PHYS 526 Homework #7

Due: Oct. 29, 2013

0. Read Chs.35-40 of Srednicki and `notes-06`.

1. Pauli Matrices

   a) Show that $e^{-i\alpha^a\sigma^a/2} = \cos(\alpha/2) - i\sigma^a(\alpha^a/\alpha)\sin(\alpha/2)$, where $\alpha = \sqrt{\alpha^a\alpha^a}$.

   b) Prove the ϵ trick.

   c) Show explicitly that $(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot{\beta}}$

   d) The space of 2×2 complex matrices forms a vector space over $\mathbb{C}$. We can use $\mathbb{I} = \sigma^0$ and the σ^i matrices as a basis for this space.

      i) Show that any matrix M in this space can be expanded in the form

      $$M = \sum_{m=0}^{3} a_m\sigma^m \ ,$$

      for some complex coefficients a_m.
      Hint: how many basis elements do you need?

      ii) Define an inner product on the space by $\langle M|N\rangle = tr(M^\dagger N)$.
      Show that the basis $\{\sigma^m\}$ is orthogonal with respect to it.

      iii) Use this fact to solve for the coefficients a_m in terms of traces.

2. $1/2 \otimes 1/2 = 0 \oplus 1$

   a) Compute $\Lambda^\mu{}_\nu\sigma^\nu$ for $\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu$ infinitesimal. Do the $\mu = 0$ and $\mu = i$ cases separately.

   b) Let $\theta^a = \frac{1}{2}\epsilon^{abc}\omega_{bc}$ and $\beta^a = \omega_{0a}$.
   Compute $M^t(\alpha^a)\epsilon\sigma^\mu\overline{M}(\alpha^a)$ for $\mu = 0$ and compare to part a).

   c) Compute $M^t(\alpha^a)\epsilon\sigma^\mu\overline{M}(\alpha^a)$ for $\mu = i$ and compare to part a).
   Hint: $2\sigma^a\sigma^b = [\sigma^a, \sigma^b] + \{\sigma^a, \sigma^b\}$
   Hint: Use $\epsilon^{abc}\epsilon^{alm} = (\delta^{bl}\delta^{cm} - \delta^{bm}\delta^{cl})$ *to solve for* ω_{ab} *in term of* θ^c.

   d) Use these results to show that $\psi\sigma^\mu\bar{\chi}$ transforms like a 4-vector under Lorentz.

3. Spinor Kinetic Terms

   a) Show that $\psi\sigma^\mu\bar{\chi} = -\bar{\chi}\bar{\sigma}^\mu\psi$. Make sure you show how the indices get moved around.

   b) Use this result to show that $\psi i\sigma^\mu\partial_\mu\bar{\chi} = \bar{\chi}i\bar{\sigma}^\mu\partial_\mu\psi$ up to total derivatives that vanish when integrated over $\int d^4x$.

   c) Prove that the 2-spinor kinetic term written in `notes-06` is real.
   Hint: $a^* = a^\dagger$ *for any complex number.*

4. γ Matrices

a) Show that the trace of an odd number of γ^μ matrices vanishes.
Hint: insert $1 = (\gamma^5)^2$ *and anticommute away.*

b) Show that $tr(\gamma^\mu \gamma^\nu) = 4\eta^{\mu\nu}$.

c) Prove $tr(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})$.
Hint: use the cyclicity of the trace and $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ *to rearrange things until you get back to where you started plus something else.*

d) Compute $\gamma^\mu \gamma^\nu \gamma_\mu$.

5. More Fun with γ Matrices.

a) Calculate $tr\,[(\not{p}_1 + m)(\not{p}_2 + m)]$, where $\not{p} = p_\mu \gamma^\mu$.

b) Find $tr\,[(\not{p}_1 + m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu]$.

c) Evaluate $tr\,[\not{p}_1 \not{p}_2 \not{p}_3 \not{p}_4 P_L]$.

d) Compute $tr\,[\not{p}_1 P_R \not{p}_2 \not{p}_3 P_L \not{p}_4]$.
Hint: think before you compute.