PHYS 528 Lecture Notes #5

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1 Fun with Fermions

We have been working with scalar, fermion, and vector fields. They correspond to particles with definite spins. They also form representations of the Lorentz group, which is generated by the set of boosts and rotations on spacetime.[1] Under this group, we have

$$\begin{aligned} x^\mu &\to \Lambda^\mu{}_\nu x^\nu \equiv x' \\ \phi(x) &\to \phi(x') \\ \psi_a(x) &\to U_a{}^b(\Lambda)\psi_b(x') \\ A^\mu &\to \Lambda^\mu{}_\nu A^\nu \end{aligned} \tag{1}$$

The fermions we have been using so far have had four components. It turns out that 4-component fermions are a reducible representation of the Lorentz group. They are built up of two irreducible components: left- and right-handed 2-component fermions. These two 2-component fermion irreps are the only "s" $= 1/2$ irreps of the Lorentz group. In the massless case, they correspond to fermions with spins anti-aligned and aligned with the direction of motion. In some cases, it is easier to work with 4-component fermions, but in many others 2-component fermions are the more sensible option. We will go over how to handle 2-component fermions in this section.

In the chiral basis, a 4-component fermion can be decomposed as

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = P_L\Psi + P_R\Psi \equiv \Psi_L + \Psi_R, \tag{2}$$

where

$$P_{L,R} = (1 \mp \gamma^5)/2, \quad \Psi_L = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}. \tag{3}$$

Applying this to the usual fermion bilinears we deal with,

$$\bar{\Psi}\gamma^\mu\Psi = \bar{\Psi}_L\gamma^\mu\Psi_L + \bar{\Psi}_R\gamma^\mu\Psi_R \tag{4}$$

as well as

$$\bar{\Psi}\Psi = \bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L. \tag{5}$$

We see from this that the kinetic terms keep the L and R components separate while the mass term we have been using mixes them.

[1] In four spacetime dimensions there three distinct rotations and three distinct boosts, so this group has dimension $d(G) = 6$.

It turns out to be very useful to develop a specific notation and procedure to handle 2-component fermions [1, 2, 3]. Let's start with the so-called left-handed irrep. Since these are two-component objects (like duh), we can write

$$\chi = \chi_\alpha, \quad \alpha = 1, 2. \tag{6}$$

Under a Lorentz transformation ($x \to x' = \Lambda x$),

$$\chi_\alpha \to \chi'_\alpha = M_\alpha{}^\beta(\Lambda)\, \chi_\beta, \tag{7}$$

where M_α^β is an $SL(2, \mathbb{C})$ matrix[2]

In QFT, we typically start with a classical field theory Lagrangian and use it to develop the quantum theory. Fermions, even at the classical level, are anti-commuting. That is given two fermion components χ_α and ξ_β we have

$$\chi_\alpha \xi_\beta = -\xi_\beta \chi_\alpha. \tag{8}$$

Such anti-commuting objects are sometimes called Grassmann numbers.

Given any two 2-component fermions, we can form a Lorentz-invariant object by contracting anti-symmetrically:

$$\epsilon^{\alpha\beta} \chi_\beta(x) \xi_\alpha(x) \to \epsilon^{\alpha\beta} \chi_\beta(x') \xi_\alpha(x'), \tag{9}$$

where

$$\epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma^2. \tag{10}$$

For this reason, it makes sense to define 2-component fermions with a raised index

$$\chi^\alpha \equiv \epsilon^{\alpha\beta} \chi_\beta. \tag{11}$$

With this definition we have

$$\chi\xi \equiv \chi^\alpha \xi_\alpha = \xi\chi \qquad \text{(Lorentz Invariant)} \tag{12}$$

It also makes sense to define a lowering operation by the inverse of $\epsilon^{\alpha\beta}$:

$$\chi_\alpha = \epsilon_{\alpha\beta} \chi^\beta \tag{13}$$

with

$$\epsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma^2. \tag{14}$$

Note the sign flip here which is chosen to give $\epsilon^{\alpha\lambda}\epsilon_{\lambda\beta} = \delta^\alpha{}_\beta$ and $\epsilon_{\alpha\lambda}\epsilon^{\lambda\beta} = \delta_\alpha{}^\beta$.

