^*$ in our theory as well. Second, when we quantize later on we will find that spinors describe fermions. It turns out that for the quantum theory to connect in a reasonable way to a classical theory, the spinors must anticommute with each other (even in the classical theory). In particular,

$$\psi_\alpha\chi_\beta = -\chi_\beta\psi_\alpha, \qquad \psi_\alpha\bar{\chi}_{\dot{\beta}} = -\bar{\chi}_{\dot{\beta}}\psi_\alpha \ . \tag{38}$$

In fancy math language, spinors are said to be *Grassmann variables.*

For future use, it will be useful to have an explicit convention for the complex conjugation of multiple classical fields, whether they be bosonic (and commuting) or fermionic (and anticommuting). For either field type, we define for a single field

$$A^\dagger := A^* \ , \tag{39}$$

as well as

$$(A_1A_2\dots A_n)^* := (A_1A_2\dots A_n)^\dagger := A_n^\dagger\dots A_2^\dagger A_1^\dagger \ . \tag{40}$$

Note that we have reversed the order with no additional signs, even for the fermion case. This convention is useful because it will match smoothly with the operation of Hermitian conjugation in the quantum theory, where we promote the fields to operators on a Hilbert space. Note as well that

$$(\chi\xi)^\dagger = (\epsilon^{\alpha\beta}\chi_\beta\xi_\alpha)^\dagger = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\xi}_{\dot{\alpha}}\bar{\chi}_{\dot{\beta}} = +\bar{\xi}\bar{\chi} \ . \tag{41}$$

At the very least, a sensible physical theory requires a kinetic term involving some spacetime derivatives. It turns out that the right form for a spinor is

$$\begin{aligned}
\mathscr{L} \ &\supset \ \frac{1}{2}\psi i\sigma^\mu\partial_\mu\bar{\psi} + \frac{1}{2}\bar{\psi}i\bar{\sigma}^\mu\partial_\mu\psi && (42)\\
&= \ \bar{\psi}i\bar{\sigma}^\mu\partial_\mu\psi = \psi i\sigma^\mu\partial_\mu\bar{\psi} && (43)
\end{aligned}$$

where you will verify the reality of the first line and the equalities in the second line in the homework.

We can also add a bilinear mass term for the spinor. If we only have a single spinor ψ (and its conjugate $\bar{\psi}$), the only Lorentz-invariant possibility is

$$\mathscr{L} \supset -\frac{1}{2}m\,\psi\psi - \frac{1}{2}m^*\bar{\psi}\bar{\psi} \ . \tag{44}$$

You might worry that these terms both vanish since ψ is anticommuting, but they do not. Note that

$$\chi\psi = \chi^\alpha\psi_\alpha = \epsilon^{\alpha\beta}\chi_\beta\psi_\alpha = -\epsilon^{\alpha\beta}\psi_\alpha\chi_\beta = \epsilon^{\beta\alpha}\psi_\alpha\chi_\beta = \psi^\beta\chi_\beta = \psi\chi \ , \tag{45}$$

where we see that the anticommutation of the spinors is cancelled by the antisymmetry of ϵ.

Having spent all that time on two-component $(1/2,0)$ and $(0,1/2)$ spinors, we turn next to study four-component objects in the $(1/2,0)\oplus(0,1/2)$ representation. While the rep is reducible under Lorentz, it is irreducible if we also impose parity which exchanges j_A and j_B in (j_A,j_B). Parity turns out to be a good symmetry of electromagnetism, and therefore we would like to build it into our fields. This is why we'll use four-component Dirac fermions to describe electrons in QED.

Consider a theory containing two $(1/2,0)$ spinors ξ and χ together with their conjugates. We will assume the theory has a global symmetry under the phase transformations

$$\xi(x) \to e^{-i\varphi}\psi(x), \qquad \chi(x) \to e^{i\varphi}\chi(x) \ . \tag{46}$$

The most general Lagrangian for this theory at bilinear order is

$$\mathscr{L} = \bar{\xi} i \bar{\sigma}^\mu \partial_\mu \xi + \bar{\chi} i \bar{\sigma}^\mu \partial_\mu \chi - m(\xi\chi + \bar{\xi}\bar{\chi}) \ . \tag{47}$$

The global symmetry allows the mixed $\chi\xi$ quadratic term, but it forbids $\chi\chi$ and $\xi\xi$.

When we quantize the theory, we will interpret m as the mass of a particle. However, since two fields are involved, it is not obvious how to relate the mass term to a specific particle. Using a four-component spinor containing both two-component spinors allows us to dodge this issue for the time being. We define the four-component *Dirac spinor* Ψ by

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \ . \tag{48}$$

The conjugate of Ψ is thus

$$\Psi^\dagger = \left(\bar{\xi}_{\dot{\alpha}},\, \chi^\alpha\right) \ . \tag{49}$$

To go along with Ψ, we also generalize the σ^μ matrices to the 4×4 Dirac matrices γ^μ,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \ , \tag{50}$$

where each of the matrix elements is itself a 2×2 matrix. Finally, let us define the barred conjugate $\bar{\Psi}$ to be

$$\bar{\Psi} = \Psi^\dagger \gamma^0 = \left(\chi^\alpha,\, \bar{\xi}_{\dot{\alpha}}\right) \ . \tag{51}$$

Note that here the bar denotes a conjugation operation, and is not part of the name of the Dirac spinor.

With these definitions in hand, we can rewrite the Lagrangian of Eq. (47) in a more compact form using Ψ. The result is

$$\mathscr{L} = \bar{\Psi} i \gamma^\mu \partial_\mu \Psi - m\bar{\Psi}\Psi \ . \tag{52}$$

The mass term looks much nicer now.

1.3 Extending Poincaré with Supersymmetry

Starting from Poincaré invariance, a natural question to ask is whether it can be extended in any reasonable way. The answer, it turns out, is almost no. This follows from the Coleman-Mandula theorem, which states that for a reasonable quantum field theory in four dimensions to have particles of non-zero mass and a non-trivial S-matrix, all bosonic symmetry generators (other than the Poincaré generators) must be Lorentz scalars [8]. Supersymmetry is a mild loophole in this argument because it is based on fermionic symmetry generators (in a sense that we will discuss below). In fact, it can be shown to be the largest possible exception [9, 10].

The generators of supersymmetric transformations in four dimensions are Weyl ((1/2,0) and (0,1/2)) fermions Q^A_α and $\overline{Q}^{\dot{\beta}}_B$, where A, $B = 1, \ldots, \mathcal{N}$ count the number $\mathcal{N}$ of independent supersymmetries. Instead of satisfying the commutation relations of a Lie algebra, they obey a *Lie superalgebra* involving both commutators and anti-commutators [3, 10]:

$$\begin{aligned} \{Q^A_\alpha, \overline{Q}_{\dot{\beta}\,B}\} &= 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A{}_B && (53) \\ \{Q^A_\alpha, Q^B_\beta\} &= 0 = \{\overline{Q}^{\dot{\alpha}}_A, \overline{Q}^{\dot{\beta}}_B\} && (54) \\ \left[P_\mu, Q^A_\alpha\right] &= 0 = [P_\mu, \overline{Q}^{\dot{\beta}}_B] \ . && (55) \end{aligned}$$

There are also commutators with the Lorentz generators $J^{\mu\nu}$, but they are pretty complicated. In this course we will mostly restrict ourselves to one set of supersymmetry generators, $\mathcal{N} = 1$. With more generators, it is not possible to get representations with chiral fermions, so the $\mathcal{N} = 1$ case seems the most relevant.

