PHYS 526 Homework #5

Due: Oct. 15, 2013

1. Interacting complex scalars.
   Let's proceed to an interacting theory by adding to the Lagrangian the term
   $$\Delta V = \frac{\lambda}{4}|\Phi|^4 \ .$$
   a) Show that this term does not violate charge conservation, and derive the classical expression for the conserved charge.
   b) Recall that $\langle\Phi_1\Phi_2\rangle = 0 = \langle\Phi_1^\dagger\Phi_2^\dagger\rangle$ and $\langle\Phi_1\Phi_2^\dagger\rangle = D_F(x_1-x_2)$. As a result, we can generalize Wick's theorem to this theory, but now with the contractions $\contraction{}{\Phi}{_1}{\Phi}\Phi_1\Phi_2^\dagger = D_F(x_1-x_2)$ and $\contraction{}{\Phi}{_1}{\Phi}\Phi_1\Phi_2 = 0 = \contraction{}{\Phi}{_1^\dagger}{\Phi}\Phi_1^\dagger\Phi_2^\dagger$. Use this together with the master formula to compute $\langle\Omega|T\{\Phi(x_1)\Phi(x_2)\Phi^\dagger(x_3)\Phi^\dagger(x_4)\}|\Omega\rangle$ at order λ^1. Can any of the other 4-point functions be non-zero?
   c) Formulate momentum space Feynman rules to compute n-point functions in this theory in a way that keeps track of charge conservation. To do so, assign arrows to propagator lines that show the direction of charge flow and figure out the arrow structure of the fundamental vertices.

2. Compute $d\sigma/d(\cos\theta)$ and the total cross section for $2\to 2$ scattering in the CM frame for the real scalar theory with $\Delta V = \lambda\phi^4/4!$ to leading non-trivial order in the coupling. Along the way, show explicitly how you evaluate the integral over phase space.

3. Consider a theory with two real scalar fields A and B:
   $$\begin{aligned}\mathscr{L} &= \frac{1}{2}(\partial A)^2 + \frac{1}{2}(\partial B)^2 - \frac{1}{2}m_A^2 A^2 - \frac{1}{2}m_B^2 B^2 \\ &\quad -\frac{g}{2}AB^2 - \frac{\lambda_A}{4!}A^4 - \frac{\lambda_B}{4!}B^4 - \frac{\lambda_{AB}}{7}A^2B^2 \ .\end{aligned}$$
   Within this theory:
   a) Compute the connected part of the momentum-space 3-point function with one A field and two B fields at leading non-trivial order.
   b) Use this result to formulate the Feynman rule for the vertex corresponding to the $g\,AB^2/2$ interaction (*i.e.* what value should you assign to it?).
   c) Compute the connected part of the momentum-space 4-point function with two A fields and two B fields at leading non-trivial order in the couplings.
   d) Use this result to formulate the Feynman rule for the vertex corresponding to the $\lambda_{AB}\,A^2B^2/7$ interaction (*i.e.* what value should you assign to it?).
   e) Show that there is a contribution to $\langle A_1A_2B_3B_4\rangle$ (in position space) from the $gAB^2/2$ coupling at order g^2, and compute the connected part of it.

4. For $m_A > 2m_B$, the decay $A \to 2B$ is possible. Compute the decay rate at leading order in the couplings (in the A rest frame). Show explicitly your evaluation of the phase space integrals.

5. For $m_A > 0$ and $m_B = 0$, compute the cross section for $BB \to AA$ to leading non-trivial order in the couplings in the CM frame. What is the minimal initial energy needed for this occur? How does the cross section behave as this limit is approached from above?