[2] $SL(2, \mathbb{C})$ is the group of 2×2 complex matrices with unit determinant.

The second 2-component fermion irrep is said to be right-handed (RH). It turns out to be a complex conjugate rep of the let-handed (LH) irrep. For this reason, given any LH fermion we can form a RH fermion by

$$\bar{\chi}_{\dot{\alpha}} \equiv (\chi_\alpha)^\dagger. \tag{15}$$

The bar on the RH fermion is part of its name, and does not imply any sort of operation as in the 4-component case. The dot on the RH index of $\dot{\alpha}$ is to distinguish it from the undotted LH index α. Similarly, given any RH fermion we can form a LH fermion by

$$\xi^\beta \equiv (\bar{\xi}^{\dot{\beta}})^\dagger. \tag{16}$$

Note that the index raising and lowering operation commutes with conjugation.

With RH fermions we can form the Lorentz invariant quantity

$$\bar{\chi}\bar{\xi} \equiv \bar{\chi}^{\dot{\alpha}}\bar{\xi}_{\dot{\alpha}} = \bar{\xi}\bar{\chi}. \tag{17}$$

The up-down arrangement of indices is chosen such that

$$(\bar{\chi}\bar{\xi})^\dagger \equiv (\bar{\chi}^{\dot{\alpha}}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\xi}^{\dot{\beta}})^\dagger = \epsilon_{\alpha\beta}(\bar{\xi}^{\dot{\beta}})^\dagger(\bar{\chi}^{\dot{\alpha}})^\dagger = +\xi\chi. \tag{18}$$

Note that conjugation reverses the order of the fermions.

A single 4-component fermion is built up from a LH and a RH 2-component fermions. In general these are different 2-component fermios, and we have

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}, \tag{19}$$

so that

$$\Psi_L = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix}, \quad \text{and} \quad \Psi_R = \begin{pmatrix} 0 \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}. \tag{20}$$

We also have for the conjugate (in the chiral rep)

$$\bar{\Psi} = \Psi^\dagger\gamma^0 = (\xi^\alpha, \, \bar{\chi}_{\dot{\alpha}}) \,. \tag{21}$$

It follows that

$$\bar{\Psi}\Psi = \xi\chi + \bar{\xi}\bar{\chi}. \tag{22}$$

Again we see that this term mixes the two 2-component fermions.

To form a kinetic term we need the γ matrices. In the chiral representation we have

$$\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot{\alpha}} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & 0 \end{pmatrix}, \tag{23}$$

where

$$\sigma^\mu = (\mathbb{I},\, \vec{\sigma}), \qquad \bar{\sigma}^\mu = (\mathbb{I},\, -\vec{\sigma}). \tag{24}$$

This leads to

$$\begin{aligned} \bar{\Psi}\gamma^\mu\Psi &= (\xi^\alpha,\, \bar{\chi}_{\dot{\alpha}}) \left(\begin{array}{cc} 0 & (\sigma^\mu)_{\alpha\dot{\alpha}} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & 0 \end{array} \right) \left(\begin{array}{c} \chi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{array} \right) \\ &= \bar{\chi}_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\chi_\alpha + \xi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}} \\ &\equiv \bar{\chi}\bar{\sigma}^\mu\chi + \xi\sigma^\mu\bar{\xi}. \end{aligned} \tag{25}$$