As with Poincaré, we will build theories using fields and states that transform linearly under supersymmetry. The fields and states in each representation can be organized into *supermultiplets* that transform into each other under SUSY. Using the SUSY superalgebra, we can deduce a few general results about these supermultiplets [1, 10]:

1. Any gauge or global symmetry generators of the theory must commute with the SUSY and Poincaré generators (up to a possible $U(1)_R$ global symmetry). Thus, all the members of a supermultiplet must have the same charges.
2. The operator P^2 commutes with all the other generators. Applying this in the rest frame of the one-particle space, this means that all the particles in a supermultiplet must have the same mass.
3. The number of bosonic and fermionic degrees of freedom (on- or off-shell) is equal within any supermultiplet.
4. Invariance under supersymmetry implies that Q_α annihilates the vacuum, $Q_\alpha \left|0\right\rangle = 0$. Applied to the vacuum (which has $\vec{P} = 0$), we also have

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\delta_{\alpha\dot{\beta}} P^0 = 2\delta_{\alpha\dot{\beta}} H \ . \tag{56}$$

   Thus, $H \left|0\right\rangle = 0$; the vacuum energy of a supersymmetric theory vanishes.

In general, we have

$$Q \left|\text{boson}\right\rangle \sim \left|\text{fermion}\right\rangle , \qquad Q \left|\text{fermion}\right\rangle \sim \left|\text{boson}\right\rangle \tag{57}$$

Infinitesimal transformations on fields take the form

$$\delta\phi \ \sim \ \xi\psi, \qquad \delta\psi \ \sim \ \sigma^\mu \bar{\xi} \partial_\mu \phi \ , \tag{58}$$

where ξ is a fermionic (Grassmann-valued) transformation parameter. Note that this parameter must be fermionic so that the transformation of a boson remains bosonic and the transformation of a fermion remains fermionic.

1.4 Supersymmetric Actions

We would like to extend the SM to include supersymmetry. For this, we need to find representations of supersymmetry on quantum fields, and to then put these representations together in a way that gives a sensible action with which to define a theory. For now, we will simply state the main results of this procedure. If there is time, we will come back later in Section 4 to give some of the underlying details. The two ($\mathcal{N} = 1$, $d = 4$) representations of SUSY we will need to supersymmetrize the SM are the *chiral* and massless *vector* supermultiplets. Let us describe both in turn.

A chiral supermultiplet Φ consists of a complex scalar ϕ, a Weyl fermion ψ, and an auxilliary complex scalar F,

$$\Phi = (\phi, \psi, F)\ . \tag{59}$$

The minimal supersymmetric action for these fields has only kinetic terms, and takes the form

$$S = \int d^4x\ \left(|\partial\phi|^2 + \bar{\psi}i\bar{\sigma}\cdot\partial\psi + F^\dagger F\right)\ . \tag{60}$$

The first two terms are just the usual kinetic pieces for a complex scalar and a Weyl fermion. The third term involving F has no derivatives, and implies that the field is non-dynamical. This means that it can be replaced by its equation of motion and does not correspond to a physical degree of freedom. Note that the number of bosonic and fermionic degrees of freedom (DOF) match up. Counting at the field level, before applying the equations of motion, we have two real bosonic DOFs from ϕ and two from F, while ψ provides four real fermionic DOFs to match. After applying the equations of motion, ϕ represents a spinless particle and antiparticle giving two *on-shell* DOFs, while ψ represents a Weyl fermion and its antiparticle each with a single helicity state. We also see that both states are massless.

Interactions can be added to the action while preserving invariance under SUSY provided they come from a *superpotential*. A superpotential $W(\Phi)$ is a function of the chiral superfield Φ that is *holomorphic*, meaning that it depends on Φ alone and not $\Phi^\dagger$. In terms of the superpotential, the interaction terms in the Lagrangian are

$$\mathscr{L}_{int} \ = \ -\frac{1}{2}\left.\frac{\partial^2 W}{\partial\Phi^2}\right|_\phi \psi\psi \ + \ F\left.\frac{\partial W}{\partial\Phi}\right|_\phi \ + \ (h.c.)\ , \tag{61}$$

where $W(\Phi)$ is to be evaluated at $\Phi \to \phi$ in all terms. The second term above involves the auxilliary field F. Together with the kinetic terms in Eq. (60), the equation of motion for F becomes

$$F^\dagger = -\left.\frac{\partial W}{\partial\Phi}\right|_\phi \tag{62}$$

Plugging back in, we get the first two kinetic terms in Eq. (60) as well as the interaction terms

$$\mathscr{L}_{int} \ = \ -\frac{1}{2}\left[\left.\frac{\partial^2 W}{\partial\Phi^2}\right|_\phi \psi\psi + (h.c.)\right] \ - \ \left|\frac{\partial W}{\partial\Phi}\right|^2_\phi \tag{63}$$

The first term gives Yukawa interactions while the second is the *F-term potential* for the scalar. Note that the scalar potential is non-negative.

e.g. 1. **The Wess-Zumino Model**

The most general renormalizable superpotential with only a single chiral multiplet is

$$W = \frac{1}{2} m\,\Phi^2 + \frac{\lambda}{3!}\Phi^3 \ . \tag{64}$$

Using our prescription above, this gives the mass and interaction terms

$$-\mathscr{L} \quad \supset \quad |m\phi + \lambda\phi^2/2|^2 \ + \ (m + \lambda\phi)\psi\psi + (h.c.) \tag{65}$$

This implies that the scalar and fermion masses are both equal to m, and that the scalar cubic and quartic interactions are directly related to the Yukawa coupling λ. These nont-trivial relations are encoded in the superpotential and are necessary to preserve SUSY.

The massless vector supermultiplet V consists of a vector field A^μ, a Weyl fermion λ and its conjugate $\bar{\lambda}$, and a real auxilliary field D,

$$V = (\lambda, A^\mu, D) \ . \tag{66}$$

The minimal supersymmetric action for them is

$$S = \int d^4x \ \left(\bar{\lambda} i\bar{\sigma}\!\cdot\!\partial\lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D^2\right) \ , \tag{67}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This theory clearly has a $U(1)$ gauge invariance with λ and D uncharged. The fermionic superpartner λ of the vector boson is often called the *gaugino*. Like the F terms above, the auxilliary D field has no kinetic terms and it is non-dynamical.

It is straightforward to generalize the vector multiplet and the corresponding action to a non-Abelian gauge invariance. In this case, the vector field G_μ^a transforms under the adjoint representation of the non-Abelian gauge group. (Recall that the adjoint of a $U(1)$ group is trivial.) To maintain SUSY, the other elements of the supermultiplet, λ^a and D^a, must also transform under the adjoint representation. The corresponding supersymmetric and gauge-invariant minimal action is

$$S = \int \ \left(\bar{\lambda}^a i\bar{\sigma}\!\cdot\! D\lambda^a - \frac{1}{4}G_{\mu\nu}^a G^{\mu\nu\,a} + \frac{1}{2}D^a D^a\right) \ , \tag{68}$$

where a is the adjoint index and

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f^{abc} G_\mu^b G_\nu^c && (69) \\ D_\mu\lambda^a &= \partial_\mu\lambda^a - g f^{abc} G_\mu^b \lambda^c \ , && (70) \end{aligned}$$

where f^{abc} are the structure constants of the gauge group and g is the gauge coupling.

