Notes #1: Context and Motivation for New Physics

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The Standard Model (SM) gives an excellent description of nearly every laboratory experiment performed to date. It provides a quantum mechanical description of all known elementary particles and their interactions with each other (except for gravity). Even so, there are many reasons why the SM cannot be the complete theory of the Universe; there must exist new physics *beyond the Standard Model* (BSM) [1]. In this course we will study some of the most promising proposals for BSM physics, with a focus on scenarios that can be tested experimentally in the forseeable future. Before getting to them, however, we will begin by discussing the many reasons why the SM description of elementary particles and interactions is incomplete. We will also present the modern interpretation of the SM as an effective theory that gives a good description of Nature up to energies

1 Motivation to Go Beyond the Standard Model

The SM is great, but there are some things it does not do. We begin by making a list of them. These shortcomings will be addressed in some of the BSM theories that will be discussed in this course.

1.1 Gravity

An obvious shortcoming of the SM is that it does not describe the gravitational force. This is almost never a problem in particle physics experiments because gravitational effects are usually completely negligible compared to the other relevant forces. The weakness of gravity can be seen in the size of Newton's constant,

$$G_N = \frac{M_{\rm Pl}^2}{8\pi} \simeq 6.9 \times 10^{-39}\ \text{GeV}^{-2}\ . \tag{1}$$

Here, $M_{\rm Pl} \simeq 2.4 \times 10^{18}\,\text{GeV}$ is the *reduced* Planck mass. The corresponding quantity for the weak force (below about 100 GeV) is the Fermi constant, $G_F = 1.16637 \times 10^{-5}\,\text{GeV}^{-2} = \sqrt{2}g^2/8m_W^2$, which is over 30 orders of magnitude larger. The electromagnetic and strong forces are even stronger than this at low energies.

The SM can be extended to include gravity in a relatively straightforward way [2].[1] Starting from the classical description of gravity, general relativity (GR), the metric field is identified as the dynamical variable and quantum mechanics is applied to it. To do this, one typically starts with a fixed background metric and expands in fluctuations around it.[2] For example, with a flat (Minkowski) background, we would write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/2M_{\rm Pl}\ . \tag{2}$$

[1] Sorry for the bad pun.

[2] You may have already done this when studying classical gravitational waves.

A quantum field theory for $h_{\mu\nu}$ can now be built using the standard techniques applied to the action for GR,

$$S = \int d^4x\,\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R(g) + \mathscr{L}_{SM}(g)\right)\ . \tag{3}$$

The quantized excitations of $h_{\mu\nu}$ can be identified with a massless spin-2 graviton interacting with the SM. This theory reproduces GR in this classical limit and agrees with experimental data. From now on, when we refer to the SM we will implicitly mean this SM plus quantized-GR theory.

Even though we have a quantum theory of gravity, it is not quite *the* quantum theory of gravity everybody wants. The problem is that this theory is non-renormalizable, with all of the interactions between the graviton and the SM involving higher-dimensional operators suppressed by powers of $M_{\rm Pl}$. As a result, this gravity-extended SM can be trusted as an effective field theory at energies far below $M_{\rm Pl}$, but it loses its predictive power for energies approaching $M_{\rm Pl}$. All current experimental measurements involve single-particle energies well below $M_{\rm Pl}$, so this breakdown of the effective theory has not been a huge problem.

Still, we would very much like to have a quantum theory of gravity that is valid up to energies approaching the Planck scale.[3] This is both a matter of theoretical principle, as well as a necessary step in understanding black holes (which we do have evidence for) and the microscopic structure of spacetime. Discovering the full quantum theory of gravity is still a work in progress, and there are many proposals for what it could be. The best-studied scenario is *superstring theory* in which the elementary constituents can be identified with one-dimensional strings instead of point-like (*i.e.* zero-dimensional) particles [3, 4, 5]. The full implications of string theory are not fully understood, but the theory has been very successful in describing the microscopic structure of black holes [6]. Incorporating the SM within string theory in a consistent way seems to require extra spatial dimensions and supersymmetry. We will examine both possibilities later in the course. Another popular attempt to formulate a quantum theory of gravity is *loop quantum gravity*, in which spacetime emerges in a more dynamical way and without reference to a fixed background metric [7].[4]

A generic expectation for a quantum theory of gravity is that it contains new states with masses near $M_{\rm Pl}$. Some of these heavy states may also have direct couplings to the SM. (String theory exhibits both features.) At energies much lower than $M_{\rm Pl}$, the heavy particles can be integrated out to generate an effective field theory consisting of the SM and graviton fields together with higher-dimensional operators connecting them. The leading terms in the action of the low-energy EFT should match up with those in Eq. (3), and there will also be additional higher-order operators suppressed by powers of $M_{\rm Pl}$. Measuring the effects of such higher-dimensional operators would give us hints about the underlying QG theory.

[3] This is what people usually have in mind when they say "quantum gravity" (QG).

[4] There are experts at PI in both strings and LQG (and other approaches to QG). If you would like to know more about QG, I would suggest speaking to them.

1.2 The Electroweak Hierarchy Problem

Electroweak symmetry breaking in the SM is induced by an elementary scalar Higgs field H. This field develops a vacuum expectation value (VEV) $\langle H \rangle = v \simeq 174$ GeV that generates masses for the weak vector bosons and the fermions of the theory. The electroweak hierarchy problem is that the VEV of the Higgs field is very sensitive to quantum corrections. Because of this, it is puzzling why the Higgs VEV has the value that it does, rather than being zero or much larger.

To see the problem more precisely, recall that the leading terms in the effective potential for the Higgs field are

$$V = -\mu^2 |H|^2 + \frac{\lambda}{2}|H|^4 \ . \tag{4}$$

The Higgs VEV corresonds to the value of $|H|$ that minimizes this potential, $v^2 = \mu^2/\lambda \simeq (174\ \text{GeV})^2$. The mass of the physical Higgs boson excitation about this minimum is $m_h = \sqrt{2\lambda\, v^2} = \sqrt{2\mu^2} \simeq 125$ GeV, which has recently been measured experimentally [8, 9].

Suppose we try to extrapolate the SM up to energies much larger than v, and let us assume there exists a very heavy new particle Ψ with mass M_Ψ and coupling y_Ψ to the Higgs. Such a particle will induce a finite quantum (loop) correction to the Higgs quadratic parameter in Eq. (4) on the order of [10]

$$\Delta\mu^2 \ \sim \ \mp \frac{y_\Psi^2}{(4\pi)^2} M_\Psi^2 \ , \tag{5}$$

where the minus (plus) sign corresponds to Ψ being a fermion (boson). This sensitivity to quantum corrections is specific to fundamental scalars and is large in the sense that it depends quadratically on the mass of the heavy particle in the loop.