As before, this term keeps separate the two 2-component pieces. Both transform as vectors under the Lorentz group. It also looks like χ and ξ have different kinetic terms. However, one can show that for any two spinors

$$\chi\sigma^\mu\bar{\xi} = -\bar{\xi}\bar{\sigma}^\mu\chi. \tag{26}$$

After integrating by parts, the kinetic terms for both χ and ξ can therefore be put in the same form. Along the way, we see that it makes sense to define the canonical kinetic term for a single 2-component fermion to be

$$\mathscr{L} \supset \bar{\chi}i\bar{\sigma}^\mu\partial_\mu\chi = \chi i\sigma^\mu\partial_\mu\bar{\chi}. \tag{27}$$

Note that both χ and $\bar{\chi}$ are needed to obtain a real-valued kinetic term.

So far we have started with 4-component fermions and have expressed them as 2-component pieces. However, in general we can simply start with some number of 2-component fermions χ_i with the free Lagrangian

$$\mathscr{L} = \sum_i \bar{\chi}_i i\bar{\sigma}^\mu\partial_\mu\chi_i - \frac{1}{2}\sum_{ij}\left(m_{ij}\chi_i\chi_j + m^*_{ij}\bar{\chi}_i\bar{\chi}_j\right). \tag{28}$$

The mass matrix here is complex and symmetric. It is always possible to rewrite this Lagrangian in terms of 4-component objects; sometimes this is useful and sometimes it isn't.

A 4-component fermion is said to be *Dirac* if the spinors making it up are different in that they aren't complex conjugates of each other. That is

$$\Psi_D = \left(\begin{array}{c} \chi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{array} \right), \tag{29}$$

with $(\bar{\xi}_{\dot{\alpha}})^\dagger \neq \chi_\alpha$. A Dirac mass term has the form $\bar{\Psi}\Psi = \chi\xi + \bar{\chi}\bar{\xi}$ and mixes these two distinct components.

A 4-component fermion is said to be *Majorana* if the spinors making it up are the same:

$$\Psi_M = \left(\begin{array}{c} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{array} \right). \tag{30}$$

Equivalently, $\Psi = \Psi^c$ where the conjugate spinor is defined (for a general 4-component fermion) to be

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix} \quad \leftrightarrow \quad \Psi^c = \begin{pmatrix} \xi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{31}$$

Note that we can also write

$$\Psi^c = C(\bar{\Psi})^t, \quad \text{with} \quad C = -i\gamma^2\gamma^0 = \begin{pmatrix} \epsilon_{\beta\alpha} & 0 \\ 0 & \epsilon^{\dot{\beta}\dot{\alpha}} \end{pmatrix}. \tag{32}$$

A Majorana mass term has the form $\overline{(\Psi^c)}\Psi = \chi\chi + \bar{\xi}\bar{\xi}$, and does not mix different fermion components. From the point of view of 2-component fermions, all these 4-component gymnastics are completely silly. A Majorana mass term is just a contraction of the same 2-component object (*e.g.* $\chi\chi$) while a Dirac mass term is a contraction of two different 2-component objects (*e.g.* $\chi\xi$).

2 Introduction to the Standard Model

We now have all the pieces we need to assemble the Standard Model (SM) [4, 5, 6, 7]. This theory provides an excellent description of the strong, weak, and electromagnetic forces, and the predictions of the theory are in excellent agreement with a very broad range of experimental measurements. Gravity is not described by the SM since this force is exceedingly weak and almost always neglible in particle physics experiments.