The next step is to couple matter fields in chiral multiplets to a vector multiplet. Let G be the non-Abelian gauge group, and t^a_{ij} the generators of the representation under which Φ transforms. The kinetic terms for the vector multiplet remain the same as in Eq. (67), while the kinetic terms for the chiral multiplet Φ take the same form as Eq. (60) but with ordinary derivatives replaced by gauge-covaraint derivatives, $\partial_\mu \to D_\mu$ where

$$[D_\mu \phi]_i = \partial_\mu \phi_i + igt^a_{ij} G^a_\mu \phi_j \ , \tag{71}$$

and similarly for ψ. Supersymmetry also requires additional interaction terms to maintain invariance. These are

$$-\mathscr{L} \ \supset \ \sqrt{2}g(\phi^\dagger t^a \psi)\lambda^a + \sqrt{2}g\bar{\lambda}^a(\bar{\psi} t^a \phi) - g(\phi^\dagger t^a \phi) D^a \ . \tag{72}$$

At this point, it is helpful to integrate out the D terms by replacing them by their algebraic equations of motion. These are

$$D^a = g(\phi^\dagger t^a \phi) \ . \tag{73}$$

Plugging back in to the action, we find an additional contribution to the scalar potential,

$$-\mathscr{L} \ \supset \ \frac{1}{2} D^a D^a \ = \ \frac{1}{2} g^2 (\phi^\dagger t^a \phi)^2 \tag{74}$$

Just like the F-term potential, this *D-term potential* is positive semi-definite.

e.g. 2. **SUSY QED**

Regular QED consists of a Dirac electron field coupled to the photon field with a $U(1)_{em}$ gauge invariance. The supersymmetrization of it requires two chiral electron multiplets $E = (\widetilde{E}, E, F_E)$ and $E^c = (\widetilde{E}^c, E^c, F_{E^c})$ with charges $Q = \mp 1$ and an Abelian vector multiplet $V = (\lambda, A_\mu, D)$. The new degrees of freedom required by SUSY are therefore a pair of complex scalar *selectrons* and a $U(1)$ gaugino fermion. Gauge invariance and supersymmetry fix all the interactions among these fields up to the superpotential, whose most general renormalizable form is

$$W = mEE^c \ . \tag{75}$$

This is nothing but a mass term for the selectrons and electrons. It gives

$$-\mathscr{L} \ \supset \ m^2 \left(|\widetilde{E}|^2 + |\widetilde{E}^c|^2 \right) + mEE^c + m\bar{E}\bar{E}^c \ . \tag{76}$$

There is also a D-term contribution to the scalar potential. The full expression is [3]

$$\begin{aligned} V \ &= \ V_F + V_D && (77) \\ &= \ m^2 \left(|\widetilde{E}|^2 + |\widetilde{E}^c|^2 \right) + \frac{e^2}{2} \left(|E|^2 - |E^c|^2 \right)^2 \ . && (78) \end{aligned}$$

This potential is clearly minimized for $|E| = |E^c| = 0$. However, for $m \to 0$ the F-terms vanish and there is a much larger space of vacua defined by $|\widetilde{E}| = |\widetilde{E}^c|$. This corresponds to $D = 0$ and is called a D-flat direction.

[3] Note that $t^a \to Q$ for Abelian theories.

1.5 (Soft) Supersymmetry Breaking

One of the predictions of exact supersymmetry is that the masses of all the states within a supermultiplet are the same. This is a problem for any supersymmetric extension of the SM because we are extremely certain that there is no scalar electron superpartner with the same mass as the electron. Thus, if we are to supersymmetrize the SM, SUSY cannot be an exact symmetry of Nature. At the same time, we would very much like to keep the very nice solution to the electroweak hierarchy problem that SUSY provides.

Both features can be achieved if SUSY is only broken *softly.* This means that all the operators in the effective Lagrangian that are not invariant under SUSY are accompanied by a factor with positive mass dimension, m_{soft}. The mass splittings between particles and their superpartners will be determined by this factor. Moreover, at energies $E \gg m_{soft}$ the effects of soft supersymmetry breaking become subleading corrections, and the theory becomes increasingly close to being exactly supersymmetric.

The most general set of (renormalizable) soft supersymmetry breaking terms for a chiral supermultiplet $\Phi = (\phi, \psi, F)$ and a vector supermultiplet $V = (\lambda^a, G^a_\mu, D^a)$ are

$$-\mathscr{L}_{soft} \;=\; m_\phi^2|\phi|^2 \;+\; \big(t\phi + b\phi^2 + a\phi^3 + c\phi^2\phi^* + h.c.\big) \;+\; (M_\lambda\,\lambda^a\lambda^a + h.c.)\ . \tag{79}$$

All of the parameters are assumed to on the order of m_{soft}^n. The last term is a gauge-invariant mass for the gaugino, allowing it to be heavier than the massless vector boson. The terms involving the chiral multiplet shift the mass of the scalar relative to the fermion and introduce new contributions to the scalar potential.

Going back to our earlier discussion of the hierarchy problem, we can apply the previous result to a softly-broken supersymmetric theory. Suppose Ψ is a fermion from a chiral multiplet and $\widetilde{\Psi}$ is its scalar superpartner. Soft SUSY breaking will modify the mass of the scalar relative to the fermion by roughly $M^2_{\widetilde{\Psi}} \simeq M_\Psi + m^2_{soft}$, but it will not significantly alter the relative size of the dimensionless coupling to the Higgs y_Ψ. With these changes from SUSY breaking, the net correction to the quadratic Higgs parameter of Eq. (1) is

$$\Delta\mu^2 \simeq \frac{y_\Psi^2}{(4\pi)^2}\, m^2_{soft}\ . \tag{80}$$

We see that the cancellation needed to stabilize the electroweak scale is preserved with softly broken SUSY provided $m_{soft} \lesssim (4\pi/y_\Psi)\mu$. This works even if the supersymmetric contribution to the mass M_Ψ is much larger than the weak scale.

2 Supersymmetrizing the Standard Model

We now have everything we need to embed the SM in a supersymmetric theory. In this section we will present the minimal supersymmetric extension of the SM (MSSM) and discuss some of its features. implications.

	$SU(3)_c \times SU(2)_L \times U(1)_Y$	Fermions	Bosons	B	L	R
$Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$	(3, 2, 1/6)	u_L, d_L	$\widetilde{u}_L, \widetilde{d}_L$	1/3	0	-1
U^c	($\bar{3}$, 1, -2/3)	$U^c = u_R^\dagger$	$\widetilde{U}^c = \widetilde{u}_R^*$	-1/3	0	-1
D^c	($\bar{3}$, 1, 1/3)	$D^c = d_R^\dagger$	$\widetilde{D}^c = \widetilde{d}_R^*$	-1/3	0	-1
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	(1, 2, -1/2)	ν_L, e_L	$\widetilde{\nu}_L, \widetilde{e}_L$	0	1	-1
E^c	(1, 1, 1)	$E^c = e_R^\dagger$	$\widetilde{E}^c = \widetilde{e}_R^*$	0	-1	-1
$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	(1, 2, 1/2)	$\widetilde{h}_u^+, \widetilde{h}_u^0$	H_u^+, H_u^0	0	0	+1
$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	(1, 2, -1/2)	$\widetilde{h}_d^0, \widetilde{h}_d^-$	H_d^0, H_d^-	0	0	+1

Table 1: Chiral supermultiplets in the MSSM.

	$SU(3)_c \times SU(2)_L \times U(1)_Y$	Fermions	Bosons	B	L	R
G^a	(8, 1, 0)	$\widetilde{g}^a$	g_μ^a	0	0	-1
W^d	(1, 3, 0)	$\widetilde{W}^d$	W_μ^d	0	0	-1
B	(1, 1, 0)	$\widetilde{B}^0$	B_μ	0	0	-1

Table 2: Vector supermultiplets in the MSSM.