The electroweak hierarchy problem comes from our expectation that there exist new states Ψ with masses much larger than μ. Such states would imply $\Delta\mu^2 \gg v^2 \sim \mu^2$ as long as y_Ψ is not too small. For example, our attempts at quantum gravity suggest new states with $M_\Psi \sim M_{\text{Pl}}$. If these heavy particles couple directly to the SM (such as can occur in string theory), we would also have $y_\Psi \sim 1$. However, even if the only coupling of the new massive states to the SM is through the massless graviton, we would still expect $y_\Psi^2 \sim (M_\Psi/M_{\text{Pl}})^4/(4\pi)^4$ [10]. In both cases, $\Delta\mu^2$ is much larger than μ^2. We will also encounter other examples of new physics that lead to large corrections to the Higgs quadratic parameter.

A quantum correction $\Delta\mu^2$ that is much larger than the observed value of μ^2 is puzzling. To achieve a very small μ^2 relative to $\Delta\mu^2$, the parameters in the underlying high-energy theory must cancel out to a very high precision. From the point of view of the low energy effective theory, there is no good reason why such a cancellation should occur. Thus, our Universe seems to be very finely tuned unless there is new physics that forces $\Delta\mu^2$ to be small.

This *electroweak hierarchy problem* motivates new physics to stabilize the Higgs μ^2

paramter. There are three main mechanisms for this, and they all suggest new particles and interactions not too far above the weak scale. They are:

1. A new (approximate) symmetry of the theory that causes the various contributions to $\Delta\mu^2$ to cancel each other out. The best example of this is *supersymmetry*.
2. The description of the Higgs field changes radicatlly at energies not too far above the weak. For instance, this will occur the Higgs is not a fundamental scalar but is instead a bound state of fermions tied together by a very strong new force. In contrast to a fundamental scalar, fermions are much less sensitive to quantum corrections due to a built-in approximate chiral symmetry, while vector bosons are protected by gauge invariance.
3. The fundamental Planck scale is much smaller than the apparent value of $M_{\rm Pl}$. In this case, quantum gravity effects will arise not too far from the weak scale implying that our effective field theory description of the SM (in which the hierarchy problem arises) is not valid anymore.

We will study extensions of the SM motivated by these three stabilization methods in much of the rest of the course. A very depressing fourth possibility is that the Higgs mass parameter just happens to be finely tuned to be small for some unknown (and untestable) reason.

1.3 Cosmology

If the SM-plus-gravity quantum effective theory of Eq. (3) is the complete theory of elementary particles at energies below $M_{\rm Pl}$, it should be able to account for the large-scale structure of the Universe. However, detailed cosmological measurements do not agree with the predictions of the SM. There are three main puzzles: the extreme flatness and spatial uniformity of the Universe, the apparent need for dark matter, and the excess of matter over antimatter. All three are strongly suggestive of new physics below the Planck scale.

The Universe is extremely uniform over very large distances. This is seen best in the cosmic microwave background (CMB) radiation, which consists of photons with a mean effective temperature of about $T \simeq 2.73\,\mathrm{K} \simeq 2.4 \times 10^{-13}$ GeV [11]. Relative to the mean value, the primordial fluctuations in the CMB temperature are very small – only about one part in 10^5. Extrapolating the CMB back in time using the SM and GR implies that the early Universe consisted of a very hot plasma of subatomic particles. The energy density of this plasma caused the Universe to expand, which in turn caused the plasma to cool. When the plasma temperature fell below $T \simeq 0.3\,\mathrm{eV}$, its charged components (mostly protons and electrons) combined to form neutral atoms. This quickly depleted the electrically-charged fraction of the plasma, a process called *recombination*, and allowed the remaining photons to travel across the Universe unimpeded. What we see today as the CMB are these leftover photons, and therefore the CMB gives us a snapshot of the Universe when it was much younger.

The uniformity of the CMB is curious because the extrapolation of the SM back in time also implies that many different regions of the CMB we see today were causally disconnected when the CMB was formed at recombination. From this point of view, it is very surprising that these regions should be so close in temperature. The leading resolution of this puzzle is *inflation*, a period of rapid exponential expansion of spacetime in the very early Universe [12]. Inflation allows a small causally connected patch of spacetime to be expanded so much that it makes up the entire visible sky today. Most theories of inflation introduce a new scalar field to the SM with a very flat potential [12].[5]. If this *inflaton* scalar field starts off with a large displacement from the minimum of its potential, the potential energy of the field can drive a period of inflation. The expansion from inflation dilutes away everything that was present in the Universe before it began. Eventually, the inflaton field rolls down to the minimum of the potential, oscillates for a bit, and transfers its energy back to radiation when it decays in a process called *reheating*. The end result of inflation and reheating is a very hot and uniform thermal plasma of subatomic particles; this is precisely what we observe.

Inflation makes other predictions that agree with cosmological observations. Measurements of the CMB show that the net spatial curvature of the Universe is zero to within a small experimental uncertainty. This is also expected from inflation, since any initial curvature would be strongly diluted by the exponential inflationary expansion. The spectrum of small temperature fluctuations in the CMB is consistent with and expected from inflation. In particular, they can be understood as coming from quantum fluctuations in the inflaton field during inflation. While these various measurements agree with the general predictions of inflation, we still do not have enough information to deduce the underlying theory of the inflaton.

Detailed studies of the CMB together with other cosmological and astrophysical observations point towards additional shortcomings of the SM. One of the most striking is that the total density of matter (*i.e.* non-relativistic particles) appears to be much larger than what can be accounted for by the SM [14, 15, 16]. The evidence for this extra *dark matter* comes from a diverse set of observations over a very broad range of distances. These include galactic rotation curves, the average motion of galaxy clusters, the structure of the distribution of matter, and the fluctuations in the CMB.

The most simple explanation for dark matter is the existence of a new massive particle with a moderate cosmological density. To account for observations, it must be electrically neutral and uncharged under the strong force, but other than that we know very little about what such a particle could be. Many theories of BSM physics predict or can accommodate a dark matter candidate. In particular, the observed DM density can plausibly be explained by a weakly-interacting massive particle (WIMP), a new particle with mass close to the weak scale and weak interactions with the SM [14, 15, 16]. Such WIMPs arise frequently in BSM theories that address the electroweak hierarchy problem. On the experimental side, searches for dark matter are underway using a wide range of techniques including particle colliders, deep underground direct dark matter detectors, and observations of cosmic rays in our galaxy [16]. None of these experiments has found anything conclusive yet.

The observed density of SM matter in the Universe is also puzzling. By energy, it is dominated by baryons (in the form of protons and light nuclei). and it consists almost

[5]The SM Higgs may also induce inflation if it has a non-minimal coupling to gravity [13]

entirely of matter rather than antimatter [17]. The origin of this *baryon asymmetry* is a mystery, and there is no known way to generate it within the SM alone. In contrast, there are a number of viable mechanisms to generate the baryon asymmetry in BSM theories [17].