The basis of the SM is gauge invariance under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. Of these factors, $SU(3)_c$ corresponds to the strong force, while $SU(2)_L \times U(1)_Y$ combine to produce the weak and electromagnetic forces. Having fixed the underlying gauge group, all we need to do is to specify the matter content and the vacuum structure. The fermionic matter content comes in three identical copies called *families*. Each family consists of the following representations under $SU(3)_c \times SU(2)_L \times U(1)_Y$:

$$\begin{aligned} Q_L &= (\mathbf{3}, \mathbf{2}, 1/6) = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\ u_R &= (\mathbf{3}, \mathbf{1}, 2/3) \\ d_R &= (\mathbf{3}, \mathbf{1}, -1/3) \\ \\ L_L &= (\mathbf{1}, \mathbf{2}, -1/2) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \\ e_R &= (\mathbf{1}, \mathbf{1}, -1) \end{aligned} \tag{33}$$

These are each 2-component fermions that we have written in 4-component notation. Note that these representations do not come in balanced LR and RH pairs, but rather the LH and RH quark and lepton fields have different gauge charges.[3] For Q_L and L_L we have written out

[3]Fermions with this property are sometimes said to be *chiral*.

the $SU(2)_L$ components explicitly. The Q_L, u_R, and d_R fields transform non-trivially under $SU(3)_c$ and are called *quarks*, while the $SU(3)_c$-neutral L_L and e_R fields are called *leptons*. Aach quark also has three colour components which we have not written out explicitly. In addition to three families of fermions, there is also a single Higgs scalar field

$$\Phi = (\mathbf{1}, \mathbf{2}, 1/2) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{34}$$

We will write the gauge fields for the $SU(3)_c \times SU(2)_L \times U(1)_Y$ factors as

$$\begin{aligned} G^a_\mu &\sim (\mathbf{8}, \mathbf{1}, 0) \\ W^p_\mu &\sim (\mathbf{1}, \mathbf{3}, 0) \\ B_\mu &\sim (\mathbf{1}, \mathbf{1}, 0) \end{aligned} \tag{35}$$

Recall that the $\mathbf{8}$ of $SU(3)_c$ is the adjoint, as is the $\mathbf{3}$ of $SU(2)_L$.

Under $SU(3)_c \times SU(2)_L \times U(1)_Y$ transformations, a given field ψ transforms according to

$$\begin{aligned} \psi_{ir} \to \psi'_{ir} &\equiv U^{(3)}_{ij} U^{(2)}_{rs} U^{(1)}\, \psi_{js} & (36) \\ &= (e^{i\alpha^a t^a_{r_c}})_{ij}\, (e^{i\beta^p t^p_{r_L}})_{rs}\, (e^{i\gamma Y} + \ldots)\, \psi_{js}. \\ &= \left[\delta_{ij}\delta_{rs} + i\alpha^a (t^a_{r_c})_{ij}\delta_{rs} + i\delta_{ij}\beta^p (t^p_{r_L})_{rs} + i\delta_{ij}\delta_{rs}\gamma\, Y\right] \psi_{rs}. & (37) \end{aligned}$$

That is, ψ carries $SU(3)_c$ (i and j) and $SU(2)_L$ (r and s) indices, and each of these product subgroups acts relative to these indices independently. The quantities α^a, β^p, and γ are the universal group transformation parameters that apply to all representations. When a field transforms as a singlet under $SU(3)_c$ or $SU(2)_L$, the corresponding representation generators vanish and we don't need to include an index for that group on the field. Thus we have

$$Q_L = (Q_L)_{ir}, \quad u_R = (u_R)_i, \quad d_R = (d_R)_i, \quad L_L = (L_L)_r, \quad e_R = (e_R). \tag{38}$$

Woohoo!

The SM Lagrangian takes the form

$$\mathscr{L} = \mathscr{L}_{gauge} + \mathscr{L}_{Higgs} + \mathscr{L}_{Yukawa}. \tag{39}$$