2.1 Fields and Interactions

In the minimal supersymmetric extension of the Standard Model (MSSM), all the SM fermions are embedded in chiral multiplets and each of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ vector bosons is embedded in vector multiplet. These multiplets are listed in Tables 1 and 2. The superpartners of the SM fermions have $s = 0$ and are called *sfermions* while the superpartners of the vector bosons have $s = 1/2$ and are called *gauginos*. For the Higgs, two chiral $SU(2)_L$ doublets are needed, with the minimal MSSM configuration involving H_u and H_d shown in Fig. 1. The $s = 1/2$ fermionic superpartners of the Higgs scalars are called *higgsinos*.

There are two reasons why at least two $SU(2)_L$ doublet chiral superfields are necessary. The first is that the SM Yukawa couplings emerge from superpotential interactions. One Higgs can supply these for both up- and down-type fermions in the SM since the Higgs can be conjugated in the Yukawa-interaction operators. This is not possible in the superpotential because of holomorphy. The second reason for at least two Higgs multiplets comes from their chiral fermion components; with only a single Higgs multiplet there would be gauge anomalies in $SU(2)_L$ and $U(1)_Y$.

With the the chiral multiplets listed in Table 1, the following superpotential couplings are consistent with gauge invariance and are included in the MSSM:

$$W \ = \ \mu\, H_u\!\cdot\! H_d + y_u\, Q\!\cdot\! H_u U^c - y_d\, Q\!\cdot\! H_d D^c - y_e\, L\!\cdot\! H_d E^c \ , \tag{81}$$

where $A\cdot B = A_a\epsilon^{ab}B_b$ is a contraction of $SU(2)_L$ indices with $\epsilon^{12} = +1 = -\epsilon^{21}$, and we have suppressed sums over flavour (generation) indices. The first term gives a mass to the higgsinos, while the other terms reproduce the Yukawa couplings of the SM. Note that holomorphy implies that two Higgs multiplets are needed to get all the Yukawas.

The MSSM superpotential interactions in Eq. (81) do not contain all the possible (renormalizable) operators consistent with gauge invariance. The renormalizable gauge-invariant operators that were not included are

$$L\cdot H_u\ ,\quad U^cD^cD^c\ ,\quad Q\cdot LD^c\ ,\quad L\cdot LE^c\ . \tag{82}$$

All of these violate baryon (B) or lepton (L) number, and are inconsistent with experiment unless they are accompanied by extremely small couplings [13]. In the MSSM, these operators are forbidden by imposing an additional symmetry called *R-parity* on the multiplets. This is a $\mathbb{Z}_2$ symmetry under which the lowest-spin element in the supermultiplet with spin s_0 is assigned the parity $R = \pm1$, with the R-parity of the higher-spin elements in the multiplet given by $R' = R\times(-1)^{2(s-s_0)}$. In the MSSM, all the SM-fermion chiral multiplets have $R = -1$ implying that the sfermions are R-odd and the SM fermions are R-even. For the gauge multiplets, the gauginos are R-odd and the vector bosons are R-even. For the Higgs chiral multiplets, the Higgs scalars are R-even and the Higgsinos are R-odd. Equivalently, the SM states are R-even and their superpartners are R-odd.

The imposition of R-parity has two important consequences. The first is that the operators listed in Eq. (82) are all odd and thus forbidden by the symmetry. This can also be seen by noting that the R-parity charges are equivalent to [1]

$$R = (-1)^{(3B-L+2s)}\ . \tag{83}$$

Two more implications of R-parity are that superpartners can only be created or destroyed and pairs, and that the lightest superpartner is stable. These two features play a key role in the phenomenology of the MSSM. In particular, the lightest superpartner (LSP) is stable and will contribute to the density of dark matter.

Consistency with experiment also requires that the MSSM superpartners be heavier than their SM counterparts. To achieve this, soft supersymmetry breaking must be added to the theory. The complete set of soft supersymmetry breaking terms in the MSSM is [1]

$$\begin{aligned}
-\mathscr{L}_{\rm soft} &= m_{H_d}^2|H_d|^2 + m_{H_u}^2\,|H_u|^2 - (B\mu\, H_u\cdot H_d + \text{h.c.})\\
&+ m_Q^2\,|\widetilde{Q}|^2 + m_U^2\,|\widetilde{U}^c|^2 + m_D^2\,|\widetilde{D}^c|^2 + m_L^2\,|\widetilde{L}|^2 + m_E^2\,|\widetilde{E}^c|^2\\
&+ \Big(y_uA_u\,\widetilde{Q}\cdot H_u\,\widetilde{U}^c - y_dA_d\,\widetilde{Q}\cdot H_d\widetilde{U}^c - y_eA_e\,\widetilde{L}\cdot H_d\widetilde{E^c} + \text{h.c.}\Big)\\
&+ \frac{1}{2}\,\Big(M_3\,\widetilde{g}^a\widetilde{g}^a + M_2\,\widetilde{W}^d\widetilde{W}^d + M_1\,\widetilde{B}^0\widetilde{B}^0 + \text{h.c.}\Big)\ .
\end{aligned} \tag{84}$$

The first two lines in Eq. (84) contain mass terms for the Higgs and sfermion scalars, the third line consists of trilinear A terms that mirror the superpotential couplings, and the fourth line contains mass terms for the gauginos.

Together, supersymmetry, gauge invariance, the matter content listed in Tables 1 and 2, the superpotential of Eq. (81), and the soft terms of Eq. (84) completely specify the MSSM.

2.2 The MSSM Mass Spectrum

Particle masses in the MSSM are determined by the soft supersymmetry breaking terms and electroweak symmetry breaking induced by the Higgs multiplets. The presence of electroweak symmetry breaking will mix states from different chiral and vector supermultiplets to produce the mass eigenstates that can be observed in experiments.

Let us begin with electroweak symmetry breaking [1, 11, 12]. For this, we should examine the effective potential for the Higgs scalars derived from H_u and H_d and find their vacuum expectation values (VEV) by minimization. These scalars have both charged (H_u^+, H_d^-) and neutral (H_u^0, H_d^0) components, but it can be shown that the (tree-level) potential is always minimized with $H_u^+ = H_d^- = 0$. This leaves a potential for the neutral components given by

$$\begin{aligned} V_H &= V_F + V_D + V_{soft} \qquad (85)\\ &= (m_{H_u}^2 + |\mu|^2)|H_u^0|^2 \;+\; (m_{H_d}^2 + |\mu|^2)|H_d^0|^2 \;-\; (B\mu\, H_u^0 H_d^0 + \text{h.c.})\\ &\quad + \frac{g^2 + g'^2}{8}\left(|H_u^0|^2 - |H_d^0|^2\right)^2 \;. \end{aligned}$$

The only term in this potential that depends on the phase of the Higgs scalars is the $B\mu$ piece. By rephasing the fields, we can assume that $B\mu$ is real and positive without loss of generality. The potential is then minimized with H_u^0 and H_d^0 both real and positive.[4]

Both H_u^0 and H_d^0 should get VEVs for electroweak symmetry breaking and to give masses to both up- and down-type fermions, and these VEVs should be finite. This imposes two necessary and sufficient conditions on the parameters:

$$(B\mu)^2 \;>\; (m_{H_u}^2 + |\mu|^2)(m_{H_d}^2 + |\mu|^2) \qquad (86)$$

$$2(B\mu) \;<\; 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \;. \qquad (87)$$