A fourth puzzle related to cosmology is the *cosmological constant problem* [18]. Most of the energy density in the Universe today (about 75%) seems to come from a positive net value of the background vacuum energy. This vacuum energy can be accommodated within the SM by adding a constant term to Eq. (3),

$$S \to S - \int d^4x\sqrt{-g}\;\Lambda_{cc}\;, \tag{6}$$

where Λ_{cc} is called the *cosmological constant* (CC). Note that without gravity, the CC would not have any physical effects. However, in GR it acts as a source for spacetime curvature and must be taken into account. The problem with the CC is its size (determined from observation), $\Lambda_{cc} \simeq (2.5\times 10^{-12}\ \text{GeV})^4$. This is absolutely miniscule compared to the natural value of M_{Pl}^4, or any other dimensionful scale within the SM for that matter. It is not clear how to explain this vast difference.

In summary, the SM (plus GR EFT) does not appear to be consistent with the observed cosmology. These disagreements could be due to new particles and interactions or they could signal deviations from general relativity as the theory of gravity. We will focus mainly on the particle-related explanations in this course.

1.4 Flavour and CP Violation

The SM has three generations of quarks and leptons with a very wide range of masses. Weak interactions induce a mixing between these different *flavours* (or *generations*) of fermions, and they allow the heavier flavours to decay to the lighter ones and mediate CP violation. While the SM is able to accomomdate this structure, it does not provide an explanation for it. Furthermore, the (strictly renormalizable) SM is also unable to account for the observed neutrino masses. Both features are suggestive of new physics.

The range of fermion masses in the SM is enormous, from sub-eV for the neutrinos, to $m_e \simeq 0.511\times 10^{-3}$ GeV for the electron, and up to $m_t \simeq 174$ GeV for the top quark. Quark mixing via the weak interactions is described by the unitary Cabbibo-Kobayashi-Maskawa (CKM) matrix, whose numerical values are [19],

$$|V_{CKM}| \;=\; \begin{pmatrix} 0.9743(2) & 0.2253(8) & 0.0041(5) \\ 0.225(8) & 0.99(2) & 0.041(1) \\ 0.008(1) & 0.040(3) & 1.02(3) \end{pmatrix}\;. \tag{7}$$

These entries are seen as being very suggestive of an underlying hierarchical structure. The CKM matrix also contains phases that give rise to CP violation.

All the charged fermion masses and mixings in the SM come from Yukawa couplings to the Higgs of the form

$$-\mathscr{L} \supset y^f_{ij}\overline{F}^i_L f^j_R \overset{(\sim)}{H} + (h.c.)\;, \tag{8}$$

where F_L is a $SU(2)_L$ doublet, f_R is a right-handed $SU(2)_L$ singlet, H is the Higgs, and i, j are flavour indices that run over the three generations. From this perspective, a theory for the underlying structure of flavour and CP is equivalent to theory for the origin of the Yukawa couplings y^f_{ij}.

In contrast, it is not possible to write a mass term for the neutrinos of the form of Eq. (8) using only SM fields. The neutrino masses must also be much smaller than those of the charged fermions, below about an eV, and their mixings appear to be anarchic with no obvious hiearchy [20]. Both features point towards new physics and a different origin for the neutrino masses compared to the charged fermions.

The multi-generational structure of the SM allows for CP violation. Observations of CP violation in quark-mediated processes is consistent with the possible phases of the CKM matrix. These phases emerge from complex values of the Yukawa couplings in Eq. (8), and at least three generations of quarks are needed for them to produce observable effects. For this reason, it is reasonable to expect that an explanation for the origin CP violation might be related to the origin of quark flavour.

The SM has another source of CP violation beyond the CKM matrix. It is the *QCD* Θ *parameter*, corresponding to the coefficient of the operator [21]

$$-\mathscr{L} \supset \frac{\Theta}{32\pi^2}\,\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}\ , \tag{9}$$

where $G^a_{\mu\nu}$ is the gluon field strength. If all the quark masses are non-zero (which appears to be the case experimentally), the Θ parameter can give rise to observable CP-violating effects. In particular, it can induce an electric dipole moment (EDM) for the neutron. Attempts to measure such an EDM have found nothing so far implying $|\Theta| \lesssim 10^{-10}$ [21]. The required smallness of this parameter has no good explanation in the SM, and is called the *strong CP problem*. The most promising solution is to introduce a new pseudoscalar *axion* particle that dynamically drives $\Theta \to 0$ [22].

1.5 And More . . .

The history of science is full of unexpected discoveries. For this reason, it is worthwhile searching very broadly for new phenomena, including those that are not necessarily motivated by a shortcoming of the SM. Many new discoveries have come about because it has become possible to probe higher energies [23]. This is a major goal of the LHC and one of the reasons why so many people are excited about it. Astrophysics and cosmology have also yielded a wide range of surprises and promise to provide even more [24]. And a third set of paths to discovery are lower-energy measurements with very high precision or intensity [25]. These may point towards new physics by finding deviations from the predictions for SM processes, or by discovering new processes that do not occur in the SM.

2 How to Think about Quantum Field Theories

Chances are you are fairly new to quantum field theory (QFT). It is a rich and complicated topic that can take many years to get an intuitive feel for. In this section we will try to accelarate this process by describing the "modern" interpretation of QFTs that is used widely in studies of the SM and beyond. The main aspects of this are dimensional analysis, symmetries, renormalization, and effective field theories.

2.1 Dimensional Analysis

Dimensional analysis (DA) is an incredibly powerful tool for estimating the sizes of things without doing any hard calculations. The idea is to keep track of the mass dimensions of everything in the problem.[6] For DA to work, it is also essential to keep track of the exact and approximate symmetries of the theory. We will illustrate this below with example.

Recall that we usually define a QFT (such as the SM) with an action in d spacetime dimensions. Consider a theory with a scalar ϕ and a fermion ψ with Lagrangian

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \overline{\psi}i\gamma^\mu\partial_\mu\psi - M\overline{\psi}\psi - \frac{A}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 - y\,\phi\,\overline{\psi}\psi \tag{10}$$

To figure out the mass dimensions, we use the fact that the action $S = \int d^dx\ \mathscr{L}$ is always dimensionless. Using a square bracket to denote the mass dimension, we also have

$$[S] = 0, \quad [x] = -1, \quad [\partial] = +1, \quad [d^dx] = -d\ . \tag{11}$$

Each term making up the action must be separately dimensionless. Applying this to the scalar and fermion kinetic terms, we find

$$0 \;=\; -d + 2[\phi] + 2 \;=\; -d + 2[\psi] + 1 \tag{12}$$

so that we have

$$[\phi] = 1 + \frac{(d-4)}{2}, \quad [\psi] = 3/2 + \frac{(d-4)}{2}\ . \tag{13}$$

We will mostly work in $d = 4$, giving $[\phi] = 1$ and $[\psi] = 3/2$. Moving on to the other terms in the Lagrangian, we find

$$[m] = [M] = 1, \quad [A] = 1 + \frac{(4-d)}{2}, \quad [\lambda] = 0 + (4-d), \quad [y] = 0 + \frac{(4-d)}{2}\ . \tag{14}$$

Note that the "mass terms" always have dimensions of mass.