The gauge piece is completely fixed by gauge invariance:

$$\begin{aligned} \mathscr{L}_{gauge} &= -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{4}(W^p_{\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2 \\ &\quad + \bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R \\ &\quad + \bar{L}_L i\gamma^\mu D_\mu L_L + \bar{e}_R i\gamma^\mu D_\mu e_R, \end{aligned} \tag{40}$$

where each covariant derivative takes the form

$$D_\mu = \partial_\mu + ig_s\, t^a_{r_c} G^a_\mu + ig\, t^p_{r_L} W^p_\mu + ig'\, Y B_\mu, \tag{41}$$

with t^a_{rc} the appropriate $SU(3)_c$ generators for the coresponding rep ($t_{r_c} = 0$ for the trivial rep), $t^p_{r_L}$ the generators for $SU(2)_L$ ($t_{r_L} = 0$ for the trivial rep), and Y is the charge of the field under $U(1)_Y$ and is called *hypercharge*. The Higgs part is

$$\mathscr{L}_{Higgs} = \left| \left(\partial_\mu + ig\,\frac{\sigma^p}{2} W^p_\mu + ig'\frac{1}{2} B_\mu \right) \Phi \right|^2 - \left(-\mu^2 |\Phi|^2 + \frac{\lambda}{2} |\Phi|^2 \right). \tag{42}$$

This potential induces spontaneous symmetry breaking whose consequences we will examine presently. Finally, the third set of terms in the SM Lagrangian corresponds to scalar-fermion *Yukawa* interactions of the form

$$\mathscr{L}_{Yukawa} = -y_u \bar{Q}_L \tilde{\Phi}\, u_R - y_d \bar{Q}_L \Phi\, d_R - y_e \bar{L}_L \Phi\, e_R + (h.c.), \tag{43}$$

where $\tilde{\Phi} \equiv i\sigma^2 \Phi = (\phi^{0*},\, -\phi^{+*})^t$ These interactions are the most general ones we can write (at the renormalizable level) while being consistent with gauge invariance given the charges of Eq. (33). Note that the gauge charges forbid fermion mass terms.

The first step in working out the implications of this Lagrangian is to determine the vacuum structure. The Higgs potential leads to spontaneous symmetry breaking and we can choose a gauge (called the *unitarity gauge*) such that

$$\Phi(x) = \begin{pmatrix} 0 \\ v + h(x)/\sqrt{2} \end{pmatrix}, \tag{44}$$

where $v = \sqrt{\mu^2/\lambda}$. The remaining h field here is called the Higgs boson. This expectation value has important consequences for the rest of the theory. From the Higgs kinetic term we obtain masses for some of the W^p_μ and B^μ gauge bosons. Inserting this form for the Higgs field into Eq. (43) we also obtain masses for the fermions.

Symmetry breaking in the SM has the same form as the $SU(2) \times U(1)$-invariant theories we considered previously. Applying an arbitrary $SU(3)_c \times SU(2)_L \times U(1)_Y$ transformation to the vacuum state chosen above, we see that this vacuum is invariant under $SU(3)_c$ as well as an Abelian subgroup of $SU(2)_L \times U(1)_Y$. The generator of this subgroup is

$$Q \equiv t^3 + Y. \tag{45}$$

We identify this unbroken subgroup with the $U(1)_{em}$ invariance of electromagnetism, so that the unbroken Q generator defined here corresponds to electric charge. Therefore there should exist a massless gauge boson corresponding to the photon.

To verify this we should construct the gauge boson mass matrix generated by the covariant kinetic term for the Higgs field. This leads to

$$|D_\mu \Phi|^2 \to \frac{1}{2}(\partial h)^2 + \frac{1}{2}\frac{v^2}{2}\left[g^2[(W^1_\mu)^2 + (W^2_\mu)^2] + (-gW^3_\mu + g'B_\mu)^2 \right]. \tag{46}$$

From this expression it is clear that two orthogonal linear combinations of W^1_μ and W^2_μ obtain equal masses. It turns out to be convenient to arrange them into the $W^\pm$ vector bosons,

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \left(W^1_\mu \mp i W^2_\mu \right). \tag{47}$$

The reason for this choice is that the two states have charges ± 1 under $U(1)_{em}$. Their equal masses are

$$m_W^2 = \frac{g^2}{2} v^2. \tag{48}$$

For W_μ^3 and B_μ we get a squared mass matrix of

$$M^2 = \frac{v^2}{2} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \tag{49}$$