The first condition comes from destabilizing the potential at the origin while the second corresponds to a positive quadratic coefficient when the quartic term vanishes along the D-flat direction $|H_u^0| = |H_d^0|$. As long as these conditions are met, both fields will get VEVs and we can write

$$\langle H_u\rangle = v_u = v \sin\beta \;, \quad \langle H_d\rangle = v_d = v\cos\beta \;, \qquad (88)$$

where $v = \sqrt{v_u^2 + v_d^2}$ and $\beta \in (0, \pi/2)$. To match experiment, we also want $v \simeq 174$ GeV. The minimization conditions that determine v_u and v_d (at tree-level) are equivalent to

$$m_Z^2 \;=\; \frac{m_{H_u}^2 - m_{H_d}^2}{\cos 2\beta} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2 \;, \qquad (89)$$

$$\sin 2\beta \;=\; \frac{2B\mu}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2} \;. \qquad (90)$$

After EWSB, one linear combination of H_u^0 and H_d^0 is eaten by the Z^0 and a related linear combination of H_u^+ and H_d^- is eaten by the $W^\pm$. The remaining physical excitations

[4]Setting H_u^0 real and positive can be arranged by a choice of gauge.

are a pair of neutral CP-even scalars h^0 and H^0, a neutral CP-odd scalar A^0, and a complex charged scalar H^+. Their masses are

$$\begin{aligned}
m^2_{h,H} &= \frac{1}{2}\left[m_Z^2 + m_A^2 \mp \sqrt{(m_A^2 - m_Z^2)^2 + 4m_Z^2 m_A^2 \sin^2 2\beta}\right] && (91)\\
m_A^2 &= \frac{2B\mu}{\sin 2\beta} = 2|\mu|^2 + m^2_{H_u} + m^2_{H_d} && (92)\\
m_{H^\pm} &= m_A^2 + m_W^2 \ . && (93)
\end{aligned}$$

In the limit $m_Z \ll m_A$ with $\tan\beta > 1$, the lighter CP-even state h^0 couples to the SM fermions and vector bosons in the same way as the SM Higgs. This is often called the *decoupling limit*.

The SM weak vector bosons and fermions also get masses from EWSB. The vector boson masses are

$$m_W^2 = \frac{g^2}{2}v^2 \ , \qquad m_Z^2 = \frac{g^2 + g'^2}{2}v^2 \ , \tag{94}$$

where g' and g are the $U(1)_Y$ and $SU(2)_L$ gauge couplings. Relative to the couplings in the MSSM superpotential, the fermion masses are

$$m_u = y_u v \sin\beta \ , \quad m_d = y_d v \cos\beta \ , \quad m_e = y_e v \cos\beta \ . \tag{95}$$

Note that the large value of the top mass ($m_t \simeq 174$ GeV) favours $\tan\beta > 1$. As $\tan\beta$ falls below this, the top Yukawa coupling y_t grows non-perturbatively large.

The superpartner mass spectrum is also modified by electroweak symmetry breaking. Among the scalars, the $\widetilde{f}_L$ and $\widetilde{f}_R = \widetilde{f^c}^*$ sfermion superpartners of the left- and right-handed fermions of the SM mix with each other. This mixing is typically proportional to the SM fermion mass, and is usually neglected except for the third generation. Thus, it is standard to hear about $\widetilde{u}_L$ and $\widetilde{u}_R$ squarks of the first generation, and $\widetilde{t}_1$ and $\widetilde{t}_2$ of the third generation, where the naming convention is $m_{\widetilde{t}_1} \leq m_{\widetilde{t}_2}$. More details can be found in Ref. [1] and Table 3.

The fermionic superpartners consist of the gauginos and the higgsinos. The $SU(3)_c$ gaugino $\widetilde{g}$ is called the *gluino*, and has a mass set by the soft term M_3. The other fermion superpartners mix with each other after electroweak symmetry breaking. The neutral *Bino* $\widetilde{B}^0$ and *Wino* $\widetilde{W}^0$ gauginos combine with the neutral higgsinos $\widetilde{H}_u^0$ and $\widetilde{H}_d^0$ to make four Majorana-fermion *neutralinos* χ_i^0 (with $|m_i| \leq |m_{i+1}|$). Similarly, the charged gaugino $\widetilde{W}^\pm$ and higgsinos $\widetilde{H}_u^+$ and $\widetilde{H}_d^-$ combine to make two Dirac-fermion *charginos* $\chi_i^\pm$ (with $|m_i| \leq |m_{i+1}|$). Again, more details can found in Ref. [1] and the summary Table 3.

Table 3 also lists a *gravitino*. It is the spin $s = 3/2$ superpartner of the graviton. The origin of its mass is more complicated, and we will discuss it in the next section in the context of supersymmetry breaking.

State	Symbol	$SU(3)_c$	Q_{em}	R
CP-even Higgs	h^0, H^0	1	0	+1
CP-odd Higgs	A^0	1	0	+1
charged Higgs	$H^\pm$	1	± 1	+1
gluino	$\widetilde{g}$	8	0	-1
neutralinos	$\widetilde{\chi}_1^0$, $\widetilde{\chi}_2^0$, $\widetilde{\chi}_3^0$, $\widetilde{\chi}_4^0$	1	0	-1
charginos	$\chi_1^\pm$, $\chi_2^\pm$	1	± 1	-1
up squarks	$\widetilde{u}_{L,R}$, $\widetilde{c}_{L,R}$, $\widetilde{t}_{1,2}$	3	2/3	-1
down squarks	$\widetilde{d}_{L,R}$, $\widetilde{s}_{L,R}$, $\widetilde{b}_{1,2}$	3	-1/3	-1
sleptons	$\widetilde{e}_{L,R}$, $\widetilde{\mu}_{L,R}$, $\widetilde{\tau}_{1,2}$	1	-1	-1
sneutrinos	$\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$	1	0	-1
gravitino	$\widetilde{G}$	1	0	-1

Table 3: Higgs and superpartner mass eigenstates in the MSSM after electroweak symmetry breaking. In writing these out we assume that left-right scalar mixing is only significant for the third generation. States are conventionally labelled in order of increasing mass: $m_i \leq m_{i+1}$.

2.3 Models of Supersymmetry Breaking

Supersymmetry breaking is needed to make the superpartners heavier than their SM counterparts. Indeed, given the current experimental bounds on superpartners, most of their mass must come from SUSY breaking. In this subsection we will discuss some of the proposals for the origin of this breaking, and what they imply for the mass spectrum of the MSSM.

Breaking supersymmetry is usually a messy business. The standard assumption is that the breaking mechanism is spontaneous. As in non-supersymmetric theories, this occurs when the underlying theory respects the symmetry but the vacuum state does not, corresponding to $Q_\alpha |0\rangle \neq 0$ for at least one α. Now, using Eq. (56) we can deduce that for a supersymmetric theory

$$\begin{aligned} 4\langle 0| H |0\rangle &= \langle 0| \left(Q_1\overline{Q}_1 + \overline{Q}_1 Q_1 + Q_2\overline{Q}_2 + \overline{Q}_2 Q_2\right) |0\rangle \\ &= \left|\overline{Q}_1 |0\rangle\right|^2 + |Q_1 |0\rangle|^2 + (1 \to 2) \end{aligned} \tag{96}$$

This vanishes if the vacuum is supersymmetric, and is positive if it is not. Comparing to our previous expressions for the F- and D-term scalar potentials, we see that $F = D = 0$ at the minimum is needed for supersymmetry, and F or D non-zero signals spontaneous supersymmetry breaking.