As a first application of dimensional analysis, suppose we have $m \gg M$ so that the decay $\phi \to \psi\bar{\psi}$ is possible, and let us compute the corresponding decay rate. In the limit $y \to 0$, the scalar and the fermions would not interact at all in this theory, so at least one power

[6] Since we are using natural units with $\hbar = c = 1$, everything can be expressed with dimensions of mass.

of y is needed in the decay amplitude. With $m \gg M$, the outgoing ψ particles will be highly relativistic with energies close to $m \gg M$. This implies that m is the only relevant dimensionful quantity for the decay rate in this limit. Since the decay rate has a mass dimension of one, the estimate from dimensional analysis is

$$\Gamma \sim y^2 m \ . \tag{15}$$

Up to factors of two and 4π and kinematic corrections that depend on $M/m \ll 1$, this is a good estimate of the full tree-level calculation.

For a second estimate, consider $\phi\phi \to \phi\phi$ scattering in the limit $y \to 0,\ A \to 0$. This process can now occur only through the $\lambda\phi^4$ operator so the scattering amplitude requires at least one power of λ. At very high energies, the only relevant dimensionful quantity is the centre-of-mass energy $s = (p_1 + p_2)^2 \gg m^2 = 4E_{cm}^2$. Since the total scattering cross section has a mass dimension of minus two, the DA estimate is

$$\sigma \sim \frac{\lambda^2}{s} \ . \tag{16}$$

This is a reasonable approximation to the full high-energy result.

If the cubic A coupling is non-zero, there is another contribution to the $\phi\phi \to \phi\phi$ scattering cross section. Two powers of the coupling are needed to make an amplitude with an even number of external states. Taking into account that A has a mass dimension of one, its contribution to the cross section (neglecting interference with the λ piece) is

$$\sigma \sim \frac{A^4}{s^3} \ . \tag{17}$$

This falls off more quickly with energy than the contribution from λ.

2.2 Renormalization

Calculations in QFT involving Feynman diagrams with loops often lead to apparent divergences. This can be unsettling at first, but it turns out that such divergences have interesting physical implications. Before getting into a few explicit examples, let us mention that divergences can arise both at very low energies (large distances - called the IR limit) and at very high energies (short distances - called the UV limit). We will concentrate on UV divergences here, since IR divergences often cancel once the sensitivity of the experiment used to measure their effect and diagrams with additional external legs are both taken into account [26, 27].

To discuss renormalization, consider a real scalar theory with the Lagrangian

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2 - \frac{\lambda_0}{4!}\phi^4 \ . \tag{18}$$

In $d = 4$, λ_0 is dimensionless and m_0 has dimensions of mass. These terms give a basic propagator of $i/(p^2 - m_0^2 + i\epsilon)$ and a 4-point vertex $-i\lambda_0$. The basic objects to be computed

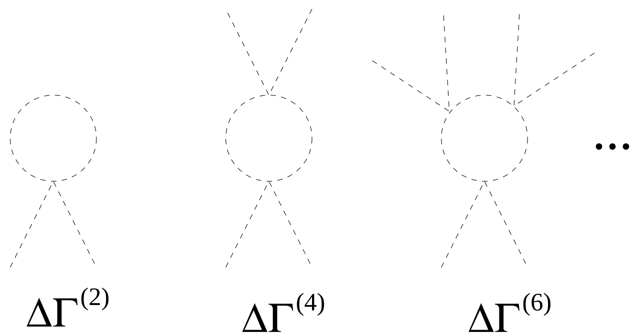

Figure 1: Corrections to 1PI n-point functions of the $\lambda\phi^4$ theory at one-loop order.

in this theory are $\Gamma^{(n)}$, the 1PI connected n-point functions with all external propagators removed. The one-loop corrections to the $(2n)$-point functions are shown in Fig. 1. Ignoring external momenta, these go like $(n \geq 1)$

$$\Delta\Gamma^{(2n)} \;\sim\; \lambda_0^n \int \frac{d^4q}{(2\pi)^4} \left(\frac{1}{q^2 - m^2 + i\epsilon} \right)^n \;\sim\; (\text{finite}) + \lim_{\Lambda\to\infty} \Lambda^{(4-2n)} \tag{19}$$

These integrals diverge in the UV ($q \to \infty$) for $n = 1$, 2, but are convergent for all higher n. (Note that "Λ^0" diverges as $\ln \Lambda$.)

What are we to do with these seemingly infinite quantum corrections to the theory? The divergences come from loops in which we sum over all possible momenta, which we have assumed can become arbitrarily large, much larger than what can be accessed experimentally. This suggests that we are asking more of the theory than what it can provide.[7] Instead, let us treat the theory as an *effective theory* valid up to some very high energy scale where we assume new physics comes in and makes the integrals finite. At first glance, this doesn't look much better; we have traded formal infinities for unknown finite quantitites and it is not immediately obvious how this will help us make testable predictions. However, it turns out that in a *renormalizable* theory the effects of the unknown new physics can be parametrized in a finite set of unknown coefficients in the low-energy theory. Once these unknown coefficients are fixed using experimental data, everything else we compute in the theory is a genuine prediction.

Let us illustrate this in the scalar theory. We will use a method called *renormalized perturbation theory.* The first step is to rewrite the Lagrangian in terms of the rescaled field

[7] Note that something similar happens in classical electrodynamics (or Newtonian gravity) when one tries to construct a point charge. The energy required to build a uniform charge distribution of radius R and total charge Q is proportional to Q^2/R, which obviously diverges as $R \to 0$.

$\phi = Z^{1/2}\widetilde{\phi}$. Plugging into the original Lagrangian and rearranging, we get

$$\begin{aligned}
\mathscr{L} &= \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2 - \frac{\lambda_0}{4!}\phi^4 && (20)\\
&= \frac{1}{2}Z(\partial\widetilde{\phi})^2 - \frac{1}{2}Zm_0^2\widetilde{\phi}^2 - \frac{\lambda_0}{4!}Z^2\widetilde{\phi}^4 && (21)\\
&= \frac{1}{2}(\partial\widetilde{\phi})^2 - \frac{1}{2}m^2\widetilde{\phi}^2 - \frac{\lambda}{4!}\widetilde{\phi}^4 && (22)\\
&\quad + \frac{1}{2}\delta Z(\partial\widetilde{\phi})^2 - \frac{1}{2}\delta m^2\widetilde{\phi}^2 - \frac{\delta\lambda}{4!}\widetilde{\phi}^4 ,
\end{aligned}$$

where

$$\delta Z = (Z-1), \quad \delta m^2 = Zm_0^2 - m^2, \quad \delta\lambda = Z^2\lambda_0 - \lambda . \tag{23}$$

The coefficients in the last line of Eq. (21) are called *counterterms.*

Using this rearranged Lagrangian, let us go back and compute various n-point functions. The strategy will be to compute with the terms in the first line of Eq. (21), and then add the counterterms as small corrections that begin at one-loop order (so that at tree level, $Z = 1$, $\delta m^2 = 0 = \delta\lambda$). The propagator of the theory is now $i/(p^2 - m^2 + i\epsilon)$ and the vertex is $-i\lambda$. We also have counterterm corrections in the form of a 2-point interaction $i(\delta Z p^2 - \delta m^2)$ and a 4-point interaction $-i\,\delta\lambda$.