As expected, this matrix has a zero eigenvalue corresponding to the photon A_μ. The other linear combination of W_μ^3 and B_μ is called the Z^0 vector boson. These mass eigenstates are related to the fields in the original basis by

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{50}$$

where the *weak mixing angle* θ_W is defined by

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \qquad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{51}$$

While the photon is massless, the Z^0 vector boson has mass

$$m_Z^2 = \left(\frac{g^2 + g'^2}{2}\right) v^2. \tag{52}$$

The longitudinal components of the massive $W^\pm$ and Z^0 vectors account for the missing NGBs from the three broken electroweak generators. Since the new mass eigenstate vector fields we have defined above are related to the original gauge eigenstates by orthogonal transformations, the kinetic terms for the mass eigenstate vectors will also be canonical.

Rewriting the gauge eigenstates in terms of mass eigenstates in the electroweak parts of the matter covariant derivatives we find

$$\begin{aligned} D_\mu &\supset igt^p W_\mu^p + ig' Y B_\mu \\ &= ig\left[\frac{1}{\sqrt{2}}(t^1 + it^2) W_\mu^+ + \frac{1}{\sqrt{2}}(t^1 - it^2) W_\mu^-\right] \\ &\qquad + i(gc_W t^3 - s_W g' Y) Z_\mu + i(g s_W t^3 + g' c_W Y) A_\mu \\ &= ig\left[\frac{1}{\sqrt{2}}(t^1 + it^2) W_\mu^+ + \frac{1}{\sqrt{2}}(t^1 - it^2) W_\mu^-\right] + i\bar{g}(t^3 - s_W^2 Q)\, Z_\mu + ie\, Q\, A_\mu\,. \end{aligned} \tag{53}$$

Along the way we have implicitely defined the couplings

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} = g s_W = g' c_W, \qquad \bar{g} = \sqrt{g^2 + g'^2}. \tag{54}$$

While the SM has many individual interaction terms, we see that they are all essentially fixed by the values of g, g', and v from the underlying gauge-invariant theory together with the output of the Higgs mechanism. Measurements of the electroweak sector of the SM find that

$$\begin{aligned} m_W &\simeq 80.4 \text{ GeV}, \quad m_Z \simeq 91.2 \text{ GeV}, \quad v \simeq 174 \text{ GeV}, \\ s_W^2 &\simeq 0.23, \quad g \simeq 0.65, \quad g' \simeq 0.45, \quad e^2/4\pi \simeq 1/137. \end{aligned} \tag{55}$$

Note that not all the values of these measurable masses and couplings are independent in the underlying theory. We will see that this allows for very stringent experimental tests of the electroweak sector of the SM.

The remaining pieces of the SM Lagrangian that we have not yet examined are the Yukawa terms. Rewriting the Higgs scalar doublet in terms the new vacuum-friendly field variables, the Yukawa interactions become

$$\begin{aligned} -\mathscr{L}_{Yukawa} &= y_u \bar{Q}_L \tilde{\Phi}\, u_R + y_d \bar{Q}_L \Phi\, d_R + y_e \bar{L}_L \Phi\, e_R + (h.c.) \qquad (56) \\ &= y_u\,(v + h/\sqrt{2})\,\bar{u}_L u_R + y_d\,(v + h/\sqrt{2})\,\bar{d}_L d_R + y_e\,(v + h/\sqrt{2})\,\bar{e}_L e_R + (h.c.). \end{aligned}$$

This expression consists of Dirac mass terms for the fermions together with fermion-Higgs boson interactions:

$$m_i = y_i\, v. \tag{57}$$

In other words, the mass of each SM fermion is proportional to how strongly it couples to the Higgs field. This would be a great thing to test if only we could find the Higgs in the first place.

References

[1] See Appendix A in:
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