It can be shown that the *supertrace* vanishes in a supersymmetric theory, whether or not SUSY is sponteneously broken. This is defined to be [1]

$$Str(\mathcal{M}^2) \;:=\; \sum_b g_b m_b^2 - \sum_f g_f m_f^2 \;=\; 0 \;, \tag{97}$$

where the sums run over all the bosons (b) and fermions (f) in the theory, and g_i is the number of real degrees of freedom of state i. This relation presents an immediate problem for trying to achieve supersymmetry breaking within the MSSM itself. Specifically, it implies that the sum of superpartner masses should match up with the sum of SM masses. Given that we have not yet seen any superpartners, we do not expect this be true.

Instead, the standard picture of supersymmetry breaking is that the spontaneous breakdown occurs in a new *hidden sector* that is very heavy and only interacts feebly with the visible MSSM sector through a set of heavy *messengers*. When the heavy hidden and messengers states are integrated out to produce a low-energy effective theory containing the MSSM, soft supersymmetry breaking operators of the form of Eq. (79) are generated. From the point of view of the effective theory, these soft terms appear as explicit breaking, even though their origin is from spontaneous breaking. The supertrace sum rule now applies to both the visible and hidden sectors together, and it is expected to be dominated by the heavy hidden states.

The amount of supersymmetry breaking can be parametrized in terms of the non-zero F- and D-terms of the hidden sector. This breaking is usually assumed to be dominated by F-terms since D-terms alone will not generate standard gaugino masses. Let us take F to correspond to the spontaneous breaking in the hidden sector, and M_* to be the mass of the messenger states. The typical size of the MSSM soft terms is then expected to be

$$m_{soft} \sim C_* \frac{F}{M_*} , \tag{98}$$

where C_* describes the strength of the connection between the visible and hidden sectors. Specific models of supersymmetry breaking and its transmission to the MSSM make predictions for F, M_*, and C_*. The three most popular scenarios are *gravity mediation*, *gauge mediation*, and *anomaly mediation*.

In gravity mediation, the hidden and visible sectors are assumed to be connected only through new states related to quantum gravity with masses on the order of $M_{\rm Pl}$. This gives visible soft terms on the order of [14]

$$m_{soft} \sim \frac{F}{M_{\rm Pl}} . \tag{99}$$

Without a specific theory of quantum gravity, it is not possible to say much more about the values of the soft terms. However, a popular simplifying assumption (with limited theoretical motivation) is called minimal supergravity (mSUGRA). In this scenario, the scalar and fermion soft terms are taken to be universal at some high scale close to $M_{GUT} \sim 10^{-2} M_{\rm Pl}$ and given by

$$M_1 = M_2 = M_3 = m_{1/2} \tag{100}$$

$$m^2_{\tilde{f}} = m_0^2 \tag{101}$$

$$A_f = A_0 . \tag{102}$$

To deduce the values near the weak scale relevant for phenomenological studies, they must be extrapolated down using the renormalization group. This evolution tends to push the masses of the $SU(3)_c$-charged superpartners above those of $SU(3)_c$-neutral states. In particular, the gaugino soft masses at low energy follow the pattern $M_3 : M_2 : M_1 \sim g_3^2 : g_2^2 : g_1^2 \sim 6 : 2 : 1$.

In gauge mediation, the source of supersymmetry breaking is assumed to couple directly to a set of heavy messenger states that carry SM gauge charges [15]. Integrating the messengers out at their mass threshold M_*, this produces

$$m_{soft} \;\sim\; \frac{g^2}{(4\pi)^2}\frac{F}{M_*} \tag{103}$$

where g is a SM gauge coupling for the groups under which the relevant MSSM superpartners are charged. For gauge mediation to dominate over gravity mediation we must have $M_* < [(4\pi)^2/g^2]M_{\rm Pl}$.

A third popular mechanism of supersymmetry breaking is anomaly mediation [16, 17]. Here, the messengers are related to the gravity supermultiplet itself that contains the graviton and the gravitino (to be discussed further below).[5] The soft terms from anomaly mediation go like

$$m_{soft} \;\sim\; \frac{g^2}{(4\pi)^2}\frac{F}{M_{\rm Pl}}\;, \tag{104}$$

where g is a SM gauge coupling for the groups under which the relevant MSSM superpartners are charged. Note that these contributions are subleading compared to those from gravity mediation. They can only dominate when the gravity-mediated pieces are suppressed, as can occur if the source of supersymmetry breaking is very *sequestered* from the visible sector. For example, the two sectors can be localized in different parts of an extra dimension [16].

There are two very important and related issues that we have not yet addressed: the goldstino and the gravitino. Supersymmetry as we have presented it so far has been a global symmetry of spacetime. In this case, the spontaneous breaking of supersymmetry implies that there exists a massless *goldstino* fermion [1]. This is the analogue of the Nambu-Goldstone boson that arises from the spontaneous breaking of a regular global symmetry, but it is now fermionic since the symmetry generators (Q and $\overline{Q}$) are as well. One can also show that the goldstino couples to the rest of the light states in the theory through a higher-dimensional operator that involves a derivative and is suppressed by $1/F$.

This is not the whole story. In much the same way that we can extend the SM to include gravity as an effective field theory, it is also possible to extend global supersymmetry to include gravity as well [14]. For this, we must embed the $s = 2$ graviton in a supermultiplet, and this leads to a *gravitino* superpartner with $s = 3/2$. The resulting theory is called *supergravity*. The gravitino of supergravity has a very important implication for the goldstino. To see this, we will have to make a brief aside.

Recall that regular gravity (GR) can be understood as a gauging of the global Poincaré symmetries of flat spacetime to a local invariance under coordinate transformations. Since

[5] Note that "gravity mediation" typically does not rely on this gravity multiplet itself. Instead, the messengers are new QG states with masses near $M_{\rm Pl}$.

global supersymmetry is an extension of Poincaré, supergravity can also be understood as a gauging of global supersymmetry. In this context, the coordinate invariance of GR emerges as a subgroup of the larger invariance under supergravity transformations. A necessary component of supergravity is therefore the graviton and its gravitino superpartner.

When supersymmetry is spontaneously broken in the context of supergravity, the massless goldstone does not appear on its own as a physical degree of freedom. Instead, it is *eaten* by the would-be massless gravitino to form a massive $s = 3/2$ graviton. Note that the counting of DOFs works out: a massless gravitino has two DOFs and the goldstino has two as well; this matches the four DOFs of a massive $s = 3/2$ particle. The eating mechanism in this case is again completely analogous to what happens when a regular gauge symmetry is spontaneously broken.

The resulting gravitino mass $m_{3/2}$ depends on all the sources of supersymmetry breaking in the theory. If this breaking is dominated by a single F-term (and the vacuum energy vanishes), the mass is

$$m_{3/2} \;=\; \frac{F}{\sqrt{3}M_{\mathrm{Pl}}}\;. \tag{105}$$

The gravitino $\widetilde{G}_\mu$ couples to SM particles and their superpartners according to the general form [1]

$$-\mathscr{L} \;\supset\; \frac{1}{M_{\mathrm{Pl}}}(\partial_\mu\widetilde{f})\overline{f}\gamma^\mu\gamma^\nu\widetilde{G}_\nu \;+\; \frac{i}{8M_{\mathrm{Pl}}}\overline{\widetilde{G}}_\mu[\gamma^\nu,\gamma^\rho]\gamma^\mu\lambda\,F_{\nu\rho} \;+\; \text{h.c.}\;, \tag{106}$$

where $\widetilde{G}_\mu$ is the graviton field, f is a SM fermion and $\widetilde{f}$ is its sfermion superpartner, and $F_{\nu\rho}$ is a vector boson field strength and λ is its gaugino superpartner. In some cases, however, the effective strength of the gravitino coupling can be much larger than the gravitational $1/M_{\mathrm{Pl}}$ factors suggest. This occurs because the longitudinal components of the massive gravitino come from the Goldstino, which couples to the SM and its superpartners with strength $1/F$. When computing matrix elements of gravitinos, these potentially enhanced coupling emerge from gravitino polarization sums.[6] For processes with characteristic energies $E \gg m_{3/2}$, this effect can be handled by making the substitution [1]

$$\widetilde{G}_\mu \to \sqrt{2/3}\;\partial_\mu\psi/m_{3/2}\;, \tag{107}$$

where ψ represents the $s = 1/2$ Goldstino field. Note that $m_{3/2}M_{\mathrm{Pl}} \sim F$, so this substitution does indeed produce a factor of $1/F$ when inserted in Eq. (106). When $E \lesssim m_{3/2}$ the full expression of Eq. (106) should be used.

Measuring the mass spectrum of the MSSM superpartners and the gravitino would provide information about the origin of supersymmetry breaking. In turn, this could tell us about new physics at very high energies and might even inform us about the underlying theory of quantum gravity. For these reasons, the discovery of supersymmetry would be a major experimental and theoretical breakthrough.

[6] A similar thing happens when massive vector bosons interact at high energies. The vector polarization sums give a factor of $(-\eta_{\mu\nu} + p_\mu p_\nu/m_V^2)$, and the second term in this expression can become numerically large for $E \gg m_V$.

3 Signals of Supersymmetry

We turn next to a give brief overview of the potential observational consequences of supersymmetry. So far, we do not have any experimental evidence for the presence of superpartners. However, the upcoming LHC run at energies approaching $\sqrt{s} = 14$ TeV will be able to probe a much wider range of superpartner candidates than ever before. We will begin this section with the various indirect constraints on the superpartner spectrum, and from there we will work up to a discussion of direct searches for supersymmetry at high-energy colliders.

3.1 Dark Matter and the LSP

A great deal of the phenomenology of supersymmetry depends on the nature of the lightest superpartner (LSP). Recall that with R-parity, this state is stable. For now, let us focus on theories where R-parity is present. In such theories, the LSP will contribute to the density of dark matter unless it is very light.

To be consistent with observations, a stable LSP must be electrically neutral. In the MSSM, this leaves the neutralinos, the sneutrinos, and the gravitino as possible LSP candidates. Of these, the most promising candidate for dark matter is the lightest neutralino χ_1^0. If this state is a moderate mixture of the Bino ($\widetilde{B}^0$) with some Wino ($\widetilde{W}^0$) or higgsino ($\widetilde{H}_{u,d}^0$), it can provide the observed DM relic density through thermal freeze-out [20] and be consistent with direct and indirect searches for DM [18]. A sneutrino LSP does not work as well because it has a relatively large weak interaction with SM matter (relative to the neutralinos) and would very likely have already been observed in direct DM searches [19].

The situation for a gravitino LSP is more complicated. Comparing the gravitino mass estimate of Eq. (105) to the soft mass scales of Eqs. (99,103,104) for various mediation mechanisms, we see that $m_{3/2} \sim m_{soft}$ in gravity mediation, $m_{3/2} < m_{soft}$ in gauge mediation, and $m_{3/2} > m_{soft}$ in anomaly mediation. A gravitino LSP can also make up the dark matter, but a more complicated *non-thermal* cosmological history is needed [18].

The possibility of a gravitino LSP has received the most attention in the context of gauge mediation. Here, the lightest SM superpartner $\widetilde{X}$ is the next-to-lightest superpartner (NLSP) and decays down to its SM counterpart and the gravitino. The corresponding decay rate is

$$\Gamma(\widetilde{X} \to X\widetilde{G}) \simeq \frac{1}{48\pi}\frac{m_{\widetilde{X}}^5}{m_{3/2}^2 M_{\rm Pl}^2} . \tag{108}$$

Note that the lighter the gravitino, the more quickly it decays. In many models of gauge mediation, the NLSP is either a mostly-Bino ($\widetilde{B}^0$) neutralino or a mostly right-handed stau ($\widetilde{\tau}_R$). These leads to the dominant decay channels $\widetilde{\chi}_1^0 \to \gamma\widetilde{G}$ or $\widetilde{\tau}_R \to \tau\widetilde{G}$.

3.2 Precision Tests

Many aspects of the SM have been studied to a very high precision. Such measurements test the validity of the SM, and they have also been applied to search for indirect evidence of new BSM physics. So far, no clear deviation from the SM has been observed, and this places important constraints on many proposals for new physics.

The most constraining precision measurements for supersymmetry are tests of quark flavour mixing and CP violation. The soft breaking terms of the MSSM can be a new source of flavour mixing in addition to the CKM matrix. For example, the $m_Q^2|\widetilde{Q}|^2$ term in Eq. (84) is a shorthand for

$$-\mathscr{L}_{soft} \supset (m_Q^2)_{ij}\widetilde{Q}_i^\dagger\widetilde{Q}_j , \tag{109}$$

where $i, j = 1, 2, 3$ are flavour (generation) indices. Non-diagonal or non-universal terms among the first two generations in $(m_Q^2)_{ij}$ or the A terms must be much smaller than the diagonal terms to be consistent with experiment for soft terms near the weak scale. If the soft terms are completely *anarchic* in flavour space, with no particular underlying structure, the superpartners must be heavier than nearly 10^6 GeV to agree with observation [21]. Similar constraints can be derived for new sources of CP violation coming from phases in the A and B terms and the gaugino masses.

These bounds have important implications for the mechanisms of supersymmetry breaking and mediation [1, 2]. Specifically, to have superpartners near the weak scale to provide a full explanation for the hierarchy problem, the flavour mixing and CP violation in the soft terms must be very small. It turns out that both gauge and anomaly mediation give diagonal and universal soft terms in their most minimal forms. Gravity mediation does not guarantee this, and may require some additional flavour structure to be viable.

3.3 Higgs Physics and Fine Tuning

The recent LHC discovery of a SM-like Higgs boson with mass near $m_h \simeq 125$ GeV is very significant for the MSSM and other supersymmetric extensions of the SM [22, 23]. A SM-like Higgs with a mass this large is challenging to obtain in the MSSM, and it implies that the theory has some degree of fine tuning. While this tuning is much less than the hierarchy problem without supersymmetry, it is also unwelcome in a theory that was motivated by its ability to remove fine tuning in the first place.

The two MSSM states that could be potentially identified with the observed Higgs boson are h^0 and H^0. Given the lack of evidence for a new scalar lighter than the Higgs, the more likely candidate of these two is the h^0 state. This particle couples to the SM in the same way as a SM Higgs boson in the *decoupling* of limit $m_A \gg m_Z$. The challenging part of connecting h^0 to the SM Higgs is the bound on its mass derived from the tree-level potential (Eq. (91)):

$$m_h^2 \leq m_Z^2 \cos^2 2\beta \leq m_Z^2 . \tag{110}$$

With $m_Z = 91.2$ GeV, this is clearly less than the observed Higgs mass. The first inequality is saturated in the decoupling limit.