With these interactions and our new interpretation of the theory, the renormalization process involves two steps. The first step is to *regulate* the would-be divergent integrals to make them finite. There are many ways to do this, but for now we will simply transform to Euclidean space $k^0 \to -ik_E^0$ and impose an upper cutoff on the Euclidean magnitude of

$$q_E^2 = (q_E^0)^2 + \vec{q}^{\,2} \leq \Lambda^2 . \tag{24}$$

The one-loop correction to the 2-point function becomes

$$\begin{aligned}
\Delta\Gamma^{(2)}(p) &= \lambda \int^{\Lambda} \frac{d^4q_E}{(2\pi)^4}\frac{1}{q_E^2 + m^2} \; - \; p^2\delta Z \; + \; \delta m^2 && (25)\\
&= p^2\left[\lambda A_1 \ln\left(\frac{\Lambda^2}{a_p p^2 + a_m m^2}\right) + \lambda A_2 - \delta Z\right] \\
&\quad - \left[\lambda B_0\Lambda^2 + \lambda B_1 m^2 \ln\left(\frac{\Lambda^2}{b_p p^2 + b_m m^2}\right) + \lambda B_2 m^2 - \delta m^2\right] ,
\end{aligned}$$

where the coefficients A_i, B_i, and a_i are finite and dimensionless. Similarly, the one-loop correction to the 4-point function takes the form

$$\begin{aligned}
\Delta\Gamma^{(4)}(s,t,u) &= \lambda^2 \int^{\Lambda} \frac{d^4q_E}{(2\pi)^4}\left(\frac{1}{q_E^2 + m^2}\right)^2 \; + \; \delta\lambda && (26)\\
&= \lambda^2\left[C_1 \ln\left(\frac{\Lambda^2}{f(s,t,u)}\right) + C_2\right] + \delta\lambda , && (27)
\end{aligned}$$

where C_1 and C_2 are finite and dimensionless, and s, t, u are the Madelstam variables. At one-loop, all the other 1PI connected n-point functions are finite.

The second step is the renormalization part itself. This amounts to fixing the counterterms by imposing *renormalization conditions* that relate the parameters m^2 and λ to experimental observables. Here, we need three renormalization conditions to fix the three counterterms, and these will require two experimental inputs.

Starting with the 2-point function, recall that the pole of the resummed propagator is the physical mass of the particle corresponding to the field ϕ:

$$(\text{Prop}) \;=\; \frac{iR}{p^2 - m_{phys}^2} + (\text{non-singular}) \;, \tag{28}$$

where R is the residue of the pole. Given this fact, a popular choice of renormalization conditions to fix is to identify m^2 with the measured particle mass, and to demand that the residue of the pole of the propagator is unity. Using our 1-loop result, the resummed propagator is

$$(\text{Prop}) \;=\; \frac{i}{p^2 - m^2 - \Delta\Gamma^{(2)}(p^2)} \;. \tag{29}$$

Applying the two renormalization conditions by expanding around $p^2 = m^2 = m_{phys}^2$,

$$0 \;=\; \Delta\Gamma^{(2)}\Big|_{p^2=m^2} \;, \tag{30}$$

$$0 \;=\; \frac{d\Delta\Gamma^{(2)}}{dp^2}\bigg|_{p^2=m^2} \;. \tag{31}$$

This gives two equations in two unknowns that allow us to solve for δZ and δm^2.

We also need to deal with $\delta\lambda$. A reasonable choice is to set the higher-order corrections to the two-to-two scattering amplitude to zero at the fixed momentum point $s = 4m^2$ and $t = u = 0$. This corresponds to

$$0 = \Delta\Gamma^{(4)}(s\!=\!4m^2, 0, 0) \;, \tag{32}$$

fixing $\delta\lambda$ and relating λ directly to an observable cross-section.

Together, we have used the three renormalization conditions of Eqs. (30,31,32) to fix the parameters in the Lagrangian and to determine the counterterms to one-loop accuracy. Including these counterterms, any other process computed to one-loop will be finite and provide an unambiguous prediction of the theory. For example, we can now predict two-to-two scattering cross sections for any other momentum values, as well as two-to-four and beyond. All of the dependence of the theory on unknown UV physics has been absorbed into the finite parameters λ and m^2, which we have fixed in terms of observables. This is the cost of renormalization: we can't predict observables starting only from the original *bare* parameters m_0^2 and λ_0^2 in Eq. (18). Instead, we are only able to predict observables in terms of a finite set of basis observables. Note as well that the renormalization conditions

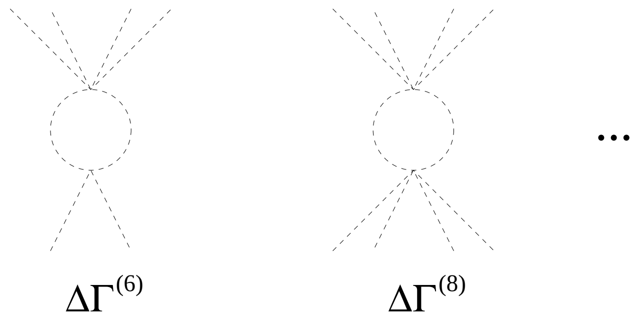

Figure 2: Corrections to $(n \geq 6)$-point functions when the $\zeta\phi^6/M^2$ operator is added.

we chose are not unique – other choices are also possible, and they would lead to a different relationship between the renormalized parameters m^2 and λ and the physical inputs.

The $\lambda\phi^4$ theory considered here is said to be *renormalizable*. This means that the renormalization picture can be extended to higher loop orders using the same finite set of counterterm interactions and renormalization conditions. That one can do this is related to the fact that only a finite number of n-point functions are UV-divergent at one-loop. A rough argument for renormalizability can be made based on dimensional analysis. Consider what a counterterm for an operator of the form ϕ^6 would look like. It would correspond to a divergence in the 6-point function and would have to have a mass dimension of minus two. To make this up, the only dimensionful quantities we have to work with are m^2, p^2, and Λ. However, we expect to have reasonably smooth limits as $m^2 \to 0$ or $p^2 \to 0$, so they should only appear as positive powers. This leaves Λ as the only quantity that can appear in the denominator, but negative powers of Λ do not correspond to UV divergences (as $\Lambda \to \infty$). Thus, we only need counterterms corresponding to the 2- and 4-point (and 0-point) functions. Theories with couplings that have exclusively non-negative mass dimensions are typically renormalizable.

