PHYS 526 Homework #11

Due: Nov. 26, 2013

0. Read Ch. 9 of Peskin and Schroeder and `notes-11`.

1. Suppose electrons and muons both interact with a new massive vector boson V^μ of mass M according to

$$-\mathscr{L}_V = g_e V^\mu \,\bar{e}\gamma^\mu e + g_\mu V^\mu \,\bar{\mu}\gamma^\mu \mu \ .$$

Compute the total unpolarized cross section for $e^+e^- \to \mu^+\mu^-$ in the CM frame and write you result in terms of $s = (p_1+p_2)^2$ and M. You may assume that the muon and electron masses are negligible compared to M and s, and that the usual photon contribution to the process is small enough to be ignored.
Hint: consult `hw-10` for how to deal with a massive vector.

2. Gaussians

   a) Compute $\int_0^\infty dx\, x\, e^{-\alpha x^2}$.

   b) Evaluate $\int_{-\infty}^\infty dx\, e^{-\alpha x^2}$ by multiplying by $\int_{-\infty}^\infty dy\, e^{-\alpha y^2}$ and changing variables to polar coordinates (r,ϕ).

   c) Derive a general formule for $\int_0^\infty dx\, x^{2n+1} e^{-\alpha x^2}$ for $n \in \mathbb{Z}^>$ by differentiating the result of a) with respect to α.

   d) Derive a general formula for $\int_{-\infty}^\infty dx\, x^{2n} e^{-\alpha x^2}$ for $n \in \mathbb{Z}^>$ by differentiating the result of b) with respect to α.

3. Complex Scalar Path Integral
Consider the free complex scalar theory

$$\mathscr{L} = |\partial\Phi|^2 - m^2|\Phi|^2 \ .$$

We can quantize the theory using path integrals. The generating functional in this case is

$$Z[J,\bar{J}] = \int [\mathscr{D}\Phi\mathscr{D}\Phi^*] \ \exp\left(iS'[\Phi,\Phi^*] + i\bar{J}\cdot\Phi + i\Phi^*\cdot J\right) \ ,$$

where $S' = S + i\epsilon\int d^4x\, |\Phi|^2$.

   a) Solve for the generating functional by evaluating the functional integral.
   Hint: there are many ways to do this. One of them is to use $\Phi = (\phi_1 + i\phi_2)/\sqrt{2}$ and to note that $[\mathscr{D}\Phi\mathscr{D}\Phi^*] = [\mathscr{D}\phi_1\mathscr{D}\phi_2]$ up to a possible overall constant that cancels out when we compute expectation values.

   b) Use your result from a) to solve for all the two-point functions. Along the way, you should show how to relate the generating functional to time-ordered operator expectation values in the vacuum.