Fortunately, loop corrections can push the mass of h^0 up to and beyond 125 GeV. The most important contribution comes from the scalar top superpartners (stops). In the absence of left-right stop mixing and in the decoupling limit, it is given at one-loop by

$$\Delta m_h^2 = \frac{12}{(4\pi)^2} m_t^2 y_t^2 \sin^2\beta \,\ln\left(\frac{m_{\tilde{t}_L} m_{\tilde{t}_R}}{m_t^2}\right) \,. \tag{111}$$

A modest additional enhancement can be obtained with stop mixing. These corrections can push the h^0 mass up to the observed value for $m_{\tilde{t}_{L,R}} \gtrsim 1.5$ TeV and $\tan\beta \gtrsim 10$.

Obtaining this enhancement comes at the cost of some fine tuning. For larger $\tan\beta$ in the decoupling limit, the minimization condition for the Higgs potential of Eq. (89) can be written as

$$m_Z^2 \simeq -2|\mu|^2 - 2m_{H_u}^2 \,. \tag{112}$$

The two terms on the right side seem to have very different origins; μ is supersymmetric while $m_{H_u}^2$ comes from supersymmetry breaking. This suggests that they do not have a good reason to cancel, and that that neither should be much larger than m_Z^2. On the other hand, the relatively heavy stops needed to push up the h^0 mass also generate a large quantum correction to $m_{H_u}^2$,

$$\Delta m_{H_u}^2 \simeq -12\frac{y_t^2}{(4\pi)^2} m_{\tilde{t}}^2 \ln\left(\frac{M_*}{m_{\tilde{t}}}\right) \,, \tag{113}$$

where $m_{\tilde{t}} \sim \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$ and M_* is the messenger scale. Taken together, the heavy stops needed to push up the Higgs mass also increase $|m_{H_u}^2|$ above the weak scale, and this implies a seemingly tuned cancellation against $|\mu|^2$, at the level of a few percent, to satisfy Eq. (112). This situation can be improved slightly by extending the Higgs sector of the MSSM, but not by much. Note as well that this expression shows how the hiearchy problem reappears as the soft masses are taken to be very large [24].

3.4 Collider Searches

Our best hope of discovering supersymmetry in the near future is the upcoming 14 TeV run of the LHC, set to begin in May, 2015. This run will collide beams of protons at higher energies than ever before, and is scheduled to collect an enormous amount of data. It is expected to be able to discover superpartners with masses up to about 3 TeV.

Superpartner production at the LHC is greatest for the gluino and the squarks of the first generation, as shown in Fig. 1. These can proceed through the strong interaction with up- and down-quarks (derived from the colliding protons) in the initial states. The next largest production rates are for the squarks of the second and third generations, which also go by the strong force. The production of states that are uncharged under $SU(3)_c$, such as charginos, neutralinos, and sleptons, tends to be much smaller.

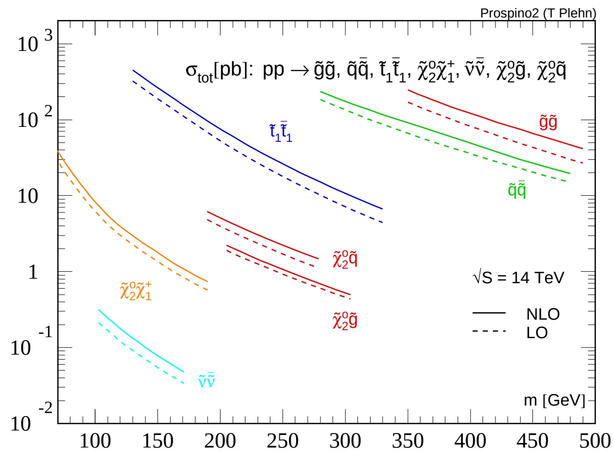

Figure 1: Superpartner production cross sections in picobarns (pb) at the 14 TeV LHC obtained computed with Prospino [25].

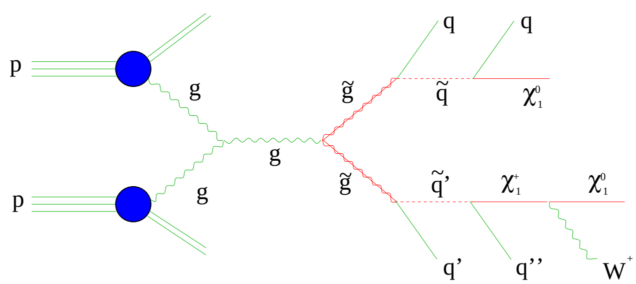

Figure 2: Creation of a pair of gluinos in a pp collision followed by their cascade decays down to a neutralino LSP.

With R-parity, superpartners are always produced in pairs. Each superpartner will then decay down to a lighter superpartner and one or more SM states. These decays continue until the LSP is reached. If the LSP is neutral and uncoloured, as motivated by dark matter and cosmology, it will quit the detectors without leaving a signal [26]. An example of such a *cascade decay* chain is shown in Fig. 2, which depicts the pair production of a pair of gluons followed by their cascade decays down to a neutralino LSP.

Despite not producing a direct signal in particle detectors, the presence of one or more LSPs in a collider event can still be deduced through momentum conservation. Since the LSPs carry off some of the momentum from the event, they can induce an imbalance in the net momentum of the visible particles in the event. The *missing transverse momentum* in a collider event is defined to be

$$\not{\vec{p}}_T = -\sum_{i\in\text{visible}} \vec{p}_{T_i} \tag{114}$$

Here, *transverse* means the component in the plane orthogonal to the beam axis. Note that

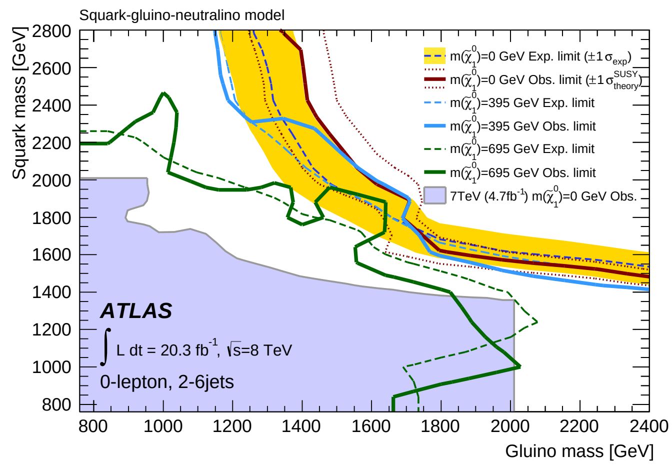

Figure 3: Limits on squarks and gluinos from ATLAS LHC searches for jets and missing energy, for Ref. [27].

it is usually not possible to apply momentum conservation along the beam axis because the initial momenta of the colliding partons (quarks or gluons within the incoming protons) is not known. The magnitude of the missing transverse momentum vector is typically called the *missing energy* $\not{E}_T$ or MET.

The strongest collider constraints on superpartner masses come from searches for jets and missing energy and apply to squarks and gluinos. In Fig. 3 we show the limits from the ATLAS experiment at the LHC using searches based on about $20\,\mathrm{fb}^{-1}$ of data taken at $\sqrt{s} = 8$, TeV [27]. The specific limits depend on how the squarks and gluinos decay and the mass of the LSP (assumed to be a neutralino here), but they can extend up to superpartner masses of nearly 2 TeV. The LHC limits on squarks of the third generation of considerably weaker, but still go up to nearly 700 GeV.

4 Supersymmetry in More Detail

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