In a renormalizable theory, a finite number of experimental inputs are needed to fix the parameters of the theory. For a non-renormalizable theory, an infinite number of experimental inputs are needed. Even so, non-renormalizable theories still be useful and predictive provided we only use them at sufficiently low energies and compute to a finite accuracy. To illustrate this, let us extend the the scalar theory of Eq. (18) with a *higher-dimensional interaction*:

$$\mathscr{L} \;\to\; \mathscr{L} - \frac{\zeta}{M^2}\,\phi^6\,, \tag{33}$$

where ζ is dimensionless and M has dimensions of mass. With this term, we now have a dimensionful quantity that can appear in denominators, and this allows the possibility of Λ appearing in numerators (or logarithms) of more n-point functions. For example, the first diagram in Fig. 2 goes like

$$\Delta\Gamma^{(6)} \;\sim\; \frac{\lambda\zeta}{M^2}\int\frac{d^4q}{(2\pi)^4}\left(\frac{1}{q^2-m^2+i\epsilon}\right)^2 \;\sim\; \frac{\lambda\zeta}{M^2}\,\ln\Lambda \tag{34}$$

This is a divergent correction to the 6-point function that needs a new counterterm of the form $\delta\zeta\,\phi^6/M^2$ to cancel off. The ϕ^6 interaction also produces new divergences in the 8-point function:

$$\Delta\Gamma^{(8)} \sim \frac{\zeta^2}{M^4}\ln\Lambda\ . \tag{35}$$

Going beyond one-loop order, the single ϕ^6 interaction of Eq. (33) produces an infinite number of new divergences. Each of them requires a counterterm and an experimental observable to fix their value. The theory is no longer renormalizable.

Things look bad at this point, but once again there is a way out of the mess provided we stick to energies much lower than the new dimensionful scale M. Consider the contribution of the ϕ^6 operator of Eq. (33) to a $2 \to 4$ scattering process with a characteristic momentum scale p^2. By dimensional analysis, the contribution to the cross must scale like

$$\Delta\sigma \sim \zeta^2\left(\frac{p}{M}\right)^4\frac{1}{p^2}\ . \tag{36}$$

Contributions from operators of higher dimension contain even more powers of (p^2/M^2). In comparison, the contribution from the renormalizable $\lambda\phi^4$ operator goes like

$$\Delta\sigma \sim \lambda^2\frac{1}{p^2}\ . \tag{37}$$

The key point to notice here is that if we focus exclusively on processes at low energies $p^2 \ll M^2$ and we are only concerned with predictions of finite accuracy, only a finite number of the higher-dimensional operators need to be considered. This means that we only need a finite number of experimental inputs, and we can make testable predictions with a well-defined theoretical uncertainty. On the other hand, the non-renormalizable theory becomes less and less useful as the characteristic energy scale p^2 approaches M^2 since more and more operators need to be included. When we discuss effective field theories below, we will argue that the dimensionful scale M can be identified with a mass scale of new physics.

2.3 Symmetries

Symmetries play a central role in all of physics, and they are particularly useful in quantum field theories. There are two specific results that you should already know from your QFT course. The first is *Noether's Theorem*, which states that for every continuous symmetry of the theory there is a corresponding conserved current. The second is *Goldstone's theorem*, that implies that for every sponteneously broken continuous symmetry of the system there exists a corresponding massless *Nambu-Goldstone Boson* (NGB) particle. We will not review these results here. Instead, we will describe how symmetries can be used to organize and guide our estimates and calculations, and illustrate some features with specific examples.

At the classical level, a symmetry is a transformation under which the action is invariant. This idea can be extended easily to quantum theories defined with a path integral provided the path integral measure is also invariant under the transformation. A transformation is said to be an *anomalous symmetry* if it leaves the action invariant but induces a non-trivial variation in the path integral measure. In this case, the anomalous symmetry is a symmetry in the classical limit of the theory but is not a symmetry of the full quantum system. Conversely, it is possible to have a transformation under which the variations of the action and the path integral measure both transform such they cancel each other out. In this case the transformation is not a symmetry of the corresponding classical theory but is a symmetry of the full quantum theory.

As a first explicit example, consider the real scalar theory with the Lagrangian of Eq. (18). The action is clearly invariant under $\phi \to -\phi$, and this implies that any n-point function (or amplitude) with an odd number of ϕ particles must vanish. Suppose we now extend this theory with a new interaction:

$$\mathscr{L} \;\to\; \mathscr{L} - \frac{A}{3!}\phi^3 + (c.t.) \;, \tag{38}$$

where $(c.t.)$ refers to any counterterms that are needed. The coupling A is a constant with dimensions of mass, and it ruins the invariance under $\phi \to -\phi$. Note, however, that the symmetry would be restored if we also had $A \to -A$. Of course, A is a fixed constant that does not transform, but pretending that it does can be extremely useful. Here, it implies that any n-point function with n odd must be proportional to (an odd number of) A. For example, at one-loop we find a correction to the 3-point vertex,

$$\Delta\Gamma^{(3)} \;\sim\; \lambda\, A\,(\ln\Lambda + \text{finite}) + \delta A \;. \tag{39}$$

The result is proportional to A, as expected. An important implication of this is that if we start with $A = 0$, quantum corrections will not generate a non-zero value. The new coupling will also correct other parameters in the Lagrangian. Its one-loop contribution to the 2-point function goes like

$$\Delta\Gamma^{(2)} \;\sim\; A^2\,(\ln\Lambda + \text{finite}) - \delta m^2 \;. \tag{40}$$

This is a correction to the mass m^2, and it respects the restored symmetry with $A \to -A$.

For a second example, let us expand our scalar theory with a Dirac fermion ψ,

$$\mathscr{L} \to \mathscr{L} + \overline{\psi} i\gamma^\mu\partial_\mu\psi - y\,\phi\overline{\psi}\psi \;. \tag{41}$$

The extended theory is symmetric under

$$\phi \to -\phi, \quad \psi \to -i\gamma^5\psi, \quad \overline{\psi} \to \overline{\psi} i\gamma^5 \;. \tag{42}$$

Correspondingly, we have $\overline{\psi}\gamma^\mu\psi \to \overline{\psi}\gamma^\mu\psi$ and $\overline{\psi}\psi \to -\overline{\psi}\psi$. This implies that a bare mass term of the form $m\overline{\psi}\psi$ is forbidden by the symmetry, as is the cubic scalar of Eq. (38). We can extend this theory further by including an explicit fermion mass term,

$$\mathscr{L} \;\to\; \mathscr{L} - M\,\overline{\psi}\psi \;. \tag{43}$$

The symmetry discussed above is now broken explicitly, but it would be restored if the field transformations were accompanied by $M \to -M$ as well. At one-loop, there is a correction to the fermion mass

$$\Delta\Gamma^{(\bar{\psi}\psi)} \sim M\,(\ln\Lambda + \text{finite}) - \delta M \ . \tag{44}$$

This is consistent with the restored symmetry for $M \to -M$. There is also a new divergence corresponding to a ϕ^3 interaction,

$$\Delta\Gamma^{(\phi^3)} \sim M\,(\ln\Lambda + \text{finite}) \ . \tag{45}$$

We need a ϕ^3 counterterm to cancel this would-be divergence, and this implies that we should have included such a term in the Lagrangian of Eq. (41). This is an example of quantum corrections "generating" a new type of interaction. In general, all possible (renormalizable) interactions that are consistent with the symmetries of the theory should be included in the Lagrangian. Note as well that this correction is consistent with the restoration of the symmetry under $A \to -A$.

For our third and final example, consider a theory of a complex scalar Φ with the Lagrangian

$$\mathscr{L} = |\partial\Phi|^2 - V(\Phi) \ , \tag{46}$$

with $V(\Phi)$ given by

$$V(\Phi) = -\mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 \ . \tag{47}$$

This theory has a continuous symmetry under $\Phi \to e^{i\alpha}\Phi$ for any constant α. Minimizing the potential, we find a degenerate circle of minima that can be parametrized by

$$\Phi|_{min} = v\,e^{i\beta} \ , \tag{48}$$

where $v = \sqrt{\mu^2/\lambda}$ and $\beta \in [0, 2\pi)$. We must choose a specific vacuum around which to expand the theory, and this will spontaneously break the symmetry. Choosing $\beta = 0$, it is convenient to rewrite the complex scalar as

$$\Phi(x) = (v + r(x)/\sqrt{2})e^{ia(x)/\sqrt{2}v} \ . \tag{49}$$

In this form, $r = a = 0$ in the ground state and we can identify these fields with small fluctuations around the vacuum. The original Lagrangian rewritten in terms of these new variables becomes

$$\mathscr{L} = \frac{1}{2}(\partial r)^2 + \frac{1}{2}(1 + r/\sqrt{2}v)^2(\partial a)^2 - \frac{1}{2}(2\lambda v^2)r^2 - \frac{\sqrt{2}\lambda}{2}vr^3 - \frac{\lambda}{8}r^4 \ . \tag{50}$$

This form yields a mass for $r(x)$ but none for $a(x)$. The theory still also has a symmetry under phase transformations, but it is now realized non-linearly in the form

$$r(x) \to r(x), \qquad a(x) \to a(x) + \sqrt{2}v\,\alpha \ . \tag{51}$$

This shift symmetry gives a second way to understand the masslessness of $a(x)$, since a mass term would be inconsistent with it. The field $a(x)$ is called a Nambu-Goldstone boson (NGB).

Let us now deform this theory with a small symmetry-breaking term,

$$V(\Phi) \to V(\Phi) - m^2(\Phi^2 + \Phi^{*2}) \ . \tag{52}$$

The new term is most negative when Φ is real, and leads to a unique minimum at

$$\Phi|_{min} = v' \ , \tag{53}$$

with $v' = \sqrt{(\mu^2 + m^2)/\lambda}$. Expanding about this minimum using the same expansion as Eq. (49) but with $v \to v'$, we find

$$V(\Phi) = \frac{1}{2}(-\mu^2 + 3\lambda v'^2)r^2 - \frac{\sqrt{2}\lambda}{2}v' r^3 - \frac{\lambda}{8}r^4 - m^2(v' + r/\sqrt{2})^2 \cos\left(\sqrt{2}a/v'\right) \ . \tag{54}$$

The last term in this expression is complicated, but it can be understood by expanding the cosine using $\cos x = 1 - x^2/2 + \ldots$. Doing this, the leading term corrects the mass of r to give $m_r^2 = -(\mu^2 + m^2) + 3\lambda v'^2 = 2\lambda v'^2$ as well as a mass for a,

$$m_a^2 = 2m^2 \ . \tag{55}$$

Note that the mass of the $a(x)$ field is proportional to the term that explicitly breaks the $U(1)$ symmetry. This should be expected since we know that $a(x)$ becomes a massless NGB in the limit of $m^2 \to 0$. Furthermore, when $m^2 \ll \mu^2 = \lambda v'^2$, the amount of $U(1)$ breaking is parametrically small and the $a(x)$ field is an approximate NGB in the sense that $m_a^2 \ll m_r^2$.

2.4 Effective Field Theories

An *effective field theory* (EFT) is a field theory that describes the low-energy dynamics of a more complicated theory in terms of only the light degrees of freedom [28]. By their construction, EFTs are usually only useful at energies well below a UV cutoff. Despite this limitation, EFTs are also very often the most convenient way to calculate experimental observables at low energies. The modern view of the SM is that it is the EFT limit of a more complicated theory.

To illustrate how EFTs work, let us begin with a simple and familiar example:

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \overline{\psi}i\gamma^\mu\partial_\mu\psi - y\,\phi\overline{\psi}\psi \ . \tag{56}$$

This theory is renormalizable, and it has a symmetry under $\phi \to -\phi$ and $\psi \to i\gamma^5\psi$ that forbids odd terms in ϕ and a mass for ψ. Suppose we are interested in studying the theory at energies that are far below the mass m of ϕ. At such energies, it is impossible to produce ϕ particles directly. However, they can still contribute to interactions among ψ particles. We would like to formulate an EFT containing only the massless ψ particles that takes this into account.

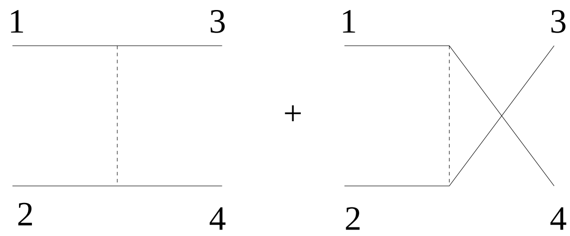

Figure 3: Contributions to $\psi\psi \to \psi\psi$ scattering mediated by the heavy scalar ϕ.

The dominant effect of ϕ on the interactions among ψ particles can be seen in the leading contributions to the scattering process $\psi\psi \to \psi\psi$ shown in Fig. 3. In the diagrams contributing to the amplitude, the massive ϕ particle only appears in the intermediate propagator. The contribution to the amplitude from the first diagram is

$$-i\mathcal{M}_1 = (-iy)^2(\bar{u}_3u_1)(\bar{u}_4u_2)\,\frac{i}{t-m^2} \tag{57}$$

$$= i\frac{g^2}{m^2}\,(\bar{u}_3u_1)\,(\bar{u}_4u_2)\,\left(1+\frac{t}{m^2}+\frac{t^2}{m^4}+\ldots\right)\,, \tag{58}$$

where p_1 and p_2 are the incoming momenta, p_3 and p_4 are the outgoing momenta, $t = (p_1-p_3)^2$, and we have expanded the propagator in powers of t/m^2 in the second line. A similar expression can be derived for the second diagram with an expansion in u/m^2, with $u = (p_1-p_4)^2$.

To formulate an EFT for the low-energy limit of the full theory defined by Eq. (56), we must construct an effective Lagrangian involving only the light ψ fields that reproduces the predictions of the full theory for $E \ll m$. This procedure is called *matching*, and we say that we have *integrated out* the scalar. All the terms in Eq. (58) can be reproduced with operators involving only ψ fields, although an infinite number of operators is needed to do so. Fortunately, we only need to reproduce a finite number of them for $|t| \ll m^2$ provided we only want to compute to a finite accuracy. The dominant first term in Eq. (58) can be reproduced with the effective operator

$$-\mathscr{L}_{EFT} \supset -\frac{y^2}{2m^2}\,(\overline{\psi}\psi)(\overline{\psi}\psi)\;. \tag{59}$$

The other terms in Eq. (58) can be generated by including similar four-fermion operators with derivatives acting on the fields and more powers of m^2 in the denominator.

An important feature of the EFT derived above is that it is non-renormalizable, even though the underlying full theory (with both ψ and ϕ) is renormalizable. Recall that in our discussion of non-renormalizable theories we argued that they can be used to make testable predictions provided they are only applied at energies that are much smaller than the mass scale that suppresses the higher-dimensional operators. The specific EFT we have derived here gives another perspective on this. The theory works very well for $E^2 \ll m^2$, but it is obvious that as E^2 approaches m^2 the EFT breaks down because the full dynamics of the heavy ϕ particle is not being taken into account.

In this example we were lucky enough to know the full, weakly-interacting (by assumption) high-energy theory. Things are not always so simple in practice. Often, we do not know the high-energy theory because we are only able to probe the light degrees of freedom experimentally. In other cases we do know the high-energy theory, but it is strongly coupled near the matching scale. A well-known example of this is QCD. At high energies, the theory can be described by quark and gluon fields interacting perturbatively through an $SU(3)_c$ gauge force. Going to lower energies, the QCD gauge coupling grows very strong near $\Lambda_{QCD} \sim 1$ GeV. Below this, $E \ll \Lambda_{QCD}$, the most useful description of the theory is in terms of scalar pion fields interacting perturbatively through higher-dimensional operators suppressed by powers of Λ_{QCD}. This is a more extreme version of matching, in which the low- and high-energy degrees of freedom are different.

These general features of EFTs have shaped the modern interpretation of the SM as the low-energy limit of a more complicated theory that describes our Universe. The SM also happens to be renormalizable, which is both good an bad. The good part of renormalizability is that it allows us to extrapolate the SM up to (nearly) arbitrarily high energies. The bad part is that this does not give us any clue as to where (in energy) new physics might be. In general, we expect heavy new physics to induce deviations from the renormalizable SM in the form of higher-dimensional (non-renormalizable) operators built from SM fields. Following the scaling arguments for operators discussed above, the most important operators at low energies are those with the lowest dimensions. Therefore the very low-energy limit of a non-renormalizable EFT is a renormalizable theory up to small corrections, and we should not be surprised that the SM as we have observed it appears to be renormalizable.

The interpretation of the renormalizable SM as the low-energy limit of an EFT also suggests that hints of new physics might be found in deviations from the SM induced by higher-dimensional operators. One potential example of this are the neutrino masses. These are forbidden in the SM at the renormalizable level, but they can be induced by an operator of the form

$$-\mathscr{L} \supset \frac{1}{M_N^2}(HL)^2 \ . \tag{60}$$

The observed neutrino mixings suggest $M_N \sim 10^{13}$ GeV. Another higher-dimensional operator that has been searched for extensively is

$$-\mathscr{L} \supset \frac{1}{M^2}\,QQQL \ . \tag{61}$$

This operator can induce proton decays such as $p \to \pi^0\,\bar{e}^+$. Despite extensive searches, proton decay has never been observed and a lifetime of $\tau_p \gtrsim 10^{32}\,yrs$ has been set, with the specific limit depending on the decay mode [19]. This translates into $M \gtrsim 10^{16}$ GeV.

2.5 EFTs and Naturalness

Naturalness is the idea that the parameters in a theory should have the values we expect for them. This is clearly a fuzzy notion, but it can be made precise in certain cases. We will illustrate the idea with a few examples.

Figure 4: Corrections to the scalar (left) and fermion (right) masses at one-loop order.

Consider again the general scalar and fermion theory defined in Eq. (10), but now take $M^2 \gg m^2$ so that the fermion is much heavier than the scalar. In this limit, let us compute the correction to the scalar mass squared due to the fermion. The leading contribution is shown in the first diagram in Fig. 4 and it gives

$$\Delta\Gamma^{(\phi^2)}(p) \;\sim\; (\text{divergent}) + \frac{y^2}{(4\pi)^2}M^2 \ln\left(\frac{p^2}{M^2}\right) + \ldots + (c.t.) \tag{62}$$

The key feature here is the finite term proportional to M^2. This is a correction to the squared mass m^2 of the scalar, and it is on the order of the fermion mass squared times a loop factor $(y^2/(4\pi)^2)$. We can always choose the counterterms to cancel off the large correction, but this result suggests that the *natural* size of the scalar mass m^2 is at least as large as M^2 times a loop factor. Arranging for m^2 to be much smaller than this would appear to require a theoretical fine-tuning.

It is also instructive to think about this correction from a low-energy EFT perspective. At low energies $E \ll M$ with $m^2 \ll M^2$, only the scalar degree of freedom is seen directly and we can formulate an EFT containing the scalar alone. To do so, we need to match the effective theory to the full theory at $p^2 \sim M^2$. One of the corrections we should account for is the fermion effect on the scalar mass parameter. This is accommodated by using a different scalar mass in the bare EFT Lagrangian relative to the full theory to make up for the absence of the fermion loop in the EFT. The difference is precisely the finite term in Eq. (62). We call this the *threshold correction* to the scalar mass from integrating out the fermion. Naturalness suggests that the scalar mass-squared parameter in the EFT should not be much smaller than a loop factor times the threshold M^2.

Suppose we take $m^2 \gg M^2$ instead, as we did earlier. What is the correction to the fermion mass from the scalar? The leading effect comes from the second diagram of Eq. (4), and gives

$$\Delta\Gamma^{(\bar{\psi}\psi)}(p) \;\sim\; (\text{divergent}) + \frac{y^2}{(4\pi)^2}M \ln\left(\frac{p^2}{m^2}\right) + \ldots + (c.t.) \tag{63}$$

The situation here is much different from the scalar in that the finite correction to the fermion mass is proportional to the fermion mass itself. This follows from the fact that the theory has a symmetry in the limit $M \to 0$ (or equivalently, $M \to -M$). An important implication of this result is that it is natural for the fermion mass to be arbitrarily